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BINOMIAL FACTORISATIONS.

GIVING

EXTENSIVE CONGRUENCE-TABLES

AND FACTORISATION-TABLES.

BY

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PREFACE.

The present Work in seven volumes is the outcome of the author's labours of the past thirty years.

The Tables were computed with the aid of a Staff of Assistants whose names will be found in Arts. 18b, 27c of the present volume, and again in the later volumes.

Most of the Tables have been printed off for years; but the Work has been greatly delayed by the War and by its aftermath.

The author's acknowledgments are due to Mr. H. J. Woodall, A.R.C.Sc., for help in reading the Proof Sheets of the Introduction, and for numerous suggestions.

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* ϕ means M.A.P.F. of, [see Art. 1].

FACTORISATION-TABLES, &c.

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BINOMIAL FACTORISATIONS.

1. *General Introduction.* This Work,—which is in several volumes—is a study of the arithmetical properties, especially of the factorisation, of certain Binomial and Trinomial forms.

The following general notation and nomenclature are used :—

Binomial n-ic ; $\mathbf{N}_n$, or $\mathbf{N}'_n$, means $(x^n \sim y^n)$, or $(x^n + y^n)$ ...... (1a).

A.P.F. means *Algebraic Prime Factor.*

M.A.P.F. means *Max. A.P.F.*

Binomial n-an ; N_n, or N'_n = M.A.P.F. of $(x^n \sim y^n)$, or $(x^n + y^n)$ .... (1b).

Extensive Tables are given of the Congruence N_n and $\mathrm{N}'_n \equiv 0$ (mod p and p^κ) and of the factorisation of N_n and N'_n (at end of this Introductory Text).

1a. *Abstract of Contents of Volumes.* This Work is projected in about* 7 volumes, dealing with following subjects :—

Vol. i ;	n-ans of degrees	n = 2, 4, 8, 16 ; 3, 6, 12, 24 ; and 4-tic Forms.	
,, ii ;	,, ,,	n = 5, 10, 15, 30.	
,, iii ;	,, ,,	n = 7, 14, 21 ; 9, 18.	
,, v ;	,, ,,	n = 11, 27 ; 13, 26.	
,, iv ;	Supplement to	Vol. i	contain the completion of many of the Congruence-Tables to modulus $p \not> 100{,}000$.
,, vi ;	,,	Vol. ii	
,, vii ;	,,	Vols. iii and v	

1b. *Other general Notation.* All symbols denote integers, (unless otherwise stated).

i, I mean *any integers ;* ϵ, e, E mean *even* numbers ; ω, Ω mean *odd* numbers.

p means an *odd prime ;* $p_1, p_2, p_3, \ldots$ mean *different odd primes.*

F_n means an odd factor of N_n.

$\tau(n)$ means *Totient* of n ;

$$\tau(p) = (p-1),\ \ \tau(p^\kappa) = p^{\kappa-1}.(p-1)\ ;\quad \tau(2) = 1,\ \ \tau(2^\kappa) = 2^{\kappa-1}\ ;$$
$$\tau(p_1p_2p_3\ldots) = \tau(p_1).\tau(p_2).\tau(p_3).\ \ldots,\ \ \tau(2\Pi p) = \tau(\Pi p)\ \ldots\ldots\ (2).$$

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* The Tables—(which are very extensive)—of all these Volumes are already in type, and printed off.

VOL. I.

[The author is indebted to Mr. H. J. Woodall, A.R.C.Sc., for help in reading the proof sheets.]

Chap. I. *Linear and 2-ic Forms.*

2. *Introduction.* This volume deals* with the arithmetical properties,—especially those connected with factorisation,—of Binomials and Trinomials of degrees $n = 2, 4, 8$; $3, 6, 12$; and with allied problems.

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* The matter in the Text of this Volume is to a great extent contained in the author's published Papers following :—

1°. *On Aurifeuillians* in *Proc. Lond. Math. Socy.*, Vol. 29, 1898, pp. 381–438.

2°. *High Primes* $(4\varpi+1)$ *and* $(6\varpi+1)$ *and Factorisations* in *Quarterly Journal of Pure and Applied Mathcs.*, Vol. 35, 1904, pp. 10–21.

3°. Six Papers in *Messenger of Mathcs.:*

(1) High Quartan Factorisations and Primes, Vol. 36, 1907, pp. 145–174.

(2) Diophantine Factorisation of Quartans, Vol. 38, 1909, pp. 81–104.

(3) ,, ,, ,, Vol. 38, 1909, pp. 145–175.

(1) High Sextan Factorisations and Primes, Vol. 39, 1910, pp. 38–63.

(2) ,, ,, ,, Vol. 39, 1910, pp. 97–128.

(3) ,, ,, ,, Vol. 40, 1911, pp. 1–36.

These last six Papers will be referred to (for shortness) as :

High 4-tans, Nos. (1), (2), (3). High Sextans, Nos. (1), (2), (3).

2a. *Nomenclature and Notation of n-ans.* The n-ans dealt with herein are symbolized and named* as follows:—

$$\begin{array}{cccc} \textit{Duans} & \textit{Quartans} & \textit{Octavans} & \&c. \\ N_{ii} = x^2+y^2, & N_{iv} = x^4+y^4, & N_{viii} = x^8+y^8, & \&c. \end{array} \quad \ldots\ldots (3a).$$

As N_{ii}, N_{iv}, N_{viii}, &c., are all *even* when xy is *odd*, their *Halves* are then dealt with under the names

Half-Duan $= \frac{1}{2}N_{ii}$, *Half-Quartan* $= \frac{1}{2}N_{iv}$, *Half-Octavan* $= \frac{1}{2}N_{viii}$, &c.

Also,—

$$\begin{array}{cccc} \textit{Cuban} & \textit{Sextan} & \textit{Duodeciman} & \&c. \\ N_{iii} = \dfrac{x^3 \mp y^3}{x \mp y}, & N_{vi} = \dfrac{x^6+y^6}{x^2+y^2}, & N_{xii} = \dfrac{x^{12}+y^{12}}{x^4+y^4}, & \&c. \end{array} \quad \ldots (3b).$$

The two kinds of Cubans are thus distinguished (when necessary)

$$N_{iii} = \frac{x^3 \sim y^3}{x \sim y}, \quad N'_{iii} = \frac{x^3+y^3}{x+y} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3c).$$

As $N_{iii} = 3I$ when $x \sim y = 3i$, and $N'_{iii} = 3I$ when $x+y = 3i$, their *third parts* are then dealt with under the names

$$\textit{Trito-Cuban} = \tfrac{1}{3}N_{iii} \text{ or } \tfrac{1}{3}N'_{iii} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3d).$$

2b. *Simple Forms.* The above n-ans (N_n) are termed *Simple n-ans* when x or $y = 1$.

2c. *Omission of subscript n.* The subscripts (n) will be omitted—for shortness' sake—when the context sufficiently shows the *degree* (n) of the n-an (N_n) dealt with.

3. *Working Condition.* In order to avoid unnecessary factors in N_n it is *usual* to assume that x, y are *prime to one another*: otherwise, if x, y have a common factor μ, then N_n will be divisible by the factor $\mu^{\tau(n)}$ when $n = \omega$, and by $\mu^{2\tau(n)}$ when $n = \epsilon$.

Similarly, in 2-ic forms such as $F = (t^2 \mp Du^2)$ it is usual to assume that t, u are prime to one another: otherwise, if t, u have a common factor μ, then F will be divisible by the factor μ^2.

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* This nomenclature and symbolism are readily extended to other degrees ($n = 5, 7, 9$, &c.), and will be adopted in succeeding volumes.

4. *Linear Form of* N_n *and* F_n. The linear forms of N_n itself, and of the divisors (F_n) of N_n, are as shown in following scheme

n	N_n	Condition	x	y	N_n	Linear Form of N_n	Linear Form of F_n
2	N_{ii}	x prime to y	ω, ϵ	ϵ, ω	Ω	$4\varpi+1$	$4\varpi+1$
	$\frac{1}{2}\mathrm{N}_{\text{ii}}$		ω	ω	Ω	$4\varpi+1$	
4	N_{iv}		ω, ϵ	ϵ, ω	Ω	$16\varpi+1$	$8\varpi+1$
	$\frac{1}{2}\mathrm{N}_{\text{iv}}$		ω	ω	Ω	$8\varpi+1$	
8	N_{viii}		ω, ϵ	ϵ, ω	Ω	$32\varpi+1$	$16\varpi+1$
	$\frac{1}{2}\mathrm{N}_{\text{viii}}$		ω	ω	Ω	$16\varpi+1$	
$e=2^\kappa$	N_e		ω, ϵ	ϵ, ω	Ω	$4e\varpi+1$	$2e\varpi+1$
	$\frac{1}{2}\mathrm{N}_e$		ω	ω	Ω	$2e\varpi+1$	

n	N_n		Condition as to x, y	N_n	Linear Form of N_n	Linear Form of F_n
3	N_{iii}	x prime to y	$x \sim y \neq 3i$	Ω	$6\varpi+1$	$6\varpi+1$
	$\frac{1}{3}\mathrm{N}_{\text{iii}}$		$x-y=3i$	Ω	$6\varpi+1$	
	$\grave{\mathrm{N}}_{\text{iii}}$		$x+y \neq 3i$	Ω	$6\varpi+1$	$6\varpi+1$
	$\frac{1}{3}\grave{\mathrm{N}}_{\text{iii}}$		$x+y=3i$	Ω	$6\varpi+1$	
6	N_{vi}		None	Ω	$12\varpi+1$	$12\varpi+1$
12	N_{xii}		None	Ω	$24\varpi+1$	$24\varpi+1$

Note that 2 and 3 are *exceptional divisors* in following cases only :—

2 (but not 4) is a divisor or N_{ii}, N_{iv}, N_{viii}, ..., N_e when x and y are *odd* .... (4a);

3 (but not 9) is a divisor of N_{iii} when $x-y=3i$, and of $\grave{\mathrm{N}}_{\text{iii}}$ when $x+y=3i$.................... (4b).

5. *2-ic Forms.* The 2-ic forms, or partitions $(t^2-\mathrm{D}u^2)$, in which n-ans (N_n), and the divisors (F_n) of n-ans, are expressible are of two kinds, *Pure* and *Impure.*

Pure 2-ic forms have D, a *mere number.*

Impure 2-ic forms have $\mathrm{D} = \delta . xy$, where δ is a *mere number.*

The two numbers t, u are styled the "*2-ic parts*" of the form, and the form itself is styled shortly the (t, u) form.

The forms dealt with in this Volume have the following determinants (D).

Pure forms, $\mathrm{D} = -1, \pm 2, \pm 3, \pm 6$.

Impure forms, $\mathrm{D} = \delta . xy$, with $\delta = -1, \pm 2, \pm 3, \pm 6$.

6. *Pure 2-ic Forms.* The pure 2-ic forms of n-ans (N_n), and of the divisors (F_n) of n-ans, occurring in this Volume are stated below along with the abbreviated symbolism for each.

D =	-1	-2	$+2$	$+2$	-3
Form :	$\mathrm{a}^2+\mathrm{b}^2$	$\mathrm{c}^2+2\mathrm{d}^2$	$\mathrm{e}^2-2\mathrm{f}^2$	$2\mathrm{f}'^2-\mathrm{e}'^2$	$\mathrm{A}^2+3\mathrm{B}^2$
Abbreviation :	(a, b)	(c, d)	(e, f)	(f′, e′)	(A, B)

D =	-3	$+3$	-6	$+6$
Form :	$\frac{1}{4}(\mathrm{L}^2+27\mathrm{M}^2)$	$\mathrm{A}'^2-3\mathrm{B}'^2$	$\mathrm{G}^2+6\mathrm{H}^2$	$\mathrm{G}'^2-6\mathrm{H}'^2$
Abbreviation :	(L, M)	(A′, B′)	(G, H)	(G′, H′)

The above 2-ic forms are reckoned *different forms* only when they are *of different determinant* (D). Thus the (e, f) and (f′, e′) forms, as also the (A, B) and (L, M) forms, being of same determinants (D = +2, −3 respectively) are not reckoned *different forms*, but merely *different expressions of the same form*, being in fact algebraically inter-convertible by the formulæ:—

$$e' = e \mp 2f,\ \ f' = e \mp f;\ \ e = 2f' \mp e',\ \ f = f' \mp e' \ldots\ldots\ldots\ldots \quad (5).$$

$$\left.\begin{array}{l} L = 2A,\ \ M = \tfrac{2}{3}B,\ \ [\text{when } B = 3\beta] \\ A = \tfrac{1}{2}L,\ \ B = \tfrac{3}{2}M,\ \ [\text{when L and M} = \epsilon] \end{array}\right\} \ldots\ldots\ldots\ldots\ldots\ldots \quad (6a).$$

$$\left.\begin{array}{l} L = A \mp 3B,\ M = \tfrac{1}{3}(A \pm B),\ \ [\text{when } A \pm B = 3\beta] \\ A = \tfrac{1}{4}(L \pm 9M),\ B = \tfrac{1}{4}(L \mp 3M),\ [\text{when L and M} = \omega] \end{array}\right\} \ldots \quad (6b).$$

The "2-ic parts" (t, u) of these forms should—by the "working condition" (Art. 3)—be *prime to one another;* except that in the form (L, M) the parts L, M are *both even* when $B = 3\beta$; but, in this case $\tfrac{1}{2}L$, $\tfrac{1}{2}M$ will be prime to one another.

But even different forms of three *different determinants* D_1, D_2, D_3 which are connected by the relation

$$\lambda^2 . D_1 D_2 = \mu^2 . D_3, \quad [\lambda, \mu \text{ any } + \text{ integers}],$$

are so related that *any two of the three forms will algebraically determine the third form.* Such sets of three forms are termed 2-*ic Triads.* The Triads occurring among the forms with the 7 determinants above named are

$$(D_1, D_2, D_3) = (\bar{1}, 2, \bar{2}), (\bar{1}, 3, \bar{3}), (\bar{1}, 6, \bar{6}), (2, \bar{3}, \bar{6}), (\bar{2}, 3, \bar{6}), (2, 3, 6), (\bar{2}, \bar{3}, 6) \ldots\ldots\ldots \quad (7).$$

6a. *Odd and even* 2-*ic parts* (t, u). The character—(as to odd or even)—of the "2-ic parts" t, u in the 2-ic forms (t, u) of n-ans (N_n) and of the factors of N_n is shown in the scheme below: it depends chiefly on the linear form of $N_n = 4i \pm 1$, or $= 2\Omega$; except in the case of (L, M), wherein it depends on the form of $B =$ or $\neq 3\beta$.

N_n	a	b	c d	e f	e′ f′	A B	A′	B′	G H	G′ H′	N_{iii}	L M
$4i+1$	ω, ϵ	ϵ, ω	ω, ϵ	ω, ϵ	ω, ω	ω, ϵ	ω, ϵ	ϵ, ω	ω, ϵ	ω, ϵ	$B \neq 3\beta$	ω, ω
$4i-1$	—	—	ω, ω	ω, ω	ω, ϵ	ϵ, ω	ϵ, ω	ω, ϵ	ω, ω	ϵ, ω	$B = 3\beta$	ϵ, ϵ
2Ω	ω	ω	ϵ, ω	ϵ, ω	ϵ, ω	—	—	—	—	—	—	—

6b. *Indefiniteness of* a, b. In consequence of the symmetry of the form (a, b), the 2-ic parts a, b are algebraically indistinguishable: the *usual convention* is to take a as the *odd* part (when one of a, b is even).

6c. *Signs of the 2-ic parts* (t, u). The 2-ic parts t, u of the form (t, u), being therein determined only from their squares, are so far *of indeterminate sign* $(\pm)$. The usual convention is to fix the $(\pm)$ sign by the following Rule:—

All *even* "parts" are taken $+$.
All *odd* "parts" are taken $+$ when of form $(4i+1)$.
,, ,, ,, ,, ,, $-$ when of form $(4i-1)$.

7. *Indefinite 2-ic forms* (t^2-Du^2). A number (N) expressible in a 2-ic form (t^2-Du^2), whose determinant D is $+$ (but $\neq \delta^2$),—[*e.g.*, D = 2, 3, 6, &c.]—is expressible in an infinite number of ways in that form, say

$$N = t_1^2 - Du_1^2 = t_2^2 - Du_2^2 = t_3^2 - Du_3^2 = \&c. \quad \ldots\ldots\ldots\ldots\ldots \quad (8),$$

but these are all (algebraically) derivable from one another by *conformal *multiplication* or *conformal *division* by some power of the *unit-form* $(\tau^2 - D\upsilon^2)^k = +1$, so that these are *not considered different forms.*

7a. *Base form.* The form $(t_1^2 - Du_1^2)$ of the above series in which t_1, u_1 are *minima* is *unique* for that number N, and is styled the *Base-form* of that series: in all applications of the form (t^2-Du^2) it is *desirable to use the Base-form.*

The necessary and sufficient condition that (t^2-Du^2) should be a Base-form is

$$t > \frac{\tau+1}{\upsilon}.u, \text{ where } \tau^2 - D.\upsilon^2 = +1, \ [\tau, \upsilon \textit{ minima}] \ldots\ldots\ldots (9).$$

If $N = t_{r-1}^2 - D.u_{r-1}^2 = t_r^2 - D.u_r^2$ be *successive* forms of the above series, whereof t_r, u_r are *given*, the form (t_r, u_r) may be

---

* *Conformal multiplication* or *division* is multiplication or division *with preservation of* 2-ic *form.* For a full statement of the formulæ required, see the Author's paper on *Connexion of Quadratic Forms* in *Proc. Lond. Math. Socy.*, Vol. 28, 1897.

reduced to the lower form (t_{r-1}, u_{r-1}) by the formulæ

$$t_{r-1} = \tau_1 t_r \sim \mathrm{D}.v_1 u_r, \quad u_{r-1} = v_1 t_r \sim \tau_1 u_r \quad \ldots\ldots\ldots\ldots \quad (9a),$$

which will give $t_{r-1} < t_r$ and $u_{r-1} < u_r$,—[if (t_r, u_r) be not itself the Base-form].

Successive application of these Reduction-formulæ will lead eventually to the Base-form (t_1, u_1).

7b. *Unit-form.* The "unit-forms" (τ_1, v_1) required in this Volume are given below, together with the value of the ratio $(\tau_1+1) \div v_1$ required in the Base-form Test (9) above.

D =	+2	+3	+6
$\tau^2 - \mathrm{D}.v^2 = +1$	$3^2-2.2^2=+1$	$2^2-3.1^2=+1$	$5^2-6.2^2=+1$
$(\tau+1) \div v =$	2	3	3

8. *2-ic forms of primes* (p). Odd primes (p), and prime powers (p^κ), and also their doubles ($2p$ and $2p^\kappa$) are *expressible in only one way* in any one "definite" 2-ic form $(t^2+\mathrm{D}u^2)$, and also in *only one way in the Base-form* of any one "indefinite" 2-ic form $(t^2-\mathrm{D}u^2)$. In this sense each of the 2-ic expressions of p, $2p$, p^κ, and $2p^\kappa$ in any 2-ic form $(t^2 \pm \mathrm{D}u^2)$ is *unique*. [The "2-ic parts" t, u herein used should be *mutually prime.*]

The converse of the above is true only for those 2-ic forms whose determinant $(\pm \mathrm{D})$ is *Idoneal:* all the determinants here used, viz. $\mathrm{D} = \bar{1}$, ∓ 2, ∓ 3, ∓ 6 are of that kind, so that—

> A number (N) expressible *in only one way* in any of the 2-ic forms (a, b), (c, d), (A, B), (L, M), (G, H) or Base-forms of (e, f), (e′, f′), (A′, B′), (G′, H′) is either p, p^κ, $2p$, or $2p^\kappa$ .... (10).

8a. *2-ic forms of composites* (N). A number (N) which is the product of r odd primes (p) or prime-powers (p^κ), each of which is expressible *in the same 2-ic form* $(t^2 \pm \mathrm{D}u^2)$, is expressible in 2^{r-1} different ways *in that same 2-ic form* $(t^2 \pm \mathrm{D}u^2)$ with t, u mutually prime. [In the case of an "indefinite" form $(t^2-\mathrm{D}u^2)$ there will be 2^{r-1} different infinite series of that form, each having its own unique Base-form.]

> Conversely, a number (N) expressible in more than one way in any 2-ic form $(t^2 \pm \mathrm{D}u^2)$ is certainly *composite*: and, if N have 2^{r-1} expressions in that form, it must have r different prime factors. [The expressions $(t^2-\mathrm{D}u^2)$ should be Base-forms] .... (10a).

9. *2-ic forms of odd divisors* (F_n) of *n-ans* (N_n). All *odd* divisors (F_n) of n-ans (N_n)—with $n = 2, 4, 8$, &c.; $3, 6, 12$, &c. —are expressible *arithmetically*,—but not usually algebraically —in the same *pure* 2-ic forms as the algebraic *pure* 2-ic forms of the n-ans (N_n) of which they are factors; (but not also in the impure 2-ic forms thereof).

10. *Duans and Half-Duans, Equivalence*, $(N_{ii} = \frac{1}{2}N'_{ii})$.

$N_{ii} = x^2 + y^2 = \Omega$, a *Duan*, $[x, y$ are, one ω, one $\epsilon]$...... (11a);

$\frac{1}{2}N'_{ii} = \frac{1}{2}(x'^2 + y'^2) = \Omega$, a *Half-Duan*, $[x', y'$ both $\omega]$...... (11b).

Duans and Half-Duans are *algebraically* interconvertible $(N_{ii} = \frac{1}{2}N'_{ii})$ by the formulæ

$x' = x \mp y,\ y' = x \pm y;\quad x = \frac{1}{2}(x' \pm y'),\ y = \frac{1}{2}(y' \mp x')$ ...... (12).

10a. *Duans and Half-Duans, 2-ic Forms.* Every Duan (N_{ii}), and every Half-Duan $(\frac{1}{2}N'_{ii})$, are *algebraically* expressible in the three 2-ic forms, one *pure* and two impure, viz.

$N_{ii} = a^2 + b^2 = x^2 + y^2$, $D = -1$ ...... (13a);

$= \gamma^2 + 2xy.\delta^2 = (x \sim y)^2 + 2xy.1^2$, $D = -2xy$ ... (13b);

$= \eta^2 - 2xy.\phi^2 = (x + y)^2 - 2xy.1^2$, $D = +2xy$ ... (13c).

$\frac{1}{2}N'_{ii} = a'^2 + b'^2 = \{\frac{1}{2}(x' \sim y')\}^2 + \{\frac{1}{2}(x' + y')\}^2$, $D = -1$ ...... (13a');

$= 2\delta'^2 + x'y'.\gamma'^2 = 2\{\frac{1}{2}(x' \sim y')\}^2 + x'y'.1^2$, $D = -2x'y'$... (13b');

$= 2\phi'^2 - x'y'.\eta'^2 = 2\{\frac{1}{2}(x' + y')\}^2 - x'y'.1^2$, $D = +2x'y'$... (13c').

Here the pure 2-ic forms of N_{ii}, $\frac{1}{2}N'_{ii}$ are the *same*, (*i.e.* of same determinant $D = -1$); whereas the impure forms, though similar, are *different 2-ic forms*, (having different determinants D).

As N_{ii}, $\frac{1}{2}N'_{ii}$ are algebraically interconvertible, it follows that each of them can be algebraically expressed in one way in the above five 2-ic forms. Hence

Every *prime* (p) of form $p = 4\varpi + 1$, and every *power* (p^κ) thereof, has one unique set of the above five forms .... (14a).

Every *composite* (N) of form $N = a^2 + b^2$ has one unique set of the above five forms for every distinct form (a, b) of N .... (14b).

11. *Quartans and Half-Quartans, 2-ic Forms.* Every Quartan (N_{iv}) and every Half-Quartan $(\frac{1}{2}N'_{iv})$ is *algebraically* expressible in three *pure* 2-ic forms, which are obtainable from the 2-ic forms of Duans and Half-Duans—(of which in fact they are merely special forms)—by writing x^2, y^2 for x, y in the 2-ic

forms of the latter two: the *impure* 2-ic forms of the latter hereby become *pure* forms.

$$\mathrm{N}_{\mathrm{iv}} = x^4 + y^4 \quad = \Omega, \text{ a } \textit{Quartan}, \ [x, y, \text{ one is } \omega, \text{ one is } \epsilon] \ \ldots \ (15);$$
$$= \mathrm{a}^2 + \mathrm{b}^2 \quad = (x^2)^2 + (y^2)^2 \ \ldots\ldots \ (15a);$$
$$= \mathrm{c}^2 + 2\mathrm{d}^2 \quad = (x^2 \sim y^2)^2 + 2(xy)^2 \ \ldots\ldots \ (15b);$$
$$= \mathrm{e}^2 - 2\mathrm{f}^2 \quad = (x^2 + y^2)^2 - 2(xy)^2 \ \ldots\ldots \ (15c);$$
$$= 2\mathrm{f}'^2 - \mathrm{e}'^2 \quad = 2(x^2 \mp xy + y^2)^2 - (x^2 \mp 2xy + y^2)^2 \ \ldots\ldots \ (15d).$$
$$\tfrac{1}{2}\mathrm{N}'_{\mathrm{iv}} = \tfrac{1}{2}(x'^4 + y'^4) = \Omega, \text{ a } \textit{Half-Quartan}, \ [x', y' \text{ both } \textit{odd}] \ \ldots\ldots \ (15');$$
$$= \mathrm{a}'^2 + \mathrm{b}'^2 \quad = \{\tfrac{1}{2}(x'^2 \sim y'^2)\}^2 + \{\tfrac{1}{2}(x'^2 + y'^2)\}^2 \ \ldots\ldots \ (15a');$$
$$= \mathrm{c}'^2 + 2\mathrm{d}'^2 \quad = (x'y')^2 + 2\{\tfrac{1}{2}(x'^2 \sim y'^2)\}^2 \ \ldots\ldots \ (15b');$$
$$= \mathrm{e}''^2 - 2\mathrm{f}''^2 \quad = (x'^2 \mp x'y' + y'^2)^2 - 2\{\tfrac{1}{2}(x'^2 \mp 2x'y' + y'^2)\}^2 \ldots \ (15c');$$
$$= 2\mathrm{f}''^2 - \mathrm{e}''^2 \quad = 2\{\tfrac{1}{2}(x'^2 + y'^2)\}^2 - (x'y')^2 \ \ldots\ldots \ (15d').$$

Note that

$$\mathrm{c} + \mathrm{e} = 2\mathrm{a} \text{ or } 2\mathrm{b}, \quad \mathrm{d} = \mathrm{f} = \tfrac{1}{2}(\mathrm{f}' - \mathrm{e}'), \quad \mathrm{e} = \tfrac{1}{2}(\mathrm{e}' + \mathrm{f}') \ \ldots\ldots \ (16a);$$
$$\mathrm{d}' = \mathrm{a}' \text{ or } \mathrm{b}', \quad \mathrm{f}'' = \mathrm{b}' \text{ or } \mathrm{a}', \quad \mathrm{c}' = \mathrm{e}'' = \mathrm{e}' \sim 2\mathrm{f}', \quad \mathrm{f}'' = \mathrm{e}' - \mathrm{f}' \quad (16b).$$

11a. *Semi-quartic Partitions.* Certain numbers (N) of form $8n \pm 1 = e^2 - 2f^2$ may be expressed in several of the semi-quartic partitions.

$$\mathrm{N} = t_1^4 - 2u_1^2 = t_2^2 - 2u_2^4 = 2u_3^4 - t_3^2 = 2u_4^2 - t_4^4.$$

These are closely connected with and may be derived from the above forms of Quartans. Thus

$$t_1^4 - 2u_1^2 = 2u_4^2 - t_4^4 \quad \text{gives} \quad \tfrac{1}{2}(t_1^4 + t_4^4) = u_1^2 + u_4^2,$$

which can be identified with Result (15a′) above. Similarly $t_2^2 - 2u_2^4 = 2u_3^4 - t_3^2$ can be identified with the same. Tables of these partitions for all numbers $\mathrm{N} < 100$ are given on pages* 144, 145. Some composite numbers can be expressed in all four of the forms: but no prime has yet been found so expressible.

12. *Octavans and Half-Octavans,*

$$\mathrm{N}_{\mathrm{viii}} = x^8 + y^8 = \Omega, \text{ an } \textit{Octavan}, \ [x, y \text{ one is } \omega, \text{ one is } \epsilon] \ldots\ldots \ (17a);$$
$$\tfrac{1}{2}\mathrm{N}'_{\mathrm{viii}} = \tfrac{1}{2}(x'^8 + y'^8) = \Omega, \text{ a } \textit{Half-Octavan}, \ [x', y' \text{ both } \textit{odd}] \ \ldots \ (17b).$$

These being special powers of N_{iv} and $\frac{1}{2}\mathrm{N}'_{\mathrm{iv}}$, their 2-ic forms may be derived from those of N_{iv} and $\frac{1}{2}\mathrm{N}'_{\mathrm{iv}}$ by writing x^2, y^2 for x, y in the 2-ic forms of the latter two.

The 2-ic forms of $\mathrm{N}_{\mathrm{xvi}}$, $\frac{1}{2}\mathrm{N}'_{\mathrm{xvi}}$, &c., may be derived in the same way from those of $\mathrm{N}_{\mathrm{viii}}$, $\frac{1}{2}\mathrm{N}'_{\mathrm{viii}}$, &c.

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* For some Errata in these Tables, see page 280.

13. *Cuban and Trito-Cuban Identities* ($\mathrm{N}_{\mathrm{iii}}$, $\frac{1}{3}\mathbf{N}_{\mathrm{iii}}$). The Cuban ($\mathrm{N}_{\mathrm{iii}}$) and Trito-Cuban ($\mathbf{N}_{\mathrm{iii}}$) have each of them *three equivalent cuban forms*, and carry with them *equivalent trinomial 2-ic forms* :—

$$\mathrm{N}_{\mathrm{iii}} = \mathrm{N}'_{\mathrm{iii}} = \mathrm{N}''_{\mathrm{iii}}\,; \quad \text{and} \quad \tfrac{1}{3}\mathbf{N}_{\mathrm{iii}} = \tfrac{1}{3}\mathbf{N}'_{\mathrm{iii}} = \tfrac{1}{3}\mathbf{N}''_{\mathrm{iii}} \;\ldots\ldots\; (18),$$

where
$$\mathrm{N}_{\mathrm{iii}} = \frac{x^3-y^3}{x-y}, \qquad \mathrm{N}'_{\mathrm{iii}} = \frac{z^3+x^3}{z+x}, \qquad \mathrm{N}''_{\mathrm{iii}} = \frac{z^3+y^3}{z+y} \;\ldots\ldots\ldots\; (19a),$$
$$= x^2+xy+y^2, \qquad = z^2-zx+x^2, \qquad = z^2-zy+y^2 \;\ldots\; (19b),$$
$$= [x, y], \qquad = [z, x], \qquad = [z, y], \ \textit{for shortness},$$

wherein
$$z = x+y, \quad [\text{one of } x,\ y,\ z \text{ is } \epsilon\,;\ \text{two are } \omega] \;\ldots\ldots\ldots\ldots\; (20a).$$

$$\tfrac{1}{3}\mathbf{N}_{\mathrm{iii}} = \tfrac{1}{3}\frac{x'^3-y'^3}{x'-y'}, \qquad \tfrac{1}{3}\mathbf{N}'_{\mathrm{iii}} = \tfrac{1}{3}\frac{z'^3+x'^3}{z'+x'}, \qquad \tfrac{1}{3}\mathbf{N}''_{\mathrm{iii}} = \tfrac{1}{3}\frac{z'^3+y'^3}{z'+y'} \;\ldots\ldots\ldots\; (19a'),$$
$$= \tfrac{1}{3}(x'^2+x'y'+y'^2), \quad = \tfrac{1}{3}(z'^2-z'x'+x'^2), \quad = \tfrac{1}{3}(z'^2-z'y'+y'^2) \;\; (19b'),$$
$$= \tfrac{1}{3}[x', y'], \qquad = \tfrac{1}{3}[z', x'], \qquad = \tfrac{1}{3}(z', y'], \ \textit{for shortness},$$
$$= x'^2-3x'\zeta'+3\zeta'^2, \quad = z'^2-3z'\eta'+3\eta'^2, \quad = z'^2-3z'\xi'+3\xi'^2$$
$$= y'^2+3y'\zeta'+3\zeta'^2, \quad = x'^2-3x'\eta'+3\eta'^2, \quad = y'^2-3y'\xi'+3\xi'^2,$$
$$= \{x', \zeta'\} = \{y', \zeta'\}, = \{z', \eta'\} = \{x', \eta'\}, = \{z', \xi'\} = \{y', \xi'\},$$
for shortness,

wherein
$$\left.\begin{array}{l} x'-y' = 3\zeta', \quad z'+x' = 3\eta', \quad z'+y' = 3\xi' \\ z' = x'+y' = \xi'+\eta', \quad \eta' = \xi'+\zeta' \end{array}\right\} \;\ldots\ldots\ldots\; (20a'),$$

and one of x', y', z' is ϵ, and two are ω; one of ξ', η', ζ' is ϵ, and two are ω.

13a. *Equivalent Cubans and Trito-Cubans.* The above 3 equivalent Cubans and their associate 3 equivalent Trito-Cubans are actually all equal to one another, thus

$$\mathrm{N}_{\mathrm{iii}} = \mathrm{N}'_{\mathrm{iii}} = \mathrm{N}''_{\mathrm{iii}} = \tfrac{1}{3}\mathbf{N}_{\mathrm{iii}} = \tfrac{1}{3}\mathbf{N}'_{\mathrm{iii}} = \tfrac{1}{3}\mathbf{N}''_{\mathrm{iii}}, \quad [\text{Cuban forms}],$$

and
$$\left.\begin{array}{l} [x, y] = [z, x] = [z, y] = \tfrac{1}{3}[x', y'] = \tfrac{1}{3}[z', x'] = \tfrac{1}{3}[z', y'] \\ = \{x', \zeta'\} = \{y', \zeta'\} = \{z', \eta'\} = \{x', \eta'\} = \{z', \xi'\} = \{y', \xi'\} \end{array}\right\}$$
$$[\text{Trinomial 2-ic forms}] \;\ldots\ldots\ldots\; (21)\,;$$

and are *algebraically* interconvertible by the formulæ :—

1°. $x' = x+2y,\ y' = x-y\,;\quad x = \frac{1}{3}(x'+2y'),\ y = \frac{1}{3}(x'-y')\ ;$
$$[x > y,\ x' > y'] \;\ldots\; (22a)\,;$$

2°. $x' = 2x+y,\ y' = y-x\,;\quad x = \frac{1}{3}(x'+y'),\ \ y = \frac{1}{3}(x'+2y')\,;$
$$[y > x,\ x' > y'] \;\ldots\; (22b)\,;$$

along with the formulæ of Art. 13 connecting z, z' with x, y, x', y' and ξ', η', ζ' with x', y', z'.

[This property of Cubans is analogous to that of equivalent Duans and Half-Duans (Art. 10). It is peculiar to Duans and Cubans; no other n-ans (with $n > 3$) possess such a property.]

13c. *Cubans and Trito-Cubans, 2-ic Forms.* Each of the three equivalent Cubans $(\mathrm{N}, \mathrm{N}', \mathrm{N}'')$ and also each of their equivalent Trito-Cubans $(\frac{1}{3}\mathbf{N}_{\text{iii}}, \frac{1}{3}\mathbf{N}'_{\text{iii}}, \frac{1}{3}\mathbf{N}''_{\text{iii}})$ is expressible in the *self-same pure 2-ic form* $(\mathrm{A}^2+3\mathrm{B}^2)$,—[the same A, and same B in each]—by the formulæ

$$\mathrm{N}_{\text{iii}} = \mathrm{N}'_{\text{iii}} = \mathrm{N}''_{\text{iii}} = \mathrm{A}^2+3\mathrm{B}^2 \quad\ldots\ldots\ldots\ldots\ldots\ldots \quad 23).$$

$$\left.\begin{array}{llll} \mathrm{A} = \tfrac{1}{2}x+y & = z-\tfrac{1}{2}x & = \tfrac{1}{2}(z+y) \\ \mathrm{B} = \tfrac{1}{2}x & = \quad \tfrac{1}{2}x & = \tfrac{1}{2}(z-y) \end{array}\right\} x=\epsilon,\ y\ \&\ z=\omega \quad\ldots\ldots\ldots \quad (23a).$$

$$\left.\begin{array}{llll} \mathrm{A} = x+\tfrac{1}{2}y & = \tfrac{1}{2}(z+x) & = z-\tfrac{1}{2}y \\ \mathrm{B} = \quad \tfrac{1}{2}y & = \tfrac{1}{2}(z-x) & = \quad \tfrac{1}{2}y \end{array}\right\} y=\epsilon,\ z\ \&\ x=\omega \quad\ldots\ldots\ldots \quad (23b).$$

$$\left.\begin{array}{llll} \mathrm{A} = \tfrac{1}{2}(x-y) & = x-\tfrac{1}{2}z & = \tfrac{1}{2}z-y \\ \mathrm{B} = \tfrac{1}{2}(x+y) & = \quad \tfrac{1}{2}z & = \tfrac{1}{2}z \end{array}\right\} z=\epsilon,\ z\ \&\ y=\omega,\ x>y \ldots \quad (23c).$$

In the case of the Trito-Cubans $(\frac{1}{3}\mathbf{N}_{\text{iii}} = \frac{1}{3}\mathbf{N}'_{\text{iii}} = \frac{1}{3}\mathbf{N}''_{\text{iii}})$, the A, B can be most neatly expressed in a single formula for all three forms with different cases according as x', y', z' is the even element of the Trito-Cuban:—

$$\mathrm{A} = \tfrac{1}{2}x',\ \ \mathrm{B} = \tfrac{1}{2}\xi',\ \text{ when } x' \text{ and } \xi' \text{ are } \epsilon \quad\ldots\ldots\ldots \quad (23a'),$$

$$\mathrm{A} = \tfrac{1}{2}y',\ \ \mathrm{B} = \tfrac{1}{2}\eta',\ \text{ when } y' \text{ and } \eta' \text{ are } \epsilon \quad\ldots\ldots\ldots \quad (23b'),$$

$$\mathrm{A} = \tfrac{1}{2}z',\ \ \mathrm{B} = \tfrac{1}{2}\xi',\ \text{ when } z' \text{ and } \xi' \text{ are } \epsilon \quad\ldots\ldots\ldots \quad (23c').$$

13d. *Equivalent 2-ic, Cuban, and Trito-Cuban forms.* By Art. 13c it is seen that—

> Every number (N) of form $\mathrm{N} = \mathrm{A}^2 + 3\mathrm{B}^2$ with definite A, B has three Cuban forms $\mathrm{N}_{\text{iii}} = \mathrm{N}'_{\text{iii}} = \mathrm{N}''_{\text{iii}}$ and three Trito-cuban forms $\frac{1}{3}\mathbf{N}_{\text{iii}} = \frac{1}{3}\mathbf{N}'_{\text{iii}} = \frac{1}{3}\mathbf{N}''_{\text{iii}}$, and has the 12 corresponding trinomial 2-ic forms shown in Art. 13a .... (24a);

and that for any particular values of A, B the whole of these forms are *unique*. Hence:—

> Every *prime* (p) of form $p = 6\varpi+1$, and every *power* (p^κ) thereof, has one unique set of the above forms, (6 cuban and 12 trinomial 2-ic).
>
> Every composite (N) of form $\mathrm{N} = \mathrm{A}^2+3\mathrm{B}^2$ has one unique set of the above forms for each distinct form (A, B) of N .... (24b).

13e. *Impure 2-ic forms of Cubans and Trito-Cubans.*

The three equivalent Cubans $\mathrm{N}_{\text{iii}} = \mathbf{N}'_{\text{iii}} = \mathrm{N}''_{\text{iii}}$ are each *algebraically* expressible in one way in one of the *impure* 2-ic forms $(t^2 \pm 3vw.u^2)$, and also in one way in one of the *impure* 2-ic forms $(\mathrm{T}^2 \mp vw.\mathrm{U}^2)$, where v, w are the cuban elements: thus

$$N_{iii} = (x-y)^2+3xy.1^2,\quad N'_{iii} = (z+x)^2-3zx.1^2,\quad N''_{iii} = (z+y)^2-3zy.1^2,$$
$$= (x+y)^2-xy.1^2.\qquad = (z-x)^2+zx.1^2.\qquad = (z-y)^2+zy.1^2$$
$$\ldots\ldots\ldots \text{(25a, b)}.$$

And the three equivalent Trito-Cubans $\frac{1}{3}\mathbf{N}_{iii} = \frac{1}{3}\mathbf{N}'_{iii} = \frac{1}{3}\mathbf{N}''_{iii}$ are each *algebraically* expressible in one way in one of the *impure* 2-ic forms $(3t'^2 \pm v'w'.u'^2)$, and also in one of the *impure* 2-ic forms $\frac{1}{3}(T'^2 \mp v'w'.U'^2)$, where v', w' are the cuban elements: thus

$$\tfrac{1}{3}\mathbf{N}_{iii} = 3\zeta'^2+x'y'.1^2,\quad \tfrac{1}{3}\mathbf{N}'_{iii} = 3\eta'^2-z'x'.1^2,\quad \tfrac{1}{3}\mathbf{N}''_{iii} = 3\xi'^2-z'y'.1^2$$
$$= \tfrac{1}{3}(z'^2-x'y'.1^2)\qquad = \tfrac{1}{3}(y'^2+z'x'.1^2),\qquad = \tfrac{1}{3}(x'^2+z'y'.1^2).$$
$$\ldots\ldots\ldots \text{(25c, d)}.$$

Here these six impure 2-ic forms are all *different 2-ic forms*, in that they are of different determinants

$$(D = -3xy,\ +3zx,\ +3zy;\ +xy,\ -zx,\ -zy).$$

Contrast this property with that of the pure 2-ic form (A^2+3B^2) which belongs to all the six cuban forms, (see Art. 13c).

Here,—(as in Art. 13d)—

Every prime (p) of form $p = 6\varpi+1$, and every power (p^κ) thereof, has one unique set of the above six impure 2-ic forms .... (26a).

Every composite (N_1) of form $N = A^2+3B^2$ has one unique set of the above forms for each distinct form (A, B) of N .... (26b).

14. *Sextans*, (N_{vi}). Every Sextan (N_{vi}) is *algebraically* expressible in seven 2-ic forms, viz. in 3 pure forms and 4 *impure* forms. The three *pure forms* may be obtained from the 2-ic forms of Cubans (N'_{iii})—(of which in fact the Sextan is only a special form)—by writing x^2, y^2 for x', y' in the 2-ic forms of the Cuban: hereby the impure forms of the latter become *pure forms*.

$$N_{vi} = \frac{x^6+y^6}{x^2+y^2} = x^4-x^2y^2+y^4 = \Omega, \text{ a } Sextan \ldots\ldots\ldots\ldots (27),$$
$$= a^2+b^2 = (x^2\sim y^2)^2+(xy)^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (27a),$$
$$= A'^2-3B'^2 = (x^2+y^2)^2-3(xy)^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (27b),$$
$$= A^2+3B^2, \text{ by one of the following formulæ} \qquad (27c),$$

$$A = \tfrac{1}{2}x^2\sim y^2,\quad B = \tfrac{1}{2}x^2;\qquad x^2 = 2B,\quad y^2 = B\mp A;\quad [x = \epsilon,\ y = \omega]$$
$$A = x^2\sim \tfrac{1}{2}y^2,\quad B = \tfrac{1}{2}y^2;\qquad x^2 = B\pm A,\quad y^2 = 2B;\quad [x = \omega,\ y = \epsilon]$$
$$A = \tfrac{1}{2}(x^2+y^2),\quad B = \tfrac{1}{2}(x^2\sim y^2);\qquad x^2 = A\pm B,\quad y^2 = A\mp B;\quad [x \text{ and } y = \omega]$$
$$\text{(27c}', \text{c}'', \text{c}''').$$

$$N_{vi} = \gamma^2 + 2xy\delta^2 \quad = (x^2 - xy + y^2)^2 + 2xy\,(x \sim y)^2 \;\ldots\ldots\ldots\; (27d).$$
$$= \eta^2 - 2xy\phi^2 \quad = (x^2 + xy + y^2)^2 - 2xy\,(x + y)^2 \;\ldots\ldots\ldots\; (27e).$$

$$N_{vi} = G^2 - 6xy.H^2 \quad = (x^2 + 3xy + y^2)^2 - 6xy.(x + y)^2 \;\ldots\ldots\ldots\; (27f),$$
$$= G'^2 + 6xy.H'^2 = (x^2 - 3xy + y^2)^2 + 6xy.(x \sim y)^2 \;\ldots\ldots\ldots\; (27g).$$

Note that—

$$A \pm B = a \text{ or } b, \text{ if } x = \epsilon; \quad B \mp A = a \text{ or } b, \text{ if } y = \epsilon \;\ldots\ldots\; (28a),$$
$$a \text{ or } b = 2B, \quad A' = 2A, \text{ if } x \,\&\, y = \omega \;\ldots\ldots\ldots\ldots\ldots\; (28b),$$
$$b \text{ or } a = B', \quad 2A' = G + G', \quad G - G' = 6b \text{ or } 6a = 6B' \;\ldots\; (28c),$$
$$H^2 + H'^2 = 2A', \quad H^2 - H'^2 = 4a \text{ or } 4b = 4B' \;\ldots\ldots\ldots\ldots\; (28d).$$

Every *composite* Sextan (N_{vi}), which is the product of r different prime factors, is *arithmetically* expressible in 2^{r-1} *different* ways in each of the *pure* 2-ic forms (a, b), (A′, B′), (A, B) including the single algebraic expression in each of those forms, as above .... (28e).

Every number (N) of form $N = 12\varpi + 1$, which is expressible in two of the three pure 2-ic forms (a, b), (A′, B′), (A, B) is a *Sextan* (N_{vi}), if the 2-ic parts a, b, A′, B′, A, B are related as above, (but not otherwise). If the number N be *composite*, it suffices that some one set of the three pure 2-ic forms should satisfy the conditions .... (28f).

14a. *Expression of given numbers* (N) *as Sextans.* Here N must be of form $N = 12\varpi + 1$, and of all the above 2-ic forms. If N be given in any one of the 2-ic forms, then the Sextan elements are readily found.

1°. Given $N = a^2 + b^2$: then $B' = b = xy$, and $(x^2 + y^2)^2 = N + 3\,(xy)^2$, whereby $x^2 + y^2$ and xy are known.

2°. Given $N = A'^2 - 3B'^2$: then $b = B' = xy$, and $(x^2 \sim y^2)^2 = N - (xy)^2$, whereby $x^2 \sim y^2$ and xy are known.

3°. Given $N = A^2 + 3B^2$: then x^2 or $y^2 = 2B$, or $(A \pm B)$.

4°. Given $N = G^2 - 6xyH^2$, [Here G & xyH^2 are given, not H]. It may be shown that $(x^2 + xy + y^2)^2 = G^2 - 4xyH^2$,
so that $(x^2 + xy + y^2)$ and $(x^2 + 3xy + y^2)$ are now known.

5°. Given $N = G'^2 + 6x'y'H'^2$, [Here G′ and $x'y'H'^2$ are given, not H′]. It may be shown that $(x'^2 - x'y' + y'^2)^2 = G'^2 + 4x'y'H'^2$,
so that $(x'^2 - x'y' + y'^2)$ and $(x'^2 + 3x'y' + y'^2)$ are now known.

15. *Duodecimans*, (N_{xii}). Every Duodeciman (N_{xii}) is *algebraically* expressible *in one way* in seven *pure* 2-ic forms of determinants $D = -1, +2, -2, +3, -3, +6, -6$; five of these forms are obtainable from the 2-ic forms of Sextans (N_{vi}) —(of which in fact 12-mans are merely special forms)—by writing x^2, y^2 in place of x, y in the 2-ic forms of N_{vi}; hereby the two *impure* 2-ic forms of N_{vi} become pure forms.

$$N_{xii} = \frac{x^{12}+y^{12}}{x^4+y^4} = x^8 - x^4y^4 + y^8 = \Omega,\ \text{a } \textit{Duodeciman} \quad \ldots\ldots \quad (29)$$

$$= a^2 + b^2 = (x^4 \sim y^4)^2 + (x^2y^2)^2 \quad \ldots\ldots \quad (29a)$$

$$= c^2 + 2d^2 = (x^4 - x^2y^2 + y^4)^2 + 2\left\{xy\,(x^2 \sim y^2)\right\}^2 \quad \ldots\ldots \quad (29b)$$

$$= e^2 - 2f^2 = (x^4 + x^2y^2 + y^4)^2 - 2\left\{xy\,(x^2 + y^2)\right\}^2 \quad \ldots\ldots \quad (22c)$$

$$= 2f'^2 - e'^2 = 2\,(x^4 - x^3y + x^2y^2 - xy^3 + y^4)^2 - (x^4 - 2x^3y + x^2y^2 - 2xy^3 + y^4)^2 \quad \ldots\ldots \quad (29d)$$

$$= A'^2 - 3B'^2 = (x^4 + y^4)^2 - 3\,(x^2y^2)^2 \quad \ldots\ldots \quad (29e)$$

$$= A^2 + 3B^2,\ \text{by one of following formulæ} \ldots\ldots \quad (29f)$$

$$A = \tfrac{1}{2}x^4 \sim y^4,\quad B = \tfrac{1}{2}x^4;\qquad x^4 = 2B,\quad y^4 = B \mp A;\quad [x = \epsilon,\ y = \omega]$$

$$A = x^4 \sim \tfrac{1}{2}y^4,\quad B = \tfrac{1}{2}y^4;\qquad x^4 = B \pm A,\quad y^4 = 2B;\quad [x = \omega,\ y = \epsilon]$$

$$A = \tfrac{1}{2}(x^4 + y^4),\quad B = \tfrac{1}{2}(x^4 \sim y^4);\qquad x^4 = A \pm B,\quad y^4 = A \mp B;\quad [x \text{ and } y = \omega]$$

$$\ldots\ldots \quad (29f',\ f'',\ f''')$$

$$N_{xii} = G^2 - 6H^2 = (x^4 + 3x^2y^2 + y^4)^2 - 6\left\{xy\,(x^2 + y^2)\right\}^2 \quad \ldots\ldots \quad (29g)$$

$$= G'^2 + 6H'^2 = (x^4 - 3x^2y^2 + y^4)^2 + 6\left\{xy\,(x^2 \sim y^2)\right\}^2 \quad \ldots\ldots \quad (29h).$$

Note that—

$$b \text{ or } a = B' = \tfrac{1}{2}(e - c),\quad \tfrac{1}{2}(c + e) = A' = \tfrac{1}{2}(G + G') \ldots\ldots \quad (30a).$$

$$e' = e - 2f,\quad f' = e - f;\quad e = 2f' - e',\quad f = f' - e' \ldots\ldots \quad (30b).$$

$$\left.\begin{array}{l} a \text{ or } b = B \pm A \text{ when } x \text{ or } y = \epsilon \\ 2A = A',\ a \text{ or } b = 2B \text{ when } x \text{ and } y = \omega \end{array}\right\} \ldots\ldots \quad (30c).$$

15a. *Expression of given numbers* (N) *as Duodecimans.* Here N must be of form $(24\varpi + 1)$ and of all the above 2-ic forms. If N be given in any one of the above 2-ic forms, its 12-man elements (x, y) can be found in the same way as for Sextans (Art. 14a).

16. *2-ic forms of large factors* (Q) of N_n. When a composite n-an (N_n) is a product of a *single* large factor (Q) by one, or more, small *prime* factors, (q_1, q_2, &c.), or powers thereof, then all these factors are capable of the same *pure* 2-ic forms as their product (N_n) itself (but see Art. 9). The 2-ic forms of N_n are given by the formulæ preceding (Arts. 10-15): the similar 2-ic forms of the small factors (q) can generally be found by trial, or taken from *Tables. The similar 2-ic forms of the large factor (Q) can then be found by the process of †*conformal division*, which is a direct and simple process.

Two Cases arise, one for 2-ic forms of $-D$, one for 2-ic forms of $+D$.

The detail given below is for a single small factor (q), so that $N_n = qQ$.

Case 1°.	*Case* 2°.
$\left.\begin{aligned} N_n &= T^2+DU^2 \\ q &= t^2+Du^2 \end{aligned}\right\}$, (*given*).	$\left.\begin{aligned} N_n &= T^2-DU^2 \\ q &= t^2-Du^2 \end{aligned}\right\}$, (*given*).
$Q = X^2+DY^2$, (*sought*).	$Q = X^2-DY^2$, (*sought*).
Then $\left.\begin{aligned} X &= (tT\mp DuU)\div q \\ Y &= (uT\pm tU)\div q \end{aligned}\right\}$ ... (31a)	$\left.\begin{aligned} X &= (tT\mp DuU)\div q \\ Y &= (uT\mp tU)\div q \end{aligned}\right\}$ ...(31b)
[opposite signs in X, Y].	[same signs in X, Y].

Here one, and only one, of the ($\pm$) signs in the brackets will give the required *integer* values of the sought X, Y.

If there be several small prime factors q_1, q_2, &c. ... q_r in N_n, so that $N_n = (q_1 q_2 \ldots q_r)\,Q$, the same process may be used, but should be applied to *only one factor* (q_1, q_2, &c.) *at a time*, (in order to ensure success in the divisions). Thus, if

$$q_1 = t_1^2 \pm Du_1^2, \quad q_2 = t_2^2 \pm Du_2^2, \quad q_3 = \text{\&c.},$$

then Step i gives the X_1, Y_1 of $N_n \div q_1 = X_1^2 \pm DY_1^2$ .... (32a),

Step ii gives the X_2, Y_2 of $(N_n \div q_1)\div q_2 = X_2^2 \pm DY_2^2$ .... (32b),

and so on, thus cancelling one factor at a time out of N_n. Proceeding in this way, integer values of (X_1, Y_1), (X_2, Y_2), &c., will be obtained at each step, (which would be otherwise uncertain).

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* The Tables of *Quadratic Partitions*, London, 1904, by the present author give all the 2-ic partitions likely to be wanted for this purpose.

† *i.e.* division *with preservation of 2-ic form*, see the present author's Paper on *Connexion of Quadratic Forms* in Proc. Lond. Math. Soc., Vol. 28, 1897, pages 295–301 (Arts. 15–23).

In Case 2° the forms $(X^2 - DY^2)$ resulting directly from the conformal are generally not *Base-forms;* so may require reduction to Base-form by multiplication by the "unit-form" $(\tau^2 - D\upsilon^2 = +1)$, see Art. 7a, b.

As to the advisability of casting out only one factor (q) at a time in the above process, note that the product $q_1 q_2$ has always *two forms of every kind* $(t^2 \pm Du^2)$. If *both* these forms be tried, integral values of X, Y will certainly result from one, (and only one) of them: but it is more trouble to effect the double trial that may be required (if the first trial fails) with the product forms of $q_1 q_2$ than it is to do the work twice by casting out each prime separately.

Ex. Given $N_{xii} = \dfrac{15^{12} + 8^{12}}{15^4 + 8^4} = 241.98433601 = qQ.$

Find the 2-ic partitions of the large factor Q which are similar to those of N_{xii}.

The scheme below shows the values of (T, U) and (t, u) in the numerator and denominator of the fraction $(N_{xii} \div q)$ forming the first step of the work, and the result-form $(X^2 \pm DY^2)$ for each of the 2-ic forms worked out.

$-D$	$\dfrac{T^2+DU^2}{t^2+Du^2}$	$= X^2+DY^2$	$+D$	$\dfrac{T^2-DU^2}{t^2-Du^2}$	$= X^2-DY^2$
$\bar{1}$	$\dfrac{46529^2+14400^2}{15^2+4^2}$	$=3135^2+124^2$			
$\bar{2}$	$\dfrac{40321^2+2.19320^2}{13^2+2.6^2}$	$=1213^2+2.2046^2$	2	$\dfrac{69121^2-2.34680^2}{21^2-2.10^2}$	$= 8901^2-2.5890^2$*
					$= 3143^2-2.132^2$
					$= 2.3011^2-2879^2$
$\bar{3}$	$\dfrac{48577^2+3.2048^2}{7^2+3.8^2}$	$=1207^2+3.1672^2$	3	$\dfrac{54721^2-3.14400^2}{17^2-3.4^2}$	$= 4577^2-3.1924^2$*
					$= 3382^2-3.729^2$
$\bar{6}$	$\dfrac{11521^2+6.19320^2}{5^2+6.6^2}$	$=3125^2+6.114^2$	6	$\dfrac{97921^2-6.34680^2}{25^2-6.8^2}$	$= 17065^2-6.6848^2$*
					$= 3149^2-6.110^2$
					$= 3.2929^2-2.2819^2$

Thus nine 2-ic forms of this large number (Q) have been quite easily worked out by help of its multiple N_{xii}. To have done this otherwise by any direct process would have been exceedingly laborious.

* These 3 forms are *not Base-forms:* their Base-forms follow just below them.

CHAP. II. *Congruence-Tables.*

This Chapter deals with the formation and use of Congruence-Tables.

17. *Simple Congruences.* Let y_n, y'_n,—or more simply y, y',—be any solutions ($< p$ or p^κ) of the Simple Congruences—

$$y_n \text{ of } \phi(y^n-1) \equiv 0, \quad y'_n \text{ of } \phi(y^n+1) \equiv 0 \pmod{p \text{ or } p^\kappa} \quad \ldots (33),$$

where ϕ means "M.A.P.F. of"—(see Introduction, Art. 1).

[The symbol y or y' is generally used; the symbol y_n or y'_n—with the subscript n in Roman figures, (as y_{ii}, y_{iii}, &c.),—is used only when required to clearly specify the Index (n)].

Each such Congruence has the following *number* of independent (*i.e.* incongruous) solutions, all $<$ the modulus (p or p^κ), viz.

$$\left.\begin{array}{r}\text{Number of each of } y, y' \text{ is } \tau(n), \text{ when } n = \omega;\\ \text{and of } y \text{ is } 2\tau(n), \text{ when } n = \epsilon\end{array}\right\} \ldots\ldots\ldots\ldots (33a),$$

and has as "general solutions"

$$Y = \lambda p \text{ or } \lambda p^\kappa + y, \quad Y' = \lambda p \text{ or } \lambda p^\kappa + y' \quad (\text{for every } y, y') \ldots (33b).$$

The solutions $(y, y') < p$ or p^κ will be styled *Least Solutions.* For purposes of record in Tables it evidently suffices to record these Least Solutions.

In this Volume, and in its continuation in Vol. IV, are given the complete sets of $\tau(n)$, $2\tau(n)$ solutions (y, y') for the following Indexes n up to the high limits of p, p^κ shown below.

n =	2, 4, 8	16, 32, 64, 128, 256	3, 6, 12	24, 48, 96, 192
p and $p^\kappa \not> $	100,000	10,000	100,000	10,000

as more fully detailed in the "Tables of Contents" of Vols. I, IV.

[It will be seen—in the "Table of Contents" of Vol. I—that the solutions y'_{iii} of $(y^3+1)/(y+1) \equiv 0 \pmod{p}$ are actually given only up to the limit of $p \not> 10{,}000$: beyond that limit they have been *omitted* to save space in printing; they can be obtained *at sight* from the printed y_{iii}, since

$$y'_{\text{iii}} = y_{\text{iii}} + 1 \textit{ always} \ldots\ldots\ldots\ldots (33c).]$$

18. *Construction of Simple Congruence-Tables.* This consists of two very distinct Steps:—

STEP I. *Finding one root* (y_1) *of the Congruence.*

STEP II. *Finding the rest of the roots* $y_2, y_3, \dots y_r$ *from* y_1.

18a. *List of Congruence-Tables.* These Tables occupy pages 1 to 97 of this Volume, and pages 1 to 160 of Vol. IV. For complete Lists of these Tables see the *Tables of Contents* at the beginning of each Volume.

18b. *Computers (of Congruence-Tables).* These were initiated by (the late) Mr. Chas. E. Bickmore: the computing was done in part by the author, but for the most part by *Assistants named below under his direct superintendence; they were checked throughout by one Assistant.

19. STEP I. *Finding one root* (y_1) *of the Congruence.* Several Methods may be used according to the data available. These are:—

METHOD i. From known factorisations of n-ans (N_n, N'_n).
,, ii. From known 2-ic, 4-tic, &c., partitions ($t^2 \mp nu^2$, &c.).
,, iii. By extracting a (modular) square root.
,, iv. From roots of lower orders (α, β, &c., factors of n).
,, v. From primitive and other roots (g, &c.).

[Note that p^κ may be substituted for p throughout Arts. 19 to 19–v.]

19a. LEMMA. *Reduction of fractions.* As the value of y—as computed from the data—appears often in *form of a fraction*, (say $N \div D$), it is convenient to show here how to reduce this to an integer.

Given $y \equiv N \div D \pmod{p}$,

then $y = (mp + N) \div D =$ an *integer* .... (34),

by adding such a multiple (mp) of p as shall give ($mp + N$) *exactly divisible* by p.

[Here N, D stand for "numerator" and "denominator" of the fraction.]

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* The Misses A. Cole, E. Cooper, B. E. Haselden, and B. B. Haselden; checking by Miss C. M. Woodward.

19—i. METHOD i. From known factorisations of n-ans (N_n, N'_n).

Given $\quad N_n$ or N'_n = M.A.P.F. of $(x^n \mp y^n) = p_1 p_2 \dots p_r$.

Hence arise at once two solutions (y_n), viz.

$$y_n = \text{integer value of } y \div x, \text{ or of } x \div y \text{ (mod each prime } p_r) \quad \dots (35)$$

of *each* of the Congruences

$$\phi(y^n \mp 1) = 0 \text{ (mod each prime } p_r).$$

The simplest case is when $x = 1$; as then y_n is given (as an integer) at sight requiring no reduction.

19—ii. METHOD ii. From known 2-ic, 4-tic, &c., partitions ($t^2 \mp nu^2$), &c., of p.

For y_{ii}. $\;p = a^2 + b^2$ gives $y_{ii} \equiv \pm a/b$, or $\pm b/a$ (mod p) (36).

For y_{iv}. $\;p = a^2 + b^2 = c^2 + 2d^2 = e^2 - 2f^2 = 2f'^2 - e'^2$.

Find y_{ii} as before, and write $y_{ii} \pm 1 = \lambda$.

Then $y_{iv} \equiv$ any of $\pm \frac{d}{c}\lambda,\; \pm \frac{c}{2d}\lambda,\; \pm \frac{f}{e}\lambda,\; \pm \frac{e}{2f}\lambda,\; \pm \frac{f'}{e'}\lambda,\; \pm \frac{e'}{2f'}\lambda,$

$$\pm\left(\frac{d}{c} \pm \frac{f}{e}\right),\; \pm\tfrac{1}{2}\left(\frac{c}{d} \pm \frac{e}{f}\right),\; \pm\left(\frac{d}{c} \pm \frac{f'}{e'}\right),\; \pm\tfrac{1}{2}\left(\frac{c}{d} \pm \frac{e'}{f'}\right) \text{ (mod } p) \quad \dots\dots\dots (37).$$

For y_{iii}, y'_{iii}. $\;p = A^2 + 3B^2$ gives

y_{iii} or $y'_{iii} \equiv (A \sim B)/2B,\; 2B/(A \sim B),\; (B + A)/(B \sim A),\; (B \sim A)/(B + A)$ (mod p) ... (38).

[The + pair are each y_{iii}; the − pair are each y'_{iii}.]

For y_{viii}. One 8-tic partition or congruence along with one 2-ic or 4-tic partition or congruence.

$$\text{Either—}\quad \left.\begin{array}{ll} & (\alpha X^2)^4 + (\beta Y^2)^4 = p, \text{ or } \equiv 0 \text{ (mod } p) \\ \text{with} & \alpha t^2 \pm \beta u^2 \quad = p, \text{ or } \equiv 0 \text{ (mod } p) \end{array}\right\} \dots\dots\dots\dots\dots (39a).$$

$$\text{Or—}\quad \left.\begin{array}{ll} & (\alpha X^4)^2 + (\beta Y^4)^2 = p, \text{ or } \equiv 0 \text{ (mod } p) \\ \text{with} & \alpha t^4 \pm \beta u^4 \quad = p, \text{ or } \equiv 0 \text{ (mod } p) \end{array}\right\} \dots\dots\dots\dots\dots (39b).$$

Either pair of above data give $y_{viii} \equiv \pm \frac{Xu}{Yt}$, or $\pm \frac{Yt}{Xu}$ (mod p) ......(39).

[This Method is not of much use for finding y_{viii}, as the data required (39a, b) are difficult to form, and no Tables thereof exist.]

19—iii. Method iii. *By extracting a (modular) square root,* [*n even*].

Given $y_{(\frac{1}{2}n)}$; then $y_{(n)} \equiv \pm\sqrt{y_{(\frac{1}{2}n)}}$ (mod p).

Hence $y_n = \pm$rational value of $\sqrt{mp + y_{(\frac{1}{2}n)}}$ .... (40).

Here such a multiple (mp) of p is to be added to $y_{(\frac{1}{2}n)}$ as will make $(mp + y_{(\frac{1}{2}n)})$ a *perfect square.*

Hereby the roots y_{iv}, y_{viii}, y_{xvi}, &c.; y_{vi}, y_{xii}, y_{xxiv}, &c., may often be found from the known values of y_{ii}, y_{iv}, y_{viii}, &c.; y_{iii}, y_{vi}, y_{xii}, &c.

19—iv. Method iv. *By multiplication of roots* $y_{(\alpha)}$, $y_{(\beta)}$, [n composite].

Given the roots $y_{(\alpha)}$, $y_{(\beta)}$ of the congruences

$$\phi\,(y^{\alpha} \mp 1) \equiv 0, \quad \phi\,(y^{\beta} \mp 1) \equiv 0 \pmod p,$$

with $n = \alpha\beta$ [α prime to β], or $n =$ L.C.M. of α, β.

Then $y_{(n)} \equiv y_{(\alpha)} \cdot y_{(\beta)}$ (mod p) .... (41).

Thus—(in the present volume)—

$y_{\text{vi}} \equiv \pm y_{\text{ii}}\, y_{\text{iii}}$, or $\pm y_{\text{ii}}\, y'_{\text{iii}}$ (mod p)................................... (41a).

$y_{\text{xii}} \equiv \pm y_{\text{iv}}\, y_{\text{iii}}$, or $\pm y_{\text{iv}}\, y'_{\text{iii}}$, or $\pm y_{\text{iv}}\, y_{\text{vi}}$ (mod p)............... .. (41b).

$y_{\text{xxiv}} \equiv \pm y_{\text{viii}}\, y_{\text{iii}}$, or $\pm y_{\text{viii}}\, y'_{\text{iii}}$, or $\pm y_{\text{viii}}\, y_{\text{vi}}$, or $\pm y_{\text{viii}}\, y_{\text{xii}}$ (mod p) (41c).

[This Method is of great use in finding $y_{(n)}$ from its known components $y_{(\alpha)}$, $y_{(\beta)}$.]

19—v. Method v. *From primitive roots* (g), *&c.*

Let g be a primitive root of $p = \lambda . n + 1$.

To find a root $y_{(n)}$ of $\phi\,(y^n \mp 1) \equiv 0$ (mod p),

take $y, y' =$ Least Residues of g^{λ}, $g^{\frac{1}{2}\lambda}$ (mod p).

Hereby $y^n \equiv +1$ and $y'^n \equiv -1$ (mod p), as required ... (42).

Instead of a primitive root (g), any Base Y whose Haupt-exponent $\xi = \lambda n$ may be used: this gives, as before,

$y =$ Least Residue of Y^{λ} or $\text{Y}^{\frac{1}{2}\lambda}$ (mod p) .... (42a).

[The use of such a Base (Y) involves much less numerical computation (for finding y) than the use of a primitive root.]

20. Step II. *Computing the set of $\tau(n)$ or $2\tau(n)$ roots (y) from one known root (y_1).*

Let y_1 be the known root, and y_ρ any sought root ($< p$ or p^κ).

Then $$y_\rho = \text{Least Residue of } y_1^\rho \pmod{p \text{ or } p^\kappa},$$ [where ρ is *prime to* n] ...... (43),

and the whole set of $\tau(n)$ or $2\tau(n)$ incongruous roots is found by assigning to ρ each of $\tau(n)-1$ or $2\tau(n)-1$ values *all prime to* n.

21. *Properties of roots* (y). There are certain important properties of the *sums* and *products* of the *Least Roots* (y, y') of same order, which are useful, some for shortening the work of finding complete sets of roots, and some as Tests of the roots.

21a. *Sums of roots.* Let the roots (y, y') be arranged in *order of magnitude*, and *re-numbered*

$$y_1, y_2, y_3, \ldots y_r; \quad y_1', y_2', y_3', \ldots y_r'.$$

(1). When $n = \omega$, $r = \tau(n)$.

$$y_1 + y_r' = y_2 + y_{r-1}' = \ldots = y_{r-1} + y_2' = y_r + y_1' = p \text{ ...... (44).}$$

$\Sigma(y) \equiv -1$, $\Sigma(y') \equiv +1 \pmod p$, if n is prime .... (44a);

$\Sigma(y) \equiv \Sigma(y') \equiv 0 \pmod p$ if $n = \alpha^\kappa$, [α prime] ...... (44b);

$\Sigma(y) \equiv +1$, $\Sigma(y') \equiv -1 \pmod p$, if $n = \alpha\beta$, [α, β primes] (44c).

If $\quad \text{Y} = \Sigma(y)$, $[y < \tfrac{1}{2}p]$, $\text{Y}' = \Sigma(y')$, $[y' < \tfrac{1}{2}p]$,

then $\quad \text{Y} - \text{Y}' \equiv -1 \pmod p$, if n is prime .... (45a);

$\text{Y} - \text{Y}' \equiv 0 \pmod p$, if $n = \alpha^\kappa$ [α prime] ...... (45b);

$\text{Y} - \text{Y}' \equiv +1 \pmod p$, if $n = \alpha\beta$ [α, β primes]... (45c).

(2). When $n = \epsilon$, $r = 2\tau(n)$.

$$y_1 + y_r = y_2 + y_{r-1} = \ldots = y_{\frac{1}{2}r-1} + y_{\frac{1}{2}r+1} = p \text{ .... (46).}$$

[All the above Results hold for p^κ as well as for p.]

Properties (44), (46) are useful for reducing by one-half the labour of computing complete sets of roots: for when one-half of the full number (r) of roots has been computed, the rest can be obtained *by simple subtraction from* p.

Properties (44a, b, c), (45a, b, c), are useful as *Tests* of the correctness of the set of roots.

21b. *Products of roots.* Let ρ, σ be two numbers $(\rho \neq \sigma)$ *prime to n.*

Then y_ρ, y_σ; y'_ρ, y'_σ, are the *Least Residues* of y_1^ρ, y_1^σ; $y_1'^\rho, y_1'^\sigma$ (mod p).

Now take ρ, σ such that $\rho+\sigma = n$ or kn, $[k=\omega]$.

Then $$y_\rho y_\sigma \equiv +1,\ y'_\rho y'_\sigma \equiv +1 \pmod p, \text{ if } n=\omega \quad \text{......... (47a)};$$
$$y_\rho y_\sigma \equiv -1 \pmod p, \text{ if } n=\epsilon \quad \text{......... (47b)}.$$

These properties are useful as Tests of the correctness of a set of roots; [they apply to p^κ as well as to p].

21c. *Table of roots* (y_ρ). The Table below shows the indices (ρ) of the roots $y_\rho \equiv y^\rho \pmod p$ by which the complete sets of roots in this Volume may be computed by formula (43). Only one-half the full number—as shown by the bar (|) are really required. See the Note at foot of Art. 21a.

n	y	Roots.	Index (ρ) of $y_\rho \equiv y^\rho$ (mod p or p^κ).	Tests by (47a, b).
2	y_{ii}	2	1 | 3;	$y_1^2 \equiv -1$, &c.
4	y_{iv}	4	1, 3 | 5, 7;	$y_1 y_3 \equiv -1$, &c.
8	y_{viii}	8	1, 3, 5, 7 | 9, 11, 13, 15;	$y_1 y_7 \equiv -1$, &c.
16	y_{xvi}	16	1, 3, 5, 7, 9, 11, 13, 15 | 17, &c., .. 31;	$y_1 y_{15} \equiv -1$, &c.
2^κ	$y_{(n)}$	n	1, 3, 5, 7, 9, 11, 13, 15, &c., ... $(n-1)$ | $(n+1)$, &c., ... $(2n-1)$;	$y_1 y_{n-1} \equiv -1$, &c.
3	y_{iii}	2	1 | 2;	$y_1 y_2 \equiv +1$, &c.
3	y'_{iii}	2	1 | 5;	$y'_1 y'_5 \equiv +1$, &c.
6	y_{vi}	4	1, 5 | 7, 11;	$y_1 y_5 \equiv -1$, &c.
12	y_{xii}	8	1, 5, 7, 11 | 13, 17, 19, 23;	$y_1 y_{11} \equiv -1$, &c.
24	y_{xxiv}	16	1, 5, 7, 11, 13, 17, 19, 23 | 25, &c., ...;	$y_1 y_{23} \equiv -1$, &c.
3.2^κ	$y_{(n)}$	$\frac{2}{3}n$	1, 5, 7, 11, 13, 17, 19, 23, &c., ... $(n-1)$ | $(n+1)$, &c., ... $(2n-1)$;	$y_1 y_{n-1} \equiv -1$, &c.

22. *General Congruences.* Let (X, Y), (X′, Y′) be a pair of *associate roots* ($< p$ or p^κ) of the general form of Congruence
$$\phi(X^n - Y^n) \equiv 0, \text{ or } \phi(X'^n + Y'^n) \equiv 0 \ (\text{mod } p \text{ or } p^\kappa) \quad .. \ ... \ (48),$$
and let y, y' be roots ($< p, p^\kappa$) of the Simple Congruence
$$\phi(y^n - 1) \equiv 0,\ \phi(y'^n + 1) \equiv 0 \ (\text{mod } p \text{ or } p^\kappa);$$

then the roots Y, Y′ *associate with* a *given* X or X′ may be found at once from the roots y, y'—(supposed known)—from the simple relation

$$\mathrm{Y} \text{ or } \mathrm{Y}' = \text{Least Residue of } \mathrm{X}y,\ \mathrm{X}'y' \pmod{p \text{ or } p^\kappa} \;\ldots\ldots\; \text{(48a)},$$

and the *number* of such incongruous roots (Y or Y′) associate with the same X or X′ is evidently

$$\text{number} = \tau(n) \text{ if } n = \omega\,; \text{ or } = \tau(2n) = 2\tau(n) \text{ if } n = \epsilon \;\ldots\; \text{(48b)},$$

and the *general form* of such roots is

$$\mathrm{Y} = mp \text{ or } mp^\kappa + \mathrm{X}y, \quad \mathrm{Y}' = mp \text{ or } mp^\kappa + \mathrm{X}'y' \;\ldots\ldots\ldots\; \text{(48c)}.$$

When one root Y or Y′ has been found as above, the rest of the complete set of incongruous roots ($< p$ or p^κ)—for that same value of X, X′ kept constant throughout—may also be found—if desired—by the same Rules as used in Art. 20 for finding the complete set of y, y'.

[It would be obviously impracticable to form Tables of the complete sets of roots of such Congruences, as each value of X, X′ would require Tables of same size as when X = 1.]

[Some Congruence Tables of the general type

$$x_1^n + x_2^n \equiv 0 \pmod{p \not> 1{,}000}$$

will be found on pages 140–160 of Vol. IV. Explanation will be given in that Volume.]

23. *Restricted General Congruence.* Using a restricted form of General Congruence

$$\phi(x^n - y^n) \equiv 0, \quad \phi(x^n + y'^n) \equiv 0 \pmod{p \text{ or } p^\kappa} \;\ldots\ldots\ldots\; \text{(49)},$$

wherein y, y' are restricted to being roots of the *Simple Congruences,*

$$\phi(y^n - 1) \equiv 0, \quad \phi(y'^n + 1) \equiv 0 \pmod{p \text{ or } p^\kappa} \;\ldots\ldots\ldots\; \text{(49a)}.$$

then the whole set of roots x, x' *associate with* y, y' in (49) may be *taken at sight* from the Tables of the Simple Congruences (49a). For it is easily seen that—

When $n = \omega$:

If (y_ρ, y_σ), (y'_ρ, y'_σ) be *any two* roots of the above *Simple Congruences* (49a), then

$$\phi(y_\rho^n - y_\sigma^n) \equiv 0, \quad \phi(y_\rho'^n - y_\sigma'^n) \equiv 0 \pmod{p \text{ or } p^\kappa} \;\ldots\ldots\; \text{(50a)};$$

$$\phi(y_\rho^n + y_\sigma'^n) \equiv 0, \quad \phi(y_\rho'^n + y_\sigma^n) \equiv 0 \pmod{p \text{ or } p^\kappa} \;\ldots\ldots\; \text{(50b)}.$$

[Here y_ρ, y_σ may $\equiv 1$; y'_ρ, y'_σ may $= p-1$.]

When $n = \epsilon$ $= 2^\kappa\mu$ $[\mu = \omega]$
- If $y_{(n)}$ be *any* root of $\phi(y^n+1) \equiv 0$,
- and $y_{(m)}$ be *any* root of either $\phi(y^m \mp 1) \equiv 0$,
- when $m = \mu, 2\mu, 4\mu, \ldots, 2^{\kappa-1}\mu$,
- then $\phi(y^n_{(m)} + y^n_{(n)}) \equiv 0 \pmod{p \text{ or } p^\kappa}$ .... (50c).
- [Here $y_{(m)}$ may $= p-1$.]

The following Table shows the roots x, x' associated with y, y' in the restricted General Congruences (50a, b, c) occurring in this Volume.

n	Roots $y_{(n)}, y'_{(n)}$.	Associate roots $x_{(n)}, x'_{(n)}$.
2	Any y_{ii}	1, $(p-1)$
4	Any y_{iv}	1, Any y_{ii}, $(p-1)$
8	Any y_{viii}	1, Any $y_{\text{ii}}, y_{\text{iv}}$, $(p-1)$
16	Any y_{xvi}	1, Any $y_{\text{ii}}, y_{\text{iv}}, y_{\text{viii}}$, $(p-1)$
2^κ	Any $y_{(n)}$	1, Any $y_{\text{ii}}, y_{\text{iv}}, y_{\text{viii}}$, &c., ..., $y_{(\frac{1}{2}n)}$, $(p-1)$
3	Any y_{iii}	1, Any y_{iii}
	Any y'_{iii}	Any y'_{iii}, $(p-1)$
3	Any y'_{iii}	Any y_{iii}, $(p-1)$
6	Any y_{vi}	1, Any $y_{\text{iii}}, y'_{\text{iii}}$, $(p-1)$
12	Any y_{xii}	1, Any $y_{\text{iii}}, y'_{\text{iii}}, y_{\text{vi}}$, $(p-1)$
24	Any y_{xxiv}	1, Any $y_{\text{iii}}, y'_{\text{iii}}, y_{\text{vi}}, y_{\text{xii}}$, $(p-1)$
3.2^κ	Any $y_{(n)}$	1, Any $y_{\text{iii}}, y'_{\text{iii}}, y_{\text{vi}}, y_{\text{xii}}$, &c., ..., $y_{(\frac{1}{2}n)}$, $(p-1)$

24. *Use of Congruence-Tables.* The chief (practical) use of Congruence-Tables is for *giving divisors* (p, p^κ) of large n-ans (N_n, N'_n). It is in fact obvious that

p or p^κ is a divisor of N_n or N'_n if N_n or $N'_n \equiv 0 \pmod{p \text{ or } p^\kappa}$ ...(51);

this latter result being shown by the roots $y_{(n)}, y'_{(n)}$ in the Tables.

[The great extent of the Congruence-Tables in the several volumes of this Work (up to p and $p^\kappa \not> 100{,}000$ for all values of $n \not> 15$, and up to 10,000 for many values of $n \not> 50$) enables the factorisation of n-ans (N_n, N'_n) of those degrees to be carried to very high limits.

The roots (y, y') in these Tables are arranged *in order of magnitude:* this enables the search for values of y, y' giving rise to N_n or N' divisible by each prime (p) to be rapidly made.]

CHAP. III. *Factorisation of Binomials.*

25. *By Factor-Tables.* A certain amount of factorisation may be effected by use of the large **Factor-Tables*—(which give the *least prime-factor* (p) of all numbers not divisible by 2, 3, 5) —up to the limit 10017000. But, in the case of n-ans (N_n, N'_n) this use is very limited—except for the cases of $n = 2, 3$—as will be seen in the scheme below, which gives the upper limit of the root y_n in the *Simple n-ans* of this Volume possible with these Tables—(in the line marked F).

The limit of the larger root (x_n) in the *Non-Simple n-ans* $\phi(x^n \mp y^n)$ is about the same, or a little lower.

N_n, N'_n	N_{ii}, $\frac{1}{2}N_{ii}$	N_{iv}, $\frac{1}{2}N_{iv}$	N_{viii}, $\frac{1}{2}N_{viii}$	N_{xvi}, $\frac{1}{2}N_{xvi}$	N_{iii}, $\frac{1}{3}N_{iii}$	N_{vi}	N_{xii}	N_{xxiv}
(F). $y \not> $	3163, 4474	56, 66	7, 8	2, 2	3164, 5481	56	7	2
(C). $y \not> $	10^5, 141425	177, 211	13, 14	2, 3	10^5, 173205	177	13	2

25a. *By Congruence-Tables.* By the use of the *Simple Congruence-Tables* in this Volume and in Vol. IV, (described in Chap. II) the factorisation of *Simple n-ans* $\phi(y^n \mp 1)$ can *always* be carried up to the high limits of the root y shown in the line marked C in the scheme above.

But factorisation of these n-ans can also be carried to *very much higher limits* whenever factors p or $p^\kappa <$ the limit (p and $p^\kappa \not> 10^5$) of the Congruence-Tables exist. [This has been *very largely done* in the Factorisation-Tables in this Volume.]

Factorisation of the *Restricted Non-Simple n-ans* $\phi(x^n \mp y^n)$, wherein the root y is one of those *occurring in the Simple Congruence Tables*, can also be effected up to about the same limits of $x_{(n)}$ as those of $y_{(n)}$ in the Simple n-ans $\phi(y^n \mp 1)$ by the method described in Art. 23.

* *Factor-Table for the first ten millions*, by D. N. Lehmer, Washington 1909.

25b. *General n-ans.* For want of suitable Congruence-Tables the factorisation of General n-ans $\phi\,(X^n \mp Y^n)$ cannot in general be effected with certainty beyond the limits of the large Factor-Tables.

25c. *Factorisation-Tables, General Symbolism.* Before referring to the Factorisation-Tables in this Volume the reader should refer to the "Explanation" (page 97) of the symbolism and notation used throughout.

26. *Duan and Cuban Factorisations.* The author has had extensive Factorisation-Tables* prepared of the *Simple Duans* $N_{ii} = (y^2+1)$ and *Simple Cubans* N_{iii} and $N'_{iii} = (y^3 \mp 1) \div (y \mp 1)$ continuous up to the high limit of $y \not> 15000$. These are so extensive that they are not produced here.

26a. *Extensive List of Primes.*

A complete List of the roots (y) of all the *primes* (p) found in those Tables of the forms named below is given on pages 237–252, up to the limit $y \not> 15000$. The *number* of primes of each form within that limit of y is subjoined.

$p =$	N_{ii}	$\frac{1}{2}N_{ii}$	$\frac{1}{5}N_{ii}$	$\frac{1}{10}N_{ii}$	$\frac{1}{13}N_{ii}$	$\frac{1}{17}N_{ii}$
Number	1199	1288	794	763	251	185
$p =$	N_{iii}	$\frac{1}{3}N_{iii}$	$\frac{1}{7}N_{iii}$	$\frac{1}{13}N_{iii}$	$\frac{1}{19}N_{iii}$	$\frac{1}{21}N_{iii}$
Number	1998	1511	770	399	269	413

A few selected types of High Duan and Cuban Factorisation only (as stated in Art. 26a, b) are given, which are of some interest in themselves, and also show the power of the auxiliary Tables.

26b. *High Duan Factorisations.*

page 99; *High Irreducible Duans.*

$$N_{ii} = (x^{\alpha})^2 + (y^{\beta})^2 > 9.10^8; \quad [x \text{ and } y \not> 11].$$

pages 100–102; *High Associate Duans.*

$$N_1 = Y_1^2 + 1, \quad N = Y^4 + 4, \quad N_2 = Y_2^2 + 1.$$

$Y_1 = y^2 - y + 1$; $y = \eta^{\gamma}$ on p. 100, $= 2^{\alpha}.\eta^{\beta}$ on pp. 101, 102; $Y_2 = y^2 + y + 1$.

Note that $N_1, N, N_2 = YY', Y'Y''.\,Y''Y$; and $N_1NN_2 = (YY'Y'')^2$.

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* As yet in MS.

26c. *High Cuban Factorisations.*

Page 150, (top); *High Numbers*, $N = Y^3+1 > 10^{19}$.

Page 150, (foot); *High Irreducible Cubans*, $N_1, N_2 > 9.10^6$.

$$N_1 = (x^3-y^3)\div(x-y),\quad N_2 = (x^3+y^3)\div(x+y);\quad x = \xi^\alpha,\ y = \eta^\beta;\ [\xi, \eta \not> 11].$$

Page 151; *High Irreducible (Simple) Cubans*, $N_1, N_2 > 9.10^6$.

$$N_1 = (y^3-1)\div(y-1),\quad N_2 = (y^3+1)\div(y+1);\quad y = \xi^\alpha.\eta^\beta,\ [\xi, \eta \not> 11].$$

26d. *Pellian Factorisations.* The successive solutions (y_r, x_r) of the Pellian Equation

$$y_r^2 - D.x_r^2 = -1$$

lead to interesting High Factorisable Duans. For the above gives

$$N_{ii} = y_r^2+1 = D.x_r^2.$$

The interesting point about these is that the large factor of the Duan N_{ii} is a *perfect square* (x_r^2), an unusual feature.

Examples: pages 106–109.

27. *Factorisations of n-ans*, $(n \not< 4)$. Extensive Tables of these important Factorisations will be found on the pages shown in the scheme below: the factorisations have been carried to the high limits of x, y shown.

	Simple n-ans.			Non-simple n-ans.		
n	N_n	y-limit	Pages	N_n	Limits x , y	Pages
4	y^4+1	1000	113–115, 119	x^4+y^4	63 , 56	120–122, 125, 220
,,	$\frac{1}{2}(y^4+1)$	1001	116–119	$\frac{1}{2}(x^4+y^4)$	61 , 53	123–125, 220
,,				x^4+y^4	$2^{10}, 3^4, \ldots\ 11^2$	126
8	y^8+1	200	140	x^8+y^8	25 , 32	142
,,	$\frac{1}{2}(y^8+1)$	199	141	$\frac{1}{2}(x^8+y^8)$	25 , 19	143
16	$y^{16}+1$	32	143	$x^{16}+y^{16}$	11 , 4	143
6	$\frac{y^6+1}{y^2+1}$	1001	157–163	$\frac{x^6+y^6}{x^2+y^2}$	57 , 60	164–170, 220
				,,	10^2 , 11^2	170
				,,	3^9 , 2^{11}	171
12	$\frac{y^{12}+1}{y^4+1}$	200	215, 216	$\frac{x^{12}+y^{12}}{x^4+y^4}$	19 , 20	217
24	$\frac{y^{24}+1}{y^8+1}$	33	217			

27a. *Tables of Primes* (from above). Complete Tables are given on the pages named in the scheme below of the primes of the forms stated found in preparing the above Factorisation-Tables. The primes are of three kinds :—

1°. General n-an primes $\phi(x^n + y^n)$. Including all up to 10^8.

2°. High Simple n-an Primes $\phi(y^n + 1)$. Including all from 10^8 to 10^{10}.

3°. High prime factors in Simple n-ans. From 10^8 to 10^{10}.

	n-ans.			High Simple n-ans.		
$p =$	$x^4 + y^4$,	$\frac{1}{2}(x^4 + y^4)$,	$\frac{x^6 + y^6}{x^2 + y^2}$	$y^4 + 1$,	$\frac{1}{2}(y^4 + 1)$,	$\frac{y^6 + 1}{y^2 + 1}$
Pages	253, 255	254, 255	256, 257	255, 281	255, 281	257, 281
Number	240	172	360	34	29	48
Limits of p	$\not> 10^8$	$\not> 10^8$	$\not> 10^8$	10^8 to 10^{10}	10^8 to 10^{10}	10^8 to 10^{10}

	High Prime Factors in Simple n-ans.			
$p =$	Aurif. Fac. in N_{vi},	In $y^4 + 1$,	In $\frac{1}{2}(y^4 + 1)$,	In $\frac{y^6 + 1}{y^2 + 1}$
Pages	286	282, 283	284, 285	287, 288
Number	69	174	120	285
Limits of p	10^8 to 10^{10}	10^8 to 10^{10}	10^8 to 10^{10}	10^8 to 10^{10}

27b. *Authorities for High Primes.* The names—so far as known to the present author—of the original authorities for the High Primes tabulated in this Volume are indicated in the Tables by the capital initials placed on extreme left, or extreme right, of the numbers according to the following scheme.

B;	Bernoulli, Jean.	J;	Jenkins, M.
B;	Bickmore, Chas. E.	Ll;	Lelasseur.
Bd;	Biddle, D., Dr.	Lo;	Looff, Dr.
Cl;	Cullen, J., S.J.	Lu;	Lucas, Ed.
D;	Desmarest, E.	Pp;	Pépin, Th., Père.
E;	Euler, Leon.	R;	Reuschle, C. G., Dr.

The present author's name is not indicated in these Tables: but, it should be clear from Chap. II that, *all* primes $\not> 10^{10}$ occurring in Simple n-ans, *i.e.* in $\phi(y^n \mp 1)$, are shown by the new Congruence Tables now published: most of the other High Primes are also either due to him, or are now confirmed by him.

27c. *Computers (of Factorisations).* These were done in part by the author himself; but for the most by the Assistants named below, under his direct superintendence, usually in original by one, and checked by another.

Duans and Cubans: Misses B. E. Haselden and B. B. Haselden.

Simple n-ans: Misses A. Cole, E. Cooper, A. L. Woodward, and Mr. R. F. Woodward.

Non-simple n-ans: Misses E. Cooper and A. L. Woodward.

[The Simple 4-tans and 6-tans were worked (as far as $y = 100$) by the late Chas. E. Bickmore and the Author jointly.]

28. *Trinomial Quartans and Half-Quartans.* These require separate development. In both cases x', y' will be written instead of x, y in the usual Quartan and Half-Quartan forms (N_{iv} and $\frac{1}{2}N'_{iv}$).

Quartans. Write

$$x' = \tfrac{1}{2}(x-y),\ \ y' = \tfrac{1}{2}(x+y);\quad x = y'+x',\ \ y = y'-x' \ \ldots \ (51),$$

where x, y are both *odd;* x', y' are one *odd*, one *even*,

giving
$$N_{iv} = x'^4 + y'^4 = \tfrac{1}{8}(x^4 + 6x^2y^2 + y^4) \ldots\ldots \ (52)$$

$$= \left\{\left(\frac{x-y}{2}\right)^2\right\}^2 + \left\{\left(\frac{x+y}{2}\right)^2\right\}^2 \ldots\ldots \ (52a)$$

$$= (xy)^2 + 2\left(\frac{x^2-y^2}{4}\right)^2 \ldots\ldots \ (52b).$$

Half-Quartans. Write

$$x' = x-y,\ \ y' = x+y;\quad x = \tfrac{1}{2}(y'+x'),\ \ y = \tfrac{1}{2}(y'-x') \ \ldots \ (51')$$

where x', y' are both *odd;* x, y are one *odd*, one *even*,

giving
$$\tfrac{1}{2}N_{iv} = \tfrac{1}{2}(x'^4 + y'^4) = x^4 + 6x^2y^2 + y^4 \ldots\ldots \ (52')$$

$$= (x^2+y^2)^2 + (2xy)^2 \ \ldots\ldots \ (52a')$$

$$= (x^2-y^2)^2 + 2(2xy)^2 \ldots\ldots \ (52b').$$

The above give the trinomial forms of Quartans and Half-Quartans, and the (a, b), (c, d) 2-ic partitions of the same.

28a. *Simple Power-Forms of above.*

Taking $x' = y^n-1$, $y' = y^n+1$, gives $y'-x' = 2$, in above ...... (53),

1°. $y=\omega$ gives $H_n = \frac{1}{16}N_{iv} = \frac{1}{16}(x'^4+y'^4) = \frac{1}{8}(y^{4n}+6y^{2n}+1)$ ...... (53a),

2°. $y=\epsilon$ gives $H_n = \frac{1}{2}N_{iv} = \frac{1}{2}(x'^4+y'^4) = (y^{4n}+6y^{2n}+1)$ ...... (53b).

Ex. See page 133. The Table (at foot) gives the factorisation of H_n for $y = 6, 10, 12$, when $n = 1, 2, 3$.

Note that $y=6$ gives $H_n = 6^{4n}+6^{2n+1}+1$ .... (54a).

Again, taking $x' = 2^{\frac{1}{2}n}-1$, $y' = 2^{\frac{1}{2}n}+1$, giving $x = 2^{\frac{1}{2}n}$, $\frac{1}{2}(y'-x') = 1$ in above, gives $H_n = \frac{1}{2}(x'^4+y'^4) = 2^{2n}+6.2^n+1$ .... (54b).

Here H_n are *real* Half-Quartans when $n=\epsilon$,

H_n may be styled *Quasi Half-Quartans* when $n=\omega$, as they have the same linear and 2-ic forms as when $n=\epsilon$.

When $n=\omega$,

$H_n = (2^n-1)^2+(2^{\frac{1}{2}(n+3)})^2 = (2^n+1)^2+2(2^{\frac{1}{2}(n+1)})^2 = (2^n+3)^2-2.2^2$ ... (54c).

When $n=\epsilon$,

$H_n = (2^n+1)^2+(2^{\frac{1}{2}n+1})^2 = (2^n-1)^2+2(2^{\frac{1}{2}n+1})^2 = (2^n+3)^2-2.2^2$ ... (54d).

Ex. (Page 130.) The top Table gives the factorisation of H_n up to $n = 17$. The Table at foot gives all the prime divisors (p) of H_n up to $p = 1{,}361$, with the exponents (n) possible to each divisor (p). The middle Tables are auxiliary Tables for finding the exponents (n) for each p.

29. *Trinomial Sextans.* Writing x', y' instead of x, y in the usual Sextan formula, and using one, or other, of the substitutions (1°, 2°)—

1°. $x' = \frac{1}{2}(x\sim y)$, $y' = \frac{1}{2}(x+y)$, giving $x = y'+x'$, $y = y'-x'$ ... (55a), where x, y are both *odd*, and x', y' are one *odd*, one *even*.

2°. $x' = x\sim y$, $y' = x+y$, giving $x = \frac{1}{2}(y'+x')$, $y = \frac{1}{2}(y'-x')$... (55b), where x', y' are both *odd*; and x, y are one *odd*, one *even*.

Then $H = \mu.N_{vi} = \mu(x'^6+y'^6)\div(x'^2+y'^2) = \mu(x^4+14x^2y^2+y^4)$ ... (56), where $\mu = 1$ when x', y' are both *odd*; $\mu = \frac{1}{16}$ when x, y are both *odd* .... (56′),

and H has the three 2-ic forms

$$H = \mu\left\{(x^2\sim y^2)^2+(4xy)^2\right\} \quad (57a)$$

$$= \mu\left\{(x^2+y^2)^2+3(2xy)^2\right\} \quad (57b)$$

$$= \mu\left\{(x^2+7y^2)^2-3(4y^2)^2\right\} \quad (57c)$$

$$= \mu\left\{7x^2+y^2)^2-3(4x^2)^2\right\} \quad (57d).$$

29a. *Trinomial Sextan Power-Forms.* In the above write

$$x' = \lambda(y^n - 1),\ \ y' = \lambda(y^n + 1),\ \text{giving}\ \ x = \tfrac{1}{2}(x' + y') = \lambda y^n,\ \ \tfrac{1}{2}(y' - x') = \lambda \quad \ldots\ldots\ldots \ (58).$$

Then $\quad \mathrm{H}_n = \mu.\mathrm{N}_{\mathrm{vi}} = \mu(x'^6 + y'^6) \div (x'^2 + y'^2) = \mu(y^{4n} + 14y^{2n} + 1) \ \ldots \quad (59),$

where $\quad \lambda = 1,\ \mu = 1$, when $y = \epsilon$; $\ \lambda = \frac{1}{2},\ \mu = \frac{1}{16}$, when $y = \omega \ \ldots$ (59a),

with the three 2-ic forms

$$\mathrm{H}_n = \mu\left\{(y^{2n} - 1)^2 + (4y^n)^2\right\} = \mu\left\{(y^{2n} + 1)^2 + 3(2y^n)^2\right\} = \mu\,(y^{2n} + 7)^2 - 3.4^2\} \quad \ldots\ldots\ldots \ (60).$$

Ex. (Page 205.) The upper Table gives the factorisation of H_n when $y = 2, 3, 5, 6, 7, 10, 14$ for various values of n. The lower Table gives the divisors (p) of H_n for $y = 2, 3$ up to $p = 601$, and the exponents (n) possible to each divisor (p).

Note that, when $y = 14$,

$$\mathrm{H}_n = 14^{4n} + 14^{2n+1} + 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (61).$$

Chap. IV. *Chains.*

30. *Chains.* A series of similarly formed composite numbers (**N**), viz.

$$N_1 = L_1 M_1, \quad N_2 = L_2 M_2, \quad \ldots, \quad N_r = L_r M_r \quad \ldots\ldots\ldots \quad (62)$$

is said to be a *Chain* when

$$M_{r-1} = L_r, \quad M_r = L_{r+1}, \quad \ldots, \quad \text{for all values of } r \quad \ldots\ldots \quad (63),$$

and N_r, N_{r+1}, &c., are said to be *Links* in the *Chain*; and L_r, M_r are termed *Link-Factors*.

The necessary and sufficient conditions for a Chain of n-ans (N_r) are—

x_r/y_r and x_{r+1}/y_{r+1} should be a pair of *associate roots* of the Congruence

$$\phi(x^n \mp y^n) \equiv 0 \ (\text{mod } M_r = L_{r+1}) \text{ at each step} \quad \ldots\ldots\ldots \quad (64).$$

30a. In the case of Simple n-ans N_n—[wherein $x = 1$]—which depend on Simple Congruences, the property $y_\rho y_\sigma = \pm 1$ of Art. 21b, show that the roots y_r, y_{r+1} of *successive* Links N_r, N_{r+1}, or N'_r, N'_{r+1}, may be selected by the relation

$$y_r y_{r+1} \equiv +1 \ (\text{mod } M_r = L_{r+1}), \text{ for } N_r, N_{r+1}, \quad [n = \omega] \ \ldots \ (64a),$$

$$y'_r y'_{r+1} \equiv +1 \ (\text{mod } M'_r = L'_{r+1}), \text{ for } N'_r, N'_{r+1}, \quad [n = \omega] \ \ldots \ (64b],$$

$$y_r y_{r+1} \equiv -1 \ (\text{mod } M_r = L_{r+1}), \text{ for } N_r, N_{r+1}, \quad [n = \epsilon] \ \ldots \ (64c);$$

whilst $y_r y'_{r+1} \equiv -1 \ (\text{mod } M_r = L'_{r+1})$ or $y'_r y_{r+1} \equiv -1 \ (\text{mod } M'_r \equiv L_{r+1})$ give Links out of the two series N_r, N'_r alternately .... (64d).

The n-ans considered in this Volume afford many examples, as will appear later.

31. *Properties of Chains.* The most salient properties are

$$M_r . L_{r+1} = \square, \text{ for all values of } r \quad \ldots\ldots\ldots \quad (65a),$$

$$(N_2 N_4 N_6 \ldots N_{2r}) \div (N_1 N_3 N_5 \ldots N_{2r-1}) = M_{2r} \div L_1 \quad \ldots\ldots\ldots \quad (65b).$$

$$N_1 N_2 N_3 \ldots N_r = L_1 (L_2 L_3 \ldots L_r)^2 M_r = L_1 (M_1 M_2 \ldots M_{r-1})^2 M_r \ \ldots \ (65c),$$

32. *Simple Chains.* Some of the most interesting Chains are those in which

$$x_r, \text{ or } y_r, \text{ or } x_r - y_r = x_0, \text{ or } y_0, \text{ or } x_0 - y_0, \textit{ a constant} \ldots\ldots \quad (66).$$

These may be styled x-Chains, y-Chains, $(x-y)$-Chains respectively.

In the case of Simple n-ans, wherein $x_0 = 1$, the formulæ (64a, b) give

$$\text{M}_{r-1} = y_{r-1}y_r - 1 = \text{L}_r, \quad \text{M}_r = y_r y_{r+1} - 1 = \text{L}_{r+1}, \; [n = \omega] \ldots (67a),$$

$$\text{M}_{r-1} = y'_{r-1}y'_r - 1 = \text{L}_r, \quad \text{M}_r = y_r y_{r+1} - 1 = \text{L}_{r+1}, \; [n = \omega] \ldots (67b),$$

$$\text{M}_{r-1} = y_{r-1}y_r + 1 = \text{L}_r, \quad \text{M}_r = y_r y_{r+1} + 1 = \text{L}_{r+1}, \; [n = \epsilon] \ldots (67c).$$

33a. *Duan and Cuban Chains.* Associate Duan and Cuban Chains may be formed as follows:—

Simple Duan Chains.

$$\text{N}_r = \text{Y}_r^2 + 1 = \text{Y}_r'^2 + 1 = \text{L}_r . \text{M}_r \ldots\ldots\ldots\ldots\ldots\ldots \quad (68).$$

$$y_{r+1} = y_r + 1, \quad y'_{r+1} = y'_r + 1\,; \quad y'_r - y_r = 1, \quad y_{r+1} = y'_r \ldots (68a).$$

$$\text{Y}_r = y_r^2 + y_r + 1 = y_r'^2 - y'_r + 1 = \text{Y}'_r \ldots\ldots\ldots\ldots\ldots\ldots \quad (68b).$$

Hence $\text{N}_r = \text{Y}_r^2 + 1 = (y_r^2 + y_r + 1)^2 + 1 = (y_r^2 + 1)(y_r^2 + 2y_r + 2) = \text{L}_r . \text{M}_r$ (68c),

whence $\text{M}_r = (y_r + 1)^2 + 1 = y_{r+1}^2 + 1 = \text{L}_{r+1}$ .... (68d);

showing that N_1, N_2, N_3, &c., is a *Chain-Series.*

33b. *Simple Cuban Chains.*

$$\text{N}_r = (\text{Y}_r^3 - 1) \div (\text{Y}_r - 1) = \text{Y}'_r \cdot \text{Y}''_r \ldots\ldots\ldots\ldots\ldots\ldots \quad (69).$$

$$y'_r - 1 = y = y''_r + 1\,; \quad y_{r+1} = y_r + 1, \quad y'_{r+1} = y'_r + 1 \ldots\ldots (69a).$$

$$\text{Y}_r = y_r^2\,; \quad \text{Y}'_r = y_r^2 + y_r + 1 = y_r'^2 - y'_r + 1\,; \quad \text{Y}''_r = y_r^2 - y_r + 1 = y_r'^2 + y'_r + 1 \ldots\ldots\ldots (69b).$$

Here $$\text{N}_r = \frac{y_r^3 - 1}{y_r - 1} \cdot \frac{y_r^3 + 1}{y_r + 1} = \text{Y}'_r . \text{Y}''_r \ldots\ldots\ldots\ldots\ldots\ldots \quad (69c).$$

Here $$\text{Y}''_r = y_r'^2 + y'_r + 1 = y_{r+1}^2 + y_r + 1 = \text{Y}'_{r+1} \ldots\ldots\ldots\ldots (69d),$$

showing that N_1, N_2, N_3, &c., is a Chain-Series.

33c. *Ex.* Examples of the above chains are given in the Tables.

Duan Chain at top of page 98; 13 Links, (N_r).

Cuban Chain at top of page 149; 18 Links, (N_r).

These Chains start of course from $y_1 = 1$. A few examples only are given in high numbers (N_r of 19 figures) to illustrate the power of the Congruence-Tables used in the factorisations.

34a. *Simple Quartan Chains.*

$$\mathrm{N}_r = 1^4 + y_r^4 = \mathrm{L}_r \mathrm{M}_r \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (70).$$

$$\left.\begin{array}{lll} y_{r+1} = y_0^2 . y_r - y_{r-1}, & \mathrm{L}_0 = 1, & \mathrm{M}_0 = 1 + y_0^4 = \mathrm{L}_1 \\ \mathrm{M}_{r-1} = 1 + y_{r-1} y_r = \mathrm{L}_r, & \mathrm{M}_r = 1 + y_r y_{r+1} = \mathrm{L}_{r+1} & \end{array}\right\} \ldots (70\mathrm{a}).$$

Examples. The upper Table on page 134 shows the elements r, x, y and the larger Chain-Factor (M_r) of the successive Links (N_0, N_1, N_2, &c.) of the Chains given by taking $y_0 = 2, 3, 4, \ldots, 10$ factorised up to very high numbers.

34b. *Simple Sextan Chains.*

$$\mathrm{N}_r = 1^4 - 1^2 . y_r^2 + y_r^4 = \mathrm{L}_r \mathrm{M}_r \quad\ldots\ldots\ldots\ldots\ldots\ldots \quad (71).$$

$$\left.\begin{array}{lll} y_{r+1} = y_0^2 . y_r - y_{r-1}, & \mathrm{L}_0 = 1, & \mathrm{M}_0 = 1 + y_0^4 = \mathrm{L}_1 \\ \mathrm{M}_{r-1} = 1 + y_{r-1} y_r = \mathrm{L}_r, & \mathrm{M}_r = 1 + y_r y_{r+1} = \mathrm{L}_{r+1} & \end{array}\right\} \ldots (71\mathrm{a}).$$

Examples. The Table on page 210 shows the elements r, x, y and the larger Chain-Factor (M_r) of the successive Links (N_0, N_1, N_2, &c.) of the Chains given by taking $y_0 = 2, 3, 4, \ldots, 11$ factorised up to very high numbers.

[Observe the formal identity of the 4-tan and 6-tan formulæ (Art. 34a, b).]

35. *Pellian Chains.* The *successive* solutions (τ_r', v_r'), (τ_r, v_r) of the *Associated* Pellian Equations

$$\tau'^2 - \mathrm{D} . v'^2 = -1, \quad \tau^2 - \mathrm{D} . v^2 = +1, \quad \text{with} \quad \mathrm{D} = \alpha^2 + \beta^2 \ \ldots \ (72),$$

give rise to *Duan Chains* $\mathrm{N}_r = v_r^2 + v_1'^2$ for every value of D when τ', v' exist.

Here (τ_1', v_1'), (τ_1, v_1) may be represented by

$$\tau_1' = \tfrac{1}{2}(y^{\frac{1}{2}} - y^{-\frac{1}{2}}), \ v_1' = \frac{1}{2\sqrt{\mathrm{D}}}(y^{\frac{1}{2}} + y^{-\frac{1}{2}}); \ \tau_1 = \tfrac{1}{2}(y + y^{-1}), \ v_1 = \frac{1}{2\sqrt{\mathrm{D}}}(y - y^{-1}) \quad\ldots\ldots\ldots (73),$$

for these satisfy the two Pellian Equations, and lead to all the well known mutual relations: they lead to

$$\tau_2' = \tfrac{1}{2}(y^{\frac{3}{2}} - y^{-\frac{3}{2}}), \ v_2' = \frac{1}{2\sqrt{\mathrm{D}}}(y^{\frac{3}{2}} + y^{-\frac{3}{2}}); \ \tau_2 = \tfrac{1}{2}(y^2 + y^{-2}), \ v_2 = \frac{1}{2\sqrt{\mathrm{D}}}(y^2 - y^{-2}) \quad\ldots\ldots\ldots (73\mathrm{a}),$$

and similarly for the r-th elements.

$$\left.\begin{array}{ll} \tau_r' = \tfrac{1}{2}(y^{r-\frac{1}{2}} - y^{-r+\frac{1}{2}}), & v_r' = \dfrac{1}{2\sqrt{\mathrm{D}}}(y^{r-\frac{1}{2}} + y^{-r+\frac{1}{2}}) \\ \tau_r = \tfrac{1}{2}(y^r + y^{-r}), & v_r = \dfrac{1}{2\sqrt{\mathrm{D}}}(y^r - y^{-r}) \end{array}\right\} \ldots\ldots (73\mathrm{b}).$$

Hence $$v_r^2 + v_r'^2 = (y^{2r} + y^{-2r} + y + y^{-1}) \div 4D,$$

and $$v'_r . v'_{r+1} = (y^{r-\frac{1}{2}} + y^{-r+\frac{1}{2}})(y^{r+\frac{1}{2}} + y^{-r-\frac{1}{2}}) \div 4D$$
$$= (y^{2r} + y^{-2r} + y + y^{-1}) \div 4D\,;$$

whence $$N_r = v_r^2 + v_1'^2 = v'_r . v'_{r+1} \quad \text{(74a).}$$

Similarly $$N_{r+1} = v_{r+1}^2 + v_1'^2 = v'_{r+1} . v'_{r+2} \quad \text{(74b).}$$

Hence the series $N_1, N_2, N_3 \ldots N_r$ is a *Duan Chain.*

The following Table gives several successive values of v', v given by D = 2, 5, 10, 13, 17 to serve as numerical examples of these Chains.

D	r =	1	2	3	4	5	6	7	8	9
2	v'_r =	1	5	29	169	985	5741	33461	195025	1136689
	v_r =	2	12	70	408	2378	13860	80782	470832	&c.
5	v'_r =	1	17	305	5473	98209	1762289	31622993		
	v_r =	4	72	1292	23184	416020	7465176	&c.		
10	v'_r =	1	37	1405	53353	2026009				
	v_r =	6	228	8658	328776	&c.				
13	v'_r =	5	6485	8417525	&c.					
	v_r =	180	233640	&c.	&c.					
17	v'_r =	1	65	4289	283009					
	v_r =	8	528	34840	2298912					

35a. *High Pellian Chains.* The production of Chains from Pellian Equations leading to very high completely factorisable numbers has been developed in great detail in the author's Memoir quoted* below. One of the most interesting cases only is here given.

Take the series of Pellians

$$Y_r^2 - D_r . X_r^2 = -1, \quad \text{with} \quad D_{r+1} = X_r\,;$$

and take $$N_r = Y_r^2 + 1 = D_r . X_r^2 = X_{r-1} . X_r^2.$$

Hereby $$N_{r+1} = X_r . X_{r+1}^2, \quad N_{r+2} = X_{r+1} . X_{r+2}^2, \quad \ldots .$$

Hence N_r, N_{r+1}, N_{r+2}, ... form a *modified Chain;* (successive Links N_r, N_{r+1}, containing the common factor X_r).

* *High Pellian Factorisations*, published in *Messenger of Mathes.*, Vol. 35, 1906.

Example. The Pellian series $\eta_r^2 - D\eta_r^2 = -1$, with $D = \eta_r^2 + 1$, has $\xi_r = 1$, and gives

$$\eta_2 = 4\eta_1^3 + 3,\quad \xi_2 = 4\eta_1^2 + 1.$$

Now take

$$D_r = y_r^2 + 1,\quad D_{r+1} = (2y_r)^2 + 1 = X_r,\quad y_{r+1} = 2y_r,\quad \text{and}\quad Y_r = 4y_r^3 + 3y_r.$$

The new series of $N_r = Y_r^2 + 1$ is a *modified Chain* of above kind.

Ex. The Tables on pages 111, 112 give the elements (r, η, α, D_r) of numerous Chains of this sort, wherein $y_r = 2^r . \eta^\alpha$, $\eta = 3, 5, 7, 11$, completely factorisable up to very high limits.

36. *Circular Chains.* If a Chain of r Links $(N_1 \dots N_r)$, viz.

$$N_1 = L_1 M_1,\ N_2 = L_2 M_2,\ N_3 = L_3 M_3,\ \dots,\ N_r = L_r M_r \ \dots \quad (75),$$

be such that

$$M_1 = L_2,\ M_2 = L_3,\ \dots,\ M_{r-1} = L_r,\ \text{and } \textit{finally}\ M_r = L_1 \dots \quad (75a),$$

it is evident that the Chain—if continued—repeats itself in periods of r Links, and is such that, if the r Links be placed at equal distances $2\pi R \div r$ round a circle of radius R, the Chain will be seen to be endless—(since $M_r = L_1$), and the Links might be re-numbered, any chosen Link being marked N_1.

The salient property of such a Chain is

$$N_1 N_2 N_3 \dots N_r = (L_1 L_2 L_3 \dots L_r)^2 = (M_1 M_2 M_3 \dots M_r)^2 \quad \dots (75b),$$

and it is evident that there must be *at least* 3 *Links* in such a Chain; (for a Chain of only 2 Links involves $N_1 = N_2$).

36a. *Circular Chains of 2-ic Forms.* Circular Chains in which all the Links (N_r) are of same binomial 2-ic form

$$N_r = T_r^2 \mp D . U_r^2 = L_r . M_r$$

are easily formed.

Take r different numbers $M_1, M_2, \dots M_r$ of that 2-ic form, say $M_r = t_r^2 \mp Du_r^2$, and write $M_1 = L_2$, $M_2 = L_3$, ... $M_{r-1} = L_r$, and finally $M_r = L_1$, and form the products $N_1 = L_1 M_1$, $N_2 = L_2 M_2$, &c. Hereby all these products N_ρ are expressible in that same form *in two different ways*, say

$$N_\rho = T_\rho^2 \mp D . U_\rho^2 = T_\rho'^2 \mp D . U_\rho'^2 = N_\rho' \ \dots\dots\dots\dots \quad (76)$$

and it is evident—from the mode of formation—that the two sets of numbers

$$(N_1, N_2, \dots, N_r),\quad (N_1', N_2', \dots, N_r')$$

form two equal and similar Circular Chains of the 2-ic forms chosen, and the continued product of the r Links of each Chain

$$= \text{the same square above stated.}$$

Hence it is seen that Circular Chains of 2-ic Forms are always *Dimorph*.

36b. *Circular Duan Chains*, $(N_\rho = x_\rho^2 + y_\rho^2)$. These are easily formed by the above process.

Ex. The smallest of such Chains (with all Link-factors > 1) is given by $N_1 = 5.13$, $N_2 = 13.17$, $N_3 = 17.5$; and the Twin Chains are

$$N_1 = 1^2+8^2, \quad N_2 = 5^2+14^2, \quad N_3 = 7^2+6^2;$$
$$N_1' = 7^2+4^2, \quad N_2' = 11^2+10^2, \quad N_3' = 9^2+2^2.$$

36c. *Circular Cuban Chains.* These are most easily formed by taking the Link-factors (M_ρ) of form $M_\rho = A_\rho^2 + 3B_\rho^2 = L_{\rho+1}$, (all different), viz.

$$M_1 = A_1^2 + 3B_1^2 = L_2, \quad M_2 = A_2^2 + 3B_2^2 = L_3, \quad \&c.,$$

and, finally, $\qquad M_r = A_r^2 + 3B_r^2 = L_1.$

Then, by conformal multiplication, every $N_\rho = L_\rho M_\rho$ can be formed *in two ways* in the 2-ic form

$$N_\rho = \mathbf{A}_\rho^2 + 3\mathbf{B}_\rho^2 = \mathbf{A}_\rho'^2 + 3B_\rho'^2 = N_\rho';$$

and, each of these 2-ic forms can now be transformed into its equivalent Cuban form

$$N_\rho = (x_\rho^3 - y_\rho^3) \div (x_\rho - y_\rho), \ (x_\rho'^3 - y_\rho'^3) \div (x_\rho' - y_\rho') = N_\rho',$$

by the formulæ of Art. 13c.

Ex. The smallest Circular Cuban Chain — (with all Link-factors L_ρ, $M_\rho > 1$) — is given by $N_1 = 7.13$, $N_2 = 13.19$, $N_3 = 19.7$, and the Twin Chains are

$$N_1 = 8^2 + 3.3^2, \quad N_2 = 2^2 + 3.9^2, \quad N_3 = 11^2 + 3.2^2;$$
$$N_1' = 4^2 + 3.5^2, \quad N_2' = 10^2 + 3.7^2, \quad N_3' = 5^2 + 3.6^2;$$

whence $\quad N_1 = \dfrac{6^3-5^3}{6-5}, \quad N_2 = \dfrac{11^3-7^3}{11-7}, \quad N_3 = \dfrac{9^3-4^3}{9-4};$

$$N_1' = \frac{9^3-1}{9-1}, \quad N_2' = \frac{14^3-3^3}{14-3}, \quad N_3' = \frac{11^3-1^3}{11-1}.$$

36d. *Circular Duo-Cuban Chains.* These have every Link N_ρ at once both Duan and Cuban. These are formed by taking every Link-factor $M_\rho = L_{\rho+1}$ of both forms $(t_\rho^2 + u_\rho^2)$, $(A_\rho^2 + 3B_\rho^2)$, and proceeding as above.

Ex. The smallest Circular Duo-Cuban Chain—(with all Link-factors L_ρ, $M_\rho > 1$)—is given by

$$N_1 = 13.37 = N_1', \quad N_2 = 37.61 = N_2', \quad N_3 = 61.13 = N_3'.$$

36e. *Circular Quartan Chains.* No general Rule has been found except in the case of the smallest, *i.e.* the Three-Link Chain ($r = 3$). In this case

Take $\mathbf{A} = A^2 \sim 3B^2$, $\mathbf{B} = 2AB$; [A, B one *odd*, one *even*] ... (77).

Take $N_1 = x^4 + y_1^4$, $N_2 = x^4 + y_2^4$, $N_3 = x^4 + y_3^4$,

[same x throughout]... (77a),

where $x = A^2 + 3B^2$, $y_1 = \mathbf{A} + \mathbf{B}$, $y_2 = \mathbf{A} \sim \mathbf{B}$, $y_3 = 2\mathbf{AB}$ ... (77b).

Then N_1, N_2, N_3 will be *found to be a Circular Quartan Chain, with

$$N_1 = L_2 L_3, \; N_2 = L_3 L_1, \; N_3 = L_1 L_2; \; N_1 N_2 N_3 = (L_1 L_2 L_3)^2 \quad (77c),$$

where $L_1 = y_2^2 + y_3^2$, $L_2 = y_3^2 + y_1^2$, $L_3 = y_1^2 + y_2^2$ .... (77d).

Ex. The Table on page 129 gives the data (A, B), and the elements (x, y_1, y_2, y_3) of a number of these Three-Link Chains, and also the resulting Link-factors (L_1, L_2, L_3). This Chain is here styled a *Nexus*.

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* See the Author's Memoir on Quartans, &c., in *Messenger*, Vol. 38, 1908, Arts. 33–33b.

Chap. V. *Aurifeuillians.*

37. *Aurifeuillians, Ant-Aurifeuillians.** Every n-an (N_n, N'_n), whose degree $n = 4i+1, 2, 3$, can be expressed *algebraically* in one or other of the IMPURE *2-ic forms* $(P^2 \mp nxy . K^2)$, the connecting sign $(\mp)$ depending on the linear form of $n = 4i+1, 2, 3$.

Under the condition—styled *Aurifeuillian condition*—

$$nxy = \square = (n\xi\eta)^2, \quad [\xi \text{ prime to } \eta] \text{ .... } (78),$$

the above *impure 2-ic form* becomes either a *difference of squares* or a *sum of squares*, as below—

N	N	n	N	n
$N_n = \phi(x^n - y^n)$	$= P^2 - Q^2$, if	$n = 4i+1$	$= P'^2 + Q'^2$, if	$n = 4i+3$ ... (79a),
$N'_n = \phi(x^n + y^n)$	$= P^2 - Q^2$, if	$n = 4i+3$	$= P'^2 + Q'^2$, if	$n = 4i+1$ ... (79b),
$N_n = \phi(x^n + y^n)$	$= P^2 - Q^2$, if	$n = 4i+2$	$= P'^2 + Q'^2$, if	$n = 4i+2$ ... (79c).

The functions N_n, N'_n thus obtained are styled †*Aurifeuillians* of order n when of form $(P^2 - Q^2)$, or †*Ant-Aurifeuillians* of order n when of form $(P'^2 + Q'^2)$. The 2-ic parts (P, Q), (P', Q') bear the following relation—

If $\quad P = f_1(x, y) \quad$ and $\quad Q = f_2(x, y)$,

then $\quad P' = f_1(x, -y) \quad$ and $\quad Q' = f_2(x, -y)$ .... (80).

The algebraic formulæ for (P, Q), (P', Q') are quite simple

* Much of this Chapter is contained in the Author's Papers—

1°. On Aurifeuillians;

2°. High Quartans, Nos. (2), (3); High Sextans, Nos. (2), (3);
the full Titles, &c., of which are given in the footnote to Art. 2.

† So named by the present Author after the late M. Aurifeuille of Toulouse, who was the first to employ them in the factorisation of large numbers; see the Memoir *Sur la Série recurrente de Fermat*, by Ed. Lucas, Rome, 1879.

when n is *small*, (as required in this volume), but become complicated when n is not quite small.

Results (79a–c) involve that—

No Aurifeuillians or Ant-Aurifeuillians exist of order $n = 4i$... (79d).

37a. *Aurifeuillians.* These functions, being a *difference of squares*, are *algebraically* resolvable into two co-factors, say L, M, so that—

$$N_n \text{ or } N'_n = P^2 - Q^2 = L.M, \text{ where } L = P - Q,\ M = P + Q \ldots (81).$$

This property is of great use in factorisation of large n-ans: many examples are given in the Tables of this Work.

The co-factors L, M are styled *Aurifeuillian Factors:* they have the following properties—

L is prime to M.. (82a).

L, M are *algebraically* expressible in the same *pure* 2-*ic forms* as the original N_n, N'_n, of which they are co-factors .... (82b).

If $M = f(\xi, \eta)$, then $L = f(\xi, -\eta)$.................................. (82c).

Result (82b) shows that

Every Aurifeuillian is always expressible *algebraically* in two ways in the same *pure* 2-*ic forms* .... (82d).

[In arithmetical work the twin factors L, M are separated by a colon (:), which may be looked on as a special sign of multiplication; see the "Explanation" of Tables on page 97, para. 6.]

Functions N_n, N'_n of degree n are susceptible of being Aurifeuillians or Ant-Aurifeuillians of one or more orders determined by the form of n, as shown below. [Here $\alpha, \beta, \gamma, \ldots$, are different *odd* primes.]

$n =$	$\alpha,\ \alpha^2,\ \alpha^\kappa$	$\alpha\beta$	$\alpha^2\beta$	$\alpha\beta\gamma$
Orders	$\alpha,\ \alpha,\ \alpha$	$\alpha,\ \beta,\ \alpha\beta$	$\alpha,\ \beta,\ \alpha\beta$	$\alpha,\ \beta,\ \gamma,\ \beta\gamma,\ \gamma\alpha,\ \alpha\beta,\ \alpha\beta\gamma$

$n =$	2	2α	$2\alpha^2$	$2\alpha^3$	$2\alpha\beta$ and $2\alpha^2\beta$
Orders	2	$2,\ 2\alpha$	$2,\ 2\alpha$	$2,\ 2\alpha$	$2,\ 2\alpha,\ 2\beta,\ 2\alpha\beta$

but the Aurifeuillian condition (78) shows that

No one n-an (N_n or N'_n)—(*i.e.* with definite x, y),—can be *explicitly* an Aurifeuillian or Ant-Aurifeuillian of more than one order (ι) .... (82e).

37b. *Quotient Aurifeuillians.* Let $\mathbf{N}$, N_1, N_2 be Aurifeuillians *of same order* (n), and

Let $\mathbf{N} = N_1 N_2$; here $\mathbf{N} = \mathbf{L}.\mathbf{M}$, $N_1 = L_1.M_1$, $N_2 = L_2.M_2$.

Then $$N_2 = \frac{\mathbf{N}}{N_1} = \frac{\mathbf{L}.\mathbf{M}}{L_1.M_1} = L_2.M_2.$$

Here $$L_2 = \frac{\mathbf{L}}{L_1} \text{ or } \frac{\mathbf{L}}{M_1}; \quad M_2 = \frac{\mathbf{M}}{M_1} \text{ or } \frac{\mathbf{M}}{L_1} \quad \ldots\ldots\ldots\ldots \quad (83).$$

[Note that here $\mathbf{L}$ contains the *whole* of L_1 or M_1, and $\mathbf{M}$ contains the *whole* of M_1 or L_1. When either L_1 or M_1 contains a *small* divisor (q), this property renders it easy to discover—(by trial division by q)—which of $\mathbf{L}$ or $\mathbf{M}$ contains the whole factors L_1 or M_1.]

The property (83) is very useful in factorisation of large Aurifeuillians $\mathbf{N}_n$, when containing an Aurifeuillian (N_1) *of same order.*

38. *Aurifeuillians in this Volume.*

The Aurifeuillians dealt with in this volume are of *three orders*, $n = 2, 3, 6$, as indicated in their special names :—

Bin-Aurifns., $n = 2$; *Trin-Aurifns.*, $n = 3$; *Sext-Aurifns.*, $n = 6$, arising from n-ans $(N_{ii}, N_{iii}, N_{vi})$ of degrees $n = 2, 3, 6$.

39. *Bin-Aurifeuillians.* These arise from n-ans (N_n) of degrees $2, 2a, 2a^2$, &c.—[a odd]—under the *Aurifeuillian condition*

$$2xy = \square = (2\xi\eta)^2, \text{ giving } x = \xi^2,\ y = 2\eta^2,\ [\xi \text{ prime to } \eta] \ \ldots \ (84).$$

39a. *Bin-Aurifeuillian Duans.* The Duan (N_{ii}), under the condition (83), becomes

$$N_{ii} = x^2 + y^2 = (x \sim y)^2 + (2\xi\eta)^2 = \xi^4 + 4\eta^4, \ [2 \text{ forms of } (a, b)] \ \ldots \ (84a)$$

$$= (x+y)^2 - (2\xi\eta)^2 = P^2 - Q^2 = L.M \ \ldots\ldots\ldots\ldots \ (84b).$$

$$P = x + y, \quad Q = 2\xi\eta; \quad L = P - Q, \quad M = P + Q \ \ldots\ldots\ldots \ (84c).$$

$$L = \xi^2 - 2\xi\eta + 2\eta^2 = (\xi \sim \eta)^2 + \eta^2; \quad M = \xi^2 + 2\xi\eta + 2\eta^2 = (\xi+\eta)^2 + \eta^2 \ \ldots \ (84d).$$

Note that the three 2-ic parts of L, M, viz. $(\xi - \eta)$, η, $(\xi + \eta)$, are in arithmetical progression.

40. *Trin-Aurifeuillians.* These arise from n-ans (N'_n) of degrees $3, 3a, 3a^2$, &c.—[a odd]—under the *Aurifeuillian condition*

$$3xy = \square = (3\xi\eta)^2, \text{ giving } x = \xi^2,\ y = 3\eta^2,\ [\xi \text{ prime to } \eta] \ \ldots \ (85).$$

and also out of other special forms of Cubans and Sextans.

40a. *Trin-Aurifeuillian Cubans.* The Cuban (N'_{iii}), under the condition (84), becomes

$$N'_{iii} = (x^3+y^3)\div(x+y) = x^2-xy+y^2 = \xi^4-3\xi^2\eta^2+9\eta^4 \quad\ldots\ldots\ldots\ldots\ (85a)$$

$$= A^2+3B^2, \text{[as in Art. 13c], and } = (x\sim y)^2+3(\xi\eta)^2,$$

$$\text{[2 forms of (A, B)]} \ \ldots\ldots\ (85b)$$

$$= (x+y)^2-(3\xi\eta)^2 = P^2-Q^2 = L.M \quad\ldots\ldots\ldots\ (85c).$$

$$P = x+y,\ \ Q = 3\xi\eta\,; \qquad L = P-Q,\ \ M = P+Q \quad\ldots\ldots\ldots\ (85d).$$

$L = \xi^2-3\xi\eta+3\eta^2,$	$M = \xi^2+3\xi\eta+3\eta^2 \quad\ldots\ldots\ldots\ (85e)$
$= (\xi^2\sim\frac{3}{2}\eta)^2+3(\frac{1}{2}\eta)^2$	$= (\xi+\frac{3}{2}\eta)^2+3(\frac{1}{2}\eta)^2, \qquad [\eta=\epsilon]\ \ldots\ (85f)$
$= (\frac{1}{2}\xi)^2+3(\frac{1}{2}\xi\sim\eta)^2$	$= (\frac{1}{2}\xi)^2+3(\frac{1}{2}\xi+\eta)^2, \qquad [\xi=\epsilon]\ \ldots\ (85f')$
$= \left(\frac{\xi\sim3\eta}{2}\right)^2+3\left(\frac{\xi\sim\eta}{2}\right)^2$	$= \left(\frac{\xi+3\eta}{2}\right)^2+3\left(\frac{\xi+\eta}{2}\right)^2, \ [\xi\eta=\omega]\ \ldots\ (85f'').$

40b. *Latent Trin-Aurifeuillian Cubans.* Although a given Cuban (N_{iii} or N'_{iii}) may not explicitly satisfy the Aurifeuillian condition, it may do so in *one of its equivalent* forms* ($N_{iii} = N'_{iii} = N''_{iii}$ of Art. 13).

$$N_{iii} = \frac{x^3-y^3}{x-y},\ \ N'_{iii} = \frac{z^3+x^3}{z+x},\ \ N''_{iii} = \frac{z^3+y^3}{z+y}, \text{ [where } z = x+y] \ldots (86).$$

Here it is seen that either of the relations

$$3zx = \square = (3\zeta\xi)^2, \ \text{ or } \ 3zy = \square = (3\zeta\eta)^2, \ \ [\zeta \text{ prime to } \xi, \text{ or to } \eta] \ \ldots (86a)$$

suffice to make the Cuban expressible in Trin-Aurifeuillian form.

39 and **40.** *Examples* (of Bin- and Trin-Aurifeuillians). A few examples only of each of these in very high numbers,—(N of 20 figures)—to show the power of the Congruence Tables (Chap. II) employed in their factorisation—

High Bin-Aurifns., page 103, (top Table); *High Trin-Aurifns.*, page 152.

41. *Common Aurifeuillian Factors*, (F = L or M). A number F may be a common Aurifeuillian Factor (L or M) of several Aurifeuillians, as detailed below, (Art. 41a–c).

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* This property is peculiar to Cubans.

41a. *Common Bin-Aurifeuillian Factors.* By equating the 2-ic parts (a, b) of a number $F = a^2 + b^2$ to the 2-ic parts of the Aurifeuillian Factors (L, M) of a Bin-Aurifeuillian, $N_r = x_r^4 + 4y_r^4$, it will be found that F is a common Aurifeuillian Factor (L or M) of the *four* different Bin-Aurifeuillians $N_1 = L_1 M_1$, $N_2 = L_2 M_2$, $N_3 = L_3 M_3$, $N_4 = L_4 M_4$ shown in scheme below

$$\begin{array}{r|c|c|c|c|l} F = & L_1 & L_2 & M_1 & M_2 & \\ x_r,\ y_r = & a+b,\ b & b+a,\ a & a-b,\ b & b-a,\ a & \ldots\ldots\ (87), \end{array}$$

and the four Bin-Aurifeuillians are connected by the following relations

$$N_1 + N_4 = N_2 + N_3 = 6F^2, \quad N_2 - N_4 = N_1 - N_3 = 8abF \ \ldots\ (87a).$$

41b. *Common Trin-Aurifeuillian Factors.* By equating the 2-ic parts (A, B) of a number $F = A^2 + 3B^2$ to the 2-ic parts of the Aurifeuillian Factors of a Trin-Aurifeuillian, $N_r = x_r^4 - 3x_r^2 y_r^2 + 9y_r^4$, it will be found that F is a common Aurifeuillian Factor (L or M) of six Trin-Aurifeuillians $N_1 = L_1 M_1$, $N_2 = L_2 M_2$, ... $N_6 = L_6 M_6$ shown in scheme below

$$\begin{array}{r|c|c|c|c|c|c|} F = & L_1 & L_2 & L_3 & M_1 & M_2 & M_3 \\ x_r, y_r = & A+3B, 2B & 2A, B+A & 3B+A, B+A & A-3B, 2B & 2A, B\sim A & 3B\sim A, A\sim B \end{array}$$
$$\ldots\ldots\ldots\ (88),$$

and the six Trin-Aurifeuillians are connected by the following relations

$$\left.\begin{array}{l} N_2 + N_4 = N_1 + N_5 = N_3 + N_6 = 14F^2 \\ N_1 - N_4 = N_2 - N_5 = \tfrac{1}{3}(N_3 - N_6) = 24ABF \end{array}\right\} \ldots\ldots\ldots\ldots\ (88a).$$

41c. *Aurifeuillian Trees.* Every Aurifeuillian $N_r = L_r . M_r$ gives rise to several new Aurifeuillians $N_{r+1} = L_{r+1} . M_{r+1}$, $N'_{r+1} = L'_{r+1} . M'_{r+1}$, &c., in each of which the L_{r+1}, L'_{r+1}, &c. $= M_r$. Similarly each of the N_{r+1} gives rise to a like number of N_{r+2}, in each of which the $L_{r+2} = M_{r+1}$; and so on in succession. The ensemble of these may be styled an *Aurifeuillian Tree*, and the several N_r are the *Branches* of order r.

41d. *Bin-Aurifeuillian Tree.* Here each N_r yields two N_{r+1}, (Art. 41a).

The Table on p. 146 shows the elements (ξ_r, η_r) of the Branches (N_r) of *four* successive steps (r = 2, 3, 4, 5) of the Tree arising from the smallest Bin-Aurifeuillian $$N_1 = 1^4 + 4.1^4 = 5.$$

41e. *Trin-Aurifeuillian Tree.* Here each N_r yields six N_{r+1}, (Art. 41b).

The top Table of p. 155 shows the elements (ξ_r, η_r) of the Branches (N_r) of *two* successive steps $(r = 2, 3)$ of the Tree arising from the smallest Trin-Aurifeuillian

$$N_1 = 1^4 - 3.1^2 + 9.1^4 = 7.$$

[Here $A_1 = 2$, $B_1 = 1$, whereby one of the N_2 has $\xi_2 = 1$, $\eta_2 = 1$, giving $N_2 = 7$, so that only five *new* Branches arise at the first step.]

42. *Aurifeuillian Sextans.* Sextans (N_{vi}) admit of Aurifeuillians of each of the three orders $n = 2, 3, 6$, [Art. 42a–e].

42a. *Bin-Aurifeuillian Sextans.* The Sextan (N_{vi}), under the Bin-Aurifeuillian condition $[2xy = \square = (2\xi\eta)^2]$, becomes

$$N_{vi} = (x^6 + y^6) \div (x^2 + y^2) = x^4 - x^2y^2 + y^4 = \xi^8 - 4\xi^4\eta^4 + 16\eta^8 \text{ .... } (89)$$

$$= (x^2 \sim y^2)^2 + (xy)^2 = (x^2 - xy + y^2)^2 + [2\xi\eta\,(x \sim y)]^2,$$
$$[2 \text{ forms of } (a, b)] \text{ ...... } (89a)$$

$$= (x^2 + y^2)^2 - 3\,(xy)^2 = (x^2 + 3xy + y^2)^2 - 3\,[2\xi\eta\,(x + y)]^2,$$
$$[2 \text{ forms of } (A', B')] \text{ ... } (89b)$$

$$= A^2 + 3B^2, \text{ (as in Art. 14) } = (x^2 - 3xy + y^2)^2 + 3\,[2\xi\eta\,(x \sim y)]^2,$$
$$[2 \text{ forms of } (A, B)] \text{ ... } (89c)$$

$$= P^2 - Q^2 = L.M \text{ .... } (89d).$$

$$P = x^2 + xy + y^2,\ \ Q = 2\xi\eta\,(x + y)\,;\ \ L = P - Q,\ \ M = P + Q \text{ ...... } (89e).$$

$$L = (\xi^2 \sim \xi\eta)^2 + (\xi\eta \sim 2\eta^2)^2,\ \ M = (\xi^2 + \xi\eta)^2 + (\xi\eta + 2\eta^2)^2 \text{ .... } (89f).$$

This is a case of a Quotient-Aurifeuillian (Art. 37b); the numerator and denominator of N_{vi} being, *each of them*, Bin-Aurifeuillians; algebraic division gives above results.

42b. *Latent Trin-Aurifeuillian Sextans.* The Sextan (N_{vi}) does not *explicitly* satisfy the Trin-Aurifeuillian condition: but, being a special form of Cuban, it may be written in the *three equivalent Cuban forms* (of Art. 13) $N_{vi} = N'_{vi} = N''_{vi}$, where

$$N_{vi} = \frac{(y^2 - x^2)^3 - (x^2)^3}{(y^2 - x^2) - x^2},\ \ N'_{vi} = \frac{(y^2 - x^2)^3 + (y^2)^3}{(y^2 - x^2) + y^2},\ \ N''_{vi} = \frac{(x^2)^3 + (y^2)^3}{x^2 + y^2},\ [y > x]\,;$$

and N'_{vi} becomes $= \dfrac{(y^2)^3 + (3z^2)^3}{y^2 + 3z^2}$, by taking $y^2 - x^2 = 3z^2$ .... (90).

Here N'_{vi} satisfies the Trin-Aurifeuillian *explicitly*, and (by Art. 13) .... (90a).

$$N'_{vi} = y^4 - 3y^2z^2 + 9z^4$$

$= A^2 + 3B^2$. (as in Art. 14), and $= (y^2 \sim 3z^2)^2 + 3(yz)^2$, [2 forms of (A, B)] ... (90b)

$$= (y^2 + 3z^2)^2 - (3yz)^2 = P^2 - Q^2 = L.M \quad \ldots\ldots (90c).$$

$$P = y^2 + 3z^2,\quad Q = 3yz;\quad L = P - Q,\quad M = P + Q \quad \ldots\ldots (90d).$$

$L = y^2 - 3yz + 3z^2,$	$M = y^2 + 3yz + 3z^2$	...... (90e)
$= (y \sim \frac{3}{2}z)^2 + 3(\frac{1}{2}z)^2,$	$= (y + \frac{3}{2}z)^2 + 3(\frac{1}{2}z)^2,$	$[z = \epsilon]$. (90f)
$= (\frac{1}{2}y)^2 + 3(\frac{1}{2}y \sim z)^2,$	$= (\frac{1}{2}y)^2 + 3(\frac{1}{2}y + z)^2,$	$[y = \epsilon]$. (90g)
$= \left(\frac{y \sim 3z}{2}\right)^2 + 3\left(\frac{y \sim z}{2}\right)^2,$	$= \left(\frac{y + 3z}{2}\right)^2 + 3\left(\frac{y + z}{2}\right)^2$	,$[yz = \omega]$. (90h).

42c. *Sext-Aurifeuillians.* These arise from n-ans of degrees $n = 6, 6a, 6a^2$, &c.,—[a odd and prime to 3]—under the condition—

$$6xy = \square = (6\xi\eta)^2,\quad [\xi \text{ prime to } \eta) \ldots\ldots (91).$$

This may be satisfied in *two ways*, giving rise to *two species* of Sext-Aurifeuillians.

$$1°.\quad x = \xi^2,\quad y = 6\eta^2;\qquad 2°.\quad x = 3\xi^2,\quad y = 2\eta^2 \ldots\ldots (91a).$$

42d. *Sext-Aurifeuillian Sextans.* The Sextan (N_{vi}) under each of the conditions (91a) becomes—

Both Species. $N_{vi} = x^4 - x^2y^2 + y^4 = P^2 - Q^2 = L.M$ ...... (91b)

$= (x^2 \sim y^2)^2 + (xy)^2 = P'^2 + Q'^2$, [2 forms of a, b] (91c).

$$P = x^2 + 3xy + y^2,\quad Q = 6\xi\eta\,(x + y);\quad L = P - Q,\quad M = P + Q \ldots (91d).$$

$$P' = x^2 - 3xy + y^2,\quad Q' = 6\xi\eta\,(x \sim y) \quad \ldots\ldots (91e).$$

In species 1°.

$$L = (\xi^2 - 3\xi\eta)^2 + (3\xi\eta \sim 6\eta^2)^2,\quad M = (\xi^2 + 3\xi\eta)^2 + (3\xi\eta + 6\eta^2)^2 \ldots (91f)$$

In species 2°.

$$L = (3\xi^2 \sim 3\xi\eta)^2 + (3\xi\eta \sim 2\eta^2)^2,\quad M = (3\xi^2 + 3\xi\eta)^2 + (3\xi\eta + 2\eta^2)^2 \ldots (91g).$$

42e. *Ex.* Extensive Factorisation-Tables of each of the three kinds—Bin-, Trin-, and Sext-Aurifeuillian Sextans — are given, as in Abstract below, ending with very high numbers.

Pages.	Aurifn.	x , y	ξ ,	x	η ,	y	N $\not>$
172, 173	Bin-	$\xi^2 = 1$, $2\eta^2$	1 ,	1	1 to 128,	2 to 32768	10^{18}
173	Bin-	$\xi^2 = 1$, $2\eta^2$	1 ,	1	160 & 201,	$\not>$ 80802	10^{20}
174 to 179	Bin-	ξ^2 , $2\eta^2$	3 to 53,	9 to 2809	1 to 37 ,	2 to 2738	10^{14}
185 to 189	Trin-	$y^2 - x^2 = 3z^2$	—	1 to 2054	—	2 to 2084	10^{14}
194	Trin-	$y^2 - x^2 = 3z^2$	—	1979 to 2701	—	2029 to 2774	10^{14}
179, 180	Sext-, 1°	$\xi^2 = 1$, $6\eta^2$	1 ,	1	1 to 75 ,	6 to 33750	10^{18}
180	Sext-, 1°	$\xi^2 = 1$, $6\eta^2$	1 ,	1	80 & 101 ,	$\not>$ 61206	10^{20}
181, 182	Sext-, 1°	ξ^2 , $6\eta^2$	5 to 53,	25 to 2809	1 to 21 ,	6 to 2646	10^{14}
183, 184	Sext-, 2°	$3\xi^2$, $2\eta^2$	1 to 31,	1 to 2883	1 to 29 ,	2 to 1682	10^{14}
194	Sext-, 2°	$3\xi^2$, $2\eta^2$	1 to 11,	3 to 363	29 to 32 ,	1682 to 2048	10^{14}

43. *Aurifeuillian Chains.* Duans, Cubans, and Sextans yield interesting cases of Aurifeuillian Chains of orders 2, 3, and 6.

44. *Aurifeuillian Duan and Cuban Chains.* Duans (N_{ii}) and Cubans (N'_{iii}) give rise to 3 cases of Bin-Aurifeuillian and 3 cases of Trin-Aurifeuillian Chains respectively of strikingly similar formation, as shown in the Abstract below:—

	Bin-Aurifn. Duans.	*Trin-Aurifn. Cubans.*	
	$N_{ii} = \xi^4 + 4\eta^4 = L.M$	$N'_{iii} = \xi^4 - 3\xi^2\eta^2 + 9\eta^4 = L.M$	(92).
	$\left.\begin{array}{l} L = \xi^2 - 2\xi\eta + 2\eta^2 \\ M = \xi^2 + 2\xi\eta + 2\eta^2 \end{array}\right\}$	$\left.\begin{array}{l} L = \xi^2 - 3\xi\eta + 3\eta^2 \\ M = \xi^2 + 3\xi\eta + 3\eta^2 \end{array}\right\}$	(92a).
Case 1°.	ξ const., $\eta_{r+1} = \eta_r + \xi$	ξ const., $\eta_{r+1} = \eta_r + \xi$ ...	(93).
Case 2°.	$\xi_{r+1} = \xi_r + 2\eta$, η const.	$\xi_{r+1} = \xi_r + 3\eta$, η const. ...	(94).
Case 3°.	$\left.\begin{array}{l} \xi'^2 - 2\eta'^2 = -C'; \\ C' = 1,\ 8i \mp 1 \end{array}\right\}$	$\left.\begin{array}{l} \xi'^2 - 3\eta'^2 = -C'; \\ C' = 2,\ 12i-1,\ 2\,(12i+1) \end{array}\right\}$	(95a).
	$\left.\begin{array}{l} \xi^2 - 2\eta^2 = +C; \\ C = 1,\ 8i \mp 1 \end{array}\right\}$	$\left.\begin{array}{l} \xi^2 - 3\eta^2 = +C; \\ C = 1,\ 12i+1,\ 2\,(12i-1) \end{array}\right\}$	(95b).

The first two Cases will be now taken up together; Case 3° later, (Art. 44d).

44a. Case 1°.

Bin-Aurifn. Duans.

$$\xi_r = \xi \text{ (const.)}, \quad \eta_{r+1} = \eta_r + \xi$$

$$L_{r+1} = \xi^2 - 2\xi\eta_{r+1} + 2\eta_{r+1}^2$$
$$= \xi^2 - 2\xi(\eta_r + \xi) + 2(\eta_r + \xi)^2$$
$$= \xi^2 + 2\xi\eta_r + 2\eta_r^2 = M_r$$

Hence $N_1, N_2, N_3, \ldots$ are *in chain*.

Trin-Aurifn. Cubans.

$$\xi_r = \xi \text{ (const.)}, \quad \eta_{r+1} = \eta_r + \xi \quad (93).$$

$$L_{r+1} = \xi^2 - 3\xi\eta_{r+1} + 3\eta_{r+1}^2 \ldots\ldots (93a)$$
$$= \xi^2 - 3\xi(\eta_r + \xi) + 3(\eta_r + \xi)^2 \ldots\ldots\ldots (93b)$$
$$= \xi^2 + 3\xi\eta_r + 3\eta_r^2 = M_r \ldots (93c).$$

Hence $N_1, N_2, N_3, \ldots$ are *in chain*.

44b. Case 2°.

Bin-Aurifn. Duans.

$$\xi_{r+1} = \xi_r + 2\eta, \quad \eta_r = \eta \text{ (const.)}$$

$$L_{r+1} = \xi_{r+1}^2 - 2\xi_{r+1}.\eta + 2\eta^2$$
$$= (\xi_r + 2\eta)^2 - 2(\xi_r + 2\eta)\eta + 2\eta^2$$
$$= \xi_r^2 + 2\xi_r\eta + 2\eta^2 = M_r$$

Hence $N_1, N_2, N_3, \ldots$ are in chain.

Trin-Aurifn. Cubans.

$$\xi_{r+1} = \xi_r + 3\eta, \quad \eta_r = \eta \text{ (const.)} \quad (94).$$

$$L_{r+1} = \xi_{r+1}^2 - 3\xi_{r+1}.\eta + 3\eta^2 \ldots (94a)$$
$$= (\xi_r + 3\eta)^2 - 3(\xi_r + 3\eta)\eta + 3\eta^2 \ldots\ldots\ldots (94b)$$
$$= \xi_r^2 + 3\xi_r\eta + 3\eta^2 = M_r \ldots (94c).$$

Hence $N_1, N_2, N_3, \ldots$ are in chain.

44c. *Ex.* It would be impracticable to give complete Tables of the above on account of the enermous extent to which they would run; a few examples of Case 2° of each kind only are given to illustrate the power of the Congruence-Tables (Chap. II) used in their factorisation.

Bin-Aurifn. Duans.

Case 2°. Page 98, $[\eta_r = 1]$.

$$N_r = y_r^4 + 4.1^4 = L_r.M_r, \quad [y_r = \xi_r].$$

$$y_{r+1} = y_r + 2;$$

$$\left.\begin{aligned} y_1 &= 49995 \text{ (mid-Table)} \\ y_1 &= 49994 \text{ (at foot)} \end{aligned}\right\}.$$

$$L_r = (y_r - 1)^2 + 1^2, \quad M_r = (y_r + 1)^2 + 1^2.$$

Trin-Aurifn. Cubans.

Case 2°. Page 149, $[\eta_r = 1]$.

$$N_r = \frac{Y_r^3 + 3^3.1}{Y_r + 3} = L_r.M_r, \quad [Y_r = y_r^2, \; y_r = \xi_r].$$

$$y_{r+1} = y_r + 3;$$

$$\left.\begin{aligned} y_1 &= 49994 \text{ (mid-Table)} \\ y_1 &= 49995 \text{ (at foot)} \end{aligned}\right\}.$$

$$L_r = y_r^2 - 3y_r + 3; \quad M_r = y_r^2 + 3y_r + 3$$
$$= y_r''^2 - y_r'' + 1; \quad = y_r'^2 + y_r' + 1.$$

44d. Case 3°. *Pellian Aurifeuillian Chains.* The successive solutions (τ_r', v_r'), (τ_r, v_r) of the Pellian Equations $\tau_r'^2 - Dv_r'^2 = \mp z_0'$, and $\tau_r^2 - Dv_r^2 = \mp z_0$ give rise to interesting Bin-Aurifeuillian Duan Chains and Trin-Aurifeuillian Cuban Chains when $D = 2, 3$ respectively. The cases when $z_0' = -1$, or -2, and $z_0 = +1$ will be considered first.

44e. *Pellian Bin-Aurifeuillian Duan Chain.* Let (τ'_ρ, v'_ρ), (τ_ρ, v_ρ) be successive solutions of the two Pellian equations

$$\tau'^2_\rho - 2v'^2_\rho = -1, \quad \tau^2_\rho - 2v^2_\rho = +1 \quad \ldots\ldots\ldots\ldots \quad (96).$$

Then, if $N'_\rho = \tau'^4_\rho + 4v'^4_\rho = L'_\rho . M'_\rho$ and $N_\rho = \tau^4_\rho + 4v^4_\rho = L_\rho . M_\rho \ldots$ (96a).

Then
$$L'_\rho = \tau'^2_\rho - 2\tau'_\rho v'_\rho + v'^2_\rho = \tau_{2\rho-1} - v_{2\rho-1} = v'_{2\rho-1} \ \ldots\ldots\ldots \ (96b),$$
$$M'_\rho = \tau'^2_\rho + 2\tau'_\rho v'_\rho + v'^2_\rho = \tau_{2\rho-1} + v_{2\rho-1} = v'_{2\rho} \ \ldots\ldots\ldots \ (96c),$$
$$L_\rho = \tau^2_\rho - 2\tau_\rho v_\rho + v^2_\rho = \tau_{2\rho} - v_{2\rho} = v'_{2\rho} \ \ldots\ldots\ldots \ (96d),$$
$$M_\rho = \tau^2_\rho + 2\tau_\rho v_\rho + v^2_\rho = \tau_{2\rho} + v_{2\rho} = v'_{2\rho+1} \ \ldots\ldots\ldots \ (96e),$$

Hence
$$\left.\begin{array}{ll} N'_\rho = v'_{2\rho-1} . v'_{2\rho}, & N_\rho = v'_{2\rho} . v'_{2\rho+1} \\ N'_{\rho+1} = v'_{2\rho+1} . v'_{2\rho+2}, & N_{\rho+1} = v'_{2\rho+2} . v'_{2\rho+3} \end{array}\right\} \ \ldots\ldots\ldots \ (96f).$$

Hence N'_1, N_1, N'_2, N_2, N'_3, N_3, ..., &c., are a Series *in chain*.

The links (N', N) being taken from either Series alternately.

44f. *Ex.* (Page 109, mid-Table.) The two series (N, N') are here ranged together into one series, the (x, y) here denoting (τ', v') and (τ, v) *alternately;* the two Pellian equations (96) being combined into one as $\tau^2_r - 2v^2_r = (-1)^r$, the *odd* values of r giving (τ'_ρ, v'_ρ), and the *even* values giving (τ_ρ, v_ρ). The M-factor of N' and N is alone recorded: (the $L_{r+1} = M_r$ being omitted to save space).

44g. *Examples of other such Chains,* $[z > 1]$. (Page 110.) The Table shows *four* such *Chains* arising from the successive solutions (x, y) of the two Pellian equations

$$\tau^2_r - 2v^2_r = (-1)^r . 7, \quad \tau^2_r - 2v^2_r = (-1)^r . 17.$$

Each Pellian has *two series* of solutions (τ_r, v_r), each of which gives rise to a *Bin-Aurifeuillian Chain,*

$$N_r = x^4_r + 4y^4_r = L_r . M_r.$$

44h. *Pellian Trin-Aurifeuillian Cuban Chain.* Let (τ'_r, v'_r), (τ_r, v_r) be successive solutions of the two Pellian equations

$$\tau'^2_r - 3v'^2_r = -2, \quad \tau^2_r - 3v^2_r = +1 \quad \ldots\ldots\ldots\ldots \quad (97).$$

Then, if
$$\left.\begin{array}{ll} N'_r = \tau'^4_r - 3\tau'^2_r v'^2_r + 9v'^4_r & = L'_r M'_r \\ N_r = \tau^4_r + 3\tau^2_r v^2_r + 9v^4_r & = L_r M_r \end{array}\right\} \ \ldots\ldots\ldots\ldots \ (97a).$$

Then

$$L'_r = \tau'^2_r + 3v'^2_r - 3\tau'_r v'_r = 2\tau_{2r-1} - 3v_{2r-1} = \tau_{2r-2}\,; \quad L'_{r+1} = \tau_{2r} \;\ldots\ldots\ldots\; (97b).$$

$$M'_r = \tau'^2_r + 3v'^2_r + 3\tau'_r v'_r = 2\tau_{2r-1} + 3v_{2r-1} = \tau_{2r} = L'_{r+1} \;\ldots\ldots\ldots\; (97c).$$

$$L_r = \tau^2_r + 3v^2_r - 3\tau_r.v_r = \tfrac{1}{2}(2\tau_{2r} - 3v_{2r}) = \tfrac{1}{2}\tau_{2r-1}, \quad L_{r+1} = \tfrac{1}{2}\tau_{2r+1} \;\ldots\; (97d).$$

$$M_r = \tau^2_r + 3v^2_r + 3\tau_r.v_r = \tfrac{1}{2}(2\tau_{2r} + 3v_{2r}) = \tfrac{1}{2}\tau_{2r+1} = L_r \;\ldots\ldots\ldots\; (97e).$$

Hence
$$\left.\begin{aligned} N'_r &= \tau_{2r-2}.\tau_{2r}, & N'_{r+1} &= \tau_{2r}.\tau_{2r+2}, & &\text{\&c.} \\ N_r &= \tfrac{1}{4}\tau_{2r-1}.\tau_{2r+1}, & N_{r+1} &= \tfrac{1}{4}\tau_{2r+1}.\tau_{2r+3}, & &\text{\&c.} \end{aligned}\right\} \ldots\ldots\ldots \;(97f).$$

Hence each of the series N'_1, N'_2, N'_3, &c., N_1, N_2, N_3, &c., is a *Chain-series:* and the Aurifeuillian factors (L'_r, M'_r), (L_r, M_r) are the members τ_{2r}, $\tau_{2r\mp 1}$ in the solutions of $\tau^2 - 3v^2_r = +1$ of *even* order, or of *odd* order for the series N'_r, N_r respectively.

Ex. (Page 153, top-Table.) This Table shows 8 terms of each of the above Chain-Series.

44i. *Examples of other such Chains*, [$z > 1$]. (Page 153, at foot.) The Table shows *four* such Chains arising from the successive solutions of the two Pellian equations

$$\tau^2_r - 3v^2_r = -11, \quad \tau^2_r - 3v^2_r = +13.$$

Each Pellian has *two series* of solutions (τ_r, v_r), each of which gives rise to a *Trin-Aurifeuillian Chain*

$$N_r = \tau^4_r - 3\tau^2_r v^2_r + 9v^4_r.$$

45. *Aurifeuillian Sextan Chains.* Sextan Chains may be formed in each order (n) of Aurifeuillians of which a Sextan is capable ($n = 2, 3, 6$), the elements (ξ, η, &c.) being taken from the associated Pellian equation in each order

$$\tau^2 - Dv^2 = (-1)^r.z, \quad [D = n = 2, 3, 6] \;\ldots\ldots\ldots\; (98).$$

45a. *Pellian Bin-Aurifeuillian Sextan Chain.* The Bin-Aurifeuillian Sextan, (Art. 42a)

$$N_{vi} = (x^6_r + y^6_r) \div (x^2_r + y^2_r) = P^2_r - Q^2_r = L_r.M_r \;\ldots\ldots\ldots\; (99),$$

where
$$x_r = \xi^2_r, \quad y_r = 2\eta^2_r \;\ldots\ldots\ldots\; (99a),$$

and gives *Chain-Series*, so that $M_r = L_{r+1}$, when ξ_r, η_r are taken from the Pellian equation,

$$\xi^2_r - 2\eta^2_r = (-1)^r.\zeta, \quad [\zeta = 8i \pm 1 = \mathit{const.}] \;\ldots\ldots\ldots\; (99b),$$

and
$$\xi_{r+1} = \xi_r + 2\eta_r, \quad \eta_{r+1} = \xi_r + \eta_r \;\ldots\ldots\ldots\; (99c).$$

When $\zeta = 1$, the Chain is *unique;* when $\zeta > 1$, it gives *two Associate Chains.*

45b. *Ex.* (T., page 195.) The Table gives the elements $(\pm\xi_r, \eta_r)$ only of r successive Links $(N_r, N_{r+1}$, &c.) of numerous Chains, *i.e.* for many values of ζ—

	$\zeta =$ 1;	7 to 47;	49 to 97;	103 to 497;
(Links)	$r =$ 10	5	4	3

The factorisation of $L_r, M_r \not> 9.10^6$ will be found in the Tables, pages 172–179, [Argt. x_r, y_r].

Full detail of the first Example only is given below:

r	=	1	2	3	4	5	6
ξ_r, η_r	=	1, 1	3, 2	7, 5	17, 12	41, 29	99, 70
x_r, y_r	=	1, 2	9, 8	49, 50	289, 288	1681, 1682	9801, 9800
L_r	=	1;	13;	421;	14281;	485113;	13.37.34261;
M_r	=	13;	421;	14281;	485113;	13.37.34261;	73.7668757;

45c. *Pellian Trin-Aurifeuillian Sextan Chain.* The Trin-Aurifeuillian Sextan (Art. 42b)

$$N_{vi} = (x_r^6 + y_r^6) \div (x_r^2 + y_r^2) = P_r^2 - Q_r^2 = L_r.M_r \text{ .... } (100),$$

wherein $$y_r^2 - 3z_r^2 = x_r^2 \text{ .... } (100a)$$

gives a *Chain-Series*, with $M_r = L_{r+1}$,

when $$x_r = +12i+1, \text{ or } -(12i-1) = \textit{const.} \text{ .... } (100b)$$

and $$y_{r+1} = 2y_r \pm 3z_r, \quad z_{r+1} = y_r \pm 2z_r \text{.................. } (100c),$$

and $$M_{r-1} = L_r = \tfrac{1}{2}\tau_{2r-1}, \quad M_r = L_{r+1} = \tfrac{1}{2}\tau_{2r+1} \text{ .... } (100d).$$

When $x_r = 1$, the Chain is *unique;* when $x_r > 1$, it gives *two Associate Chains.*

45d. *Ex.* (T., page 197.) The Table gives the elements (x, y_r, z_r) only of r successive Links $(N_1', N_2', N_3'$, &c.) of numerous Chains, *i.e.* for many values of x—

	$x =$ 1;	−11;	13, −23;	37 to −143;	157 to 251;
(Links)	$r =$ 7	6	5	4	3

The factorisation of the factors $L_r, M_r \not> 9.10^6$ will be found in the Tables, pages 172–179. Fuller detail of the first Example only $(x = 1)$ will be found on page 153 (mid-Table).

45e. *Pellian Sext-Aurifeuillian Chains.* The Sextan (N_{vi}) has *two species* (Art. 42c) of Sext-Aurifeuillians

$$N_{vi} = (x^6+y^6)\div(x^2+y^2) = P_r^2 - Q_r^2 = L_r.M_r \;\ldots\ldots\ldots\; (101),$$

where,
$$\left.\begin{array}{lll}\text{in Species } 1^\circ, & x_r = \xi_r^2, & y_r = 6\eta_r^2 \\ \text{in Species } 2^\circ, & x_r = 3\xi_r^2, & y_r = 2\eta_r^2\end{array}\right\}\;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\;(101a).$$

These give Chain-Series, so that $M_r = L_{r+1}$, when ξ_r, η_r are taken *alternately* from the two Associate Pellian Equations, thus

Species 1°. *Species* 2°.

$$\xi_{2\rho}^2 - 6\eta_{2\rho}^2 = 3\xi_{2\rho\pm1}^2 - 2\eta_{2\rho\pm1}^2 = \zeta = const. = 6i+1 \text{ or } -(6i-1) \ldots\; (101b).$$

The ξ_r, η_r of either species determine the successive ξ_r, η_r of the *other* species by the relations

$r = 2\rho = \epsilon$ (Species 2°). $r = 2\rho\pm1 = \omega$ (Species 1°).

$$\left.\begin{array}{ll}\xi_{2\rho} = 3\xi_{2\rho-1} + 2\eta_{2\rho-1}, & \xi_{2\rho+1} = \xi_{2\rho} + 2\eta_{2\rho} \\ \eta_{2\rho} = \xi_{2\rho-1} + \eta_{2\rho-1}, & \eta_{2\rho+1} = \xi_{2\rho} + 3\eta_{2\rho}\end{array}\right\}\;\ldots\ldots\ldots\;(101c).$$

45f. *Ex.* (T., page 196.) The Table gives the elements ($\pm\xi_r, \eta_r$) only of r successive Links (N_1, N_2, N_3, &c.) of numerous Chains, *i.e.* for many values of ζ—

$\zeta =$	1 to -53;	67, -71;	73 to -101;	115 to -215;
(Links) $r =$	6	5	4	3

The factorisation of the factors $L_r, M_r \not> 9.10^6$ will be found in the Tables, pages 172–179. Fuller details of the first Example only are given below.

$r =$	1	2	3	4	5
Species	2°	1°	2°	1°	2°
$\xi_r, \eta_r =$	1, 1	5, 2	9, 11	49, 20	89, 90
$x_r, y_r =$	3, 2	25, 24	243, 242	2401, 2400	23763, 23762
$L_r =$	1;	61;	13.457;	37.15733;	13.4387837;
$M_r =$	61;	13.457;	37.15733;	13.4387837;	73.76568797?

45g. *Sexto-Trin-Aurifeuillian Sextan Chains.* Sextan Chains may also be composed by combining the two Aurifeuillian Series of orders $n = 3$ and 6, taking the Links from each Series alternately.

In what follows Trin-Aurifeuillians and Sext-Aurifeuillians are denoted by the letters T, S respectively. Four cases arise, two due to the two kinds of solution (see below) of the Pellian equation $y^2 - 3z^2 = x^2$ which determine T (Art. 42b), and two to the two species of S (Art. 42c).

Examples. (Pages 201–204.) The Tables give such of the elements—to Argument $\rho = 1, 3, 5$, &c.—

$$t,\ u,\ x,\ y \text{ for T (Art. 42b)}; \quad \xi,\ \eta,\ x,\ y \text{ for S (Art. 42c)};$$

as are necessary for determining the Links (N_1, N_2, N_3, &c.) of the Chains.

In all four Tables, $\rho = \omega$, $r = \rho + 1$.* Here follows an Abstract of the formulæ of the four Cases. The factorisations of the L, M (when $r \not> 9.10^6$) will be found on pages 185–189 and 194 for T, and on pages 179–184 and 194 for S.

		Case I, p. 201.	Case II, p. 202.
Elements		u, y, ξ, η	t, y, ξ, η
T_ρ	x_ρ	$\frac{1}{2}(t_\rho^2 - 3u_\rho^2)$	$(t_\rho^2 - 3u_\rho^2)$
	y_ρ	$\frac{1}{2}(t_\rho^2 + 3u_\rho^2)$	$(t_\rho^2 + 3u_\rho^2)$
	z_ρ	$t_\rho u_\rho$	$2t_\rho u_\rho$
	T_ρ	$L_\rho M_\rho$	$L_\rho . M_\rho$
S_r	x_r	$\xi_r^2 = \xi^2$	ξ_r^2
	y_r	$6\eta_r^2$	$6\eta_r^2 = 6\eta^2$
	S_r	$L_r . M_r$	$L_r . M_r$
		$t_\rho = \xi_r$ (*const.*)	$u_r = \eta_\rho$ (const.)
		$u_{\rho+2} = u_\rho + 2t$	$t_{\rho+2} = t_\rho + 6u$
		$u_\rho = (\eta_r + \eta_{r-2})$	$t_\rho = \frac{1}{2}(\xi_{r-2} + \xi_r)$
		$\eta_r = \frac{1}{4}(u_\rho + u_{\rho+2})$	$\xi_r = \frac{1}{2}(t_\rho + t_{\rho+2})$
Chain		$M_\rho = L_r$ and $M_r = L_{\rho+2}$	
		T_ρ, S_r, $T_{\rho+2}$, S_{r+2}, &c.	

		Case III, p. 203.	Case IV, p. 204.
Elements		ξ, η, t, y	ξ, η, u, y
T_r	x_r	$\frac{1}{2}(t_r^2 - 3u_r^2)$	$(t_r^2 - 3u_r^2)$
	y_r	$\frac{1}{2}(t_r^2 + 3u_r^2)$	$(t_r^2 + 3u_r^2)$
	z_r	$t_r u_r$	$2t_r u_r$†
	T_r	$L_r . M_r$	$L_r . M_r$
S_ρ	x_ρ	$3\xi_\rho^2 = 3\xi^2$	$3\xi_\rho^2$
	y_ρ	$2\eta_\rho^2$	$2\eta_\rho^2 = 2\eta^2$
	S_ρ	$L_\rho . M_\rho$	$L_\rho . M_\rho$
		$u_r = \xi_\rho$ (const.)	$t_r = \eta_\rho$ (const.)
		$\eta_{\rho+2} = \eta_\rho + 3\xi$	$u_{r+2} = u_r + 2t$
		$t_r = \eta_\rho + \eta_{\rho+2}$	$u_r = \frac{1}{2}(\xi_\rho + \xi_{\rho+2})$
		$\eta_\rho = \frac{1}{4}(t_r + t_{r-2})$	$\xi_\rho = \frac{1}{2}(u_r + u_{r-2})$
		$t_{r+2} = t_r + 6u$	$\xi_{\rho+2} = \xi_\rho + 2\eta$
Chain		$M_\rho = L_r$ and $M_r = L_{\rho+2}$	
		S_ρ, T_r, $S_{\rho+2}$, T_{r+2}, &c.	

* Erratum on p. 203, l. 7, and on p. 204, l. 7; *for* $\rho + 1 = 2$, *read* $\rho + 1 = \epsilon$.

† Erratum on p. 204, l. 6; *for* $z_r = t_r u_r$, *read* $z_r = 2t_r u_r$.

45h. *Detailed Examples.* The full detail of the *first set* of Examples of each Case I–IV (of which the data only are given in the Tables, pages 201–204) is given in the four Tables following, clearly showing the resulting Chains.

Case I, page 201. Chain T_ρ, S_r, $T_{\rho+2}$, S_{r+2}, &c.

	ρ	1	3	5	7	9	11
	t, u	5, $\bar{3}$	5, 7	5, 17	5, 27	5, 37	5, 47
T_ρ	x, y	$\bar{1}$, 26	61, 86	421, 446	1081, 1106	2041, 2066	3301, 3326
	L	181;	13.157;	106861;	601.1381;	3224401;	13.683317;
	M	2521;	20101;	13.25717;	13.132757;	5517661;	13572781?
	r	2	4	6	8	10	12
	ξ, η	5, 1	5, 6	5, 11	5, 16	5, 21	5, 26
S_r	x, y	25, 6	25, 216	25, 726	25, 1536	25, 2646	25, 4056
	L	181;	20101;	13.25717;	13.132757;	5517661;	13572781?
	M	13.157;	106861;	601.1381;	3224401;	13.683317;	181.110161;

Case II, page 202. Chain T_ρ, S_r, $T_{\rho+2}$, S_{r+2}, &c.

	ρ	1	3	5	7	9	11	13
	t, u	$\bar{2}$, 1	4, 1	10, 1	16, 1	22, 1	28, 1	34, 1
T_ρ	x, y	1, 7	13, 19	97, 103	253, 259	481, 487	781, 787	1153, 1159
	L	13;	97;	13.433;	45289;	178693;	13.38197;	13.86209;
	M	181;	1009;	17989;	13.7309;	307261;	760993;	1593589;
	r	2	4	6	8	10	12	14
	ξ, η	1, 1	7, 1	13, 1	19, 1	25, 1	31, 1	37, 1
S_r	x, y	1, 6	49, 6	169, 6	361, 6	625, 6	961, 6	1369, 6
	L	13;	1009;	17989;	13.7309;	307261;	760993;	1593589;
	M	97;	13.433;	45289;	178693;	13.38197;	13.86209;	73.109.277;

	ρ	15	17	19
	t, u	40, 1	46, 1	52, 1
T_ρ	x, y	1597, 1603	2113, 2119	2701, 2707
	L	73.109.277;	3930709;	2137.3049;
	M	13.228733;	5100397;	373.21997;
	r	16	18	20
	ξ, η	43, 1	49, 1	55, 1
S_r	x, y	1849, 6	2401, 6	3025, 6
	L	13.228733;	5100397;	373.21997;
	M	3930709;	2137.3049;	10205341?

Case III, page 203. Chain S_ρ, T_r, $S_{\rho+2}$, T_{r+2}, &c.

S_ρ	ρ	1	3	5	7	9	11	13
	ξ, η	1, 1	1, 4	1, 7	1, 10	1, 13	1, 16	1, 19
	x, y	3, 2	3, 32	3, 98	3, 200	3, 338	3, 512	3, 722
	L	1;	13·37;	13·13·37;	29629;	90697;	13·73·229;	445141;
	M	61;	2161;	14737;	13·4153;	37·389;	316201;	13·46957;
T_r	r	2	4	6	8	10	12	14
	t, u	5, 1	11, 1	17, 1	23, 1	29, 1	35, 1	41, 1
	x, y	11, 14	59, 62	143, 146	263, 266	419, 422	611, 614	839, 842
	L	61;	2161;	14737;	13·4153;	37·3889;	316201;	13·46957;
	M	13·37;	13·13·37;	29629;	90697;	13·73·229;	445141;	457·1789;

S_ρ	ρ	15	17	19	21	23	25
	ξ, η	1, 22	1, 25	1, 28	1, 31	1, 34	1, 37
	x, y	3, 968	3, 1250	3, 1568	3, 1922	3, 2312	3, 2738
	L	457·1789;	1385809;	13·169909;	3353341;	193·25357;	1321·5233;
	M	13·82609;	73·24133;	2736673;	97·41953;	349·16729;	13·625369;
T_r	r	16	18	20	22	24	26
	t, u	47, 1	53, 1	59, 1	65, 1	71, 1	77, 1
	x, y	1103, 1106	1403, 1406	1739, 1742	2111, 2114	2519, 2522	2963, 2966
	L	13·82609;	73·24133;	2736673;	97·41953;	349·16729;	13·625369;
	M	1385809;	13·169909;	3353341;	193·25357;	1321·5233;	9500089;

Case IV, page 204. Chain S_ρ, T_r, $S_{\rho+2}$, T_{r+2}, &c.

S_ρ	ρ	1	3	5	7	9	11	13
	ξ, η	1, 1	3, 1	5, 1	7, 1	9, 1	11, 1	13, 1
	x, y	3, 2	27, 2	75, 2	147, 2	243, 2	363, 2	507, 2
	L	1;	373;	3769;	13·1249;	13·3637;	61·1801;	61·3613;
	M	61;	13·109;	8389;	28753;	37·1993;	13·12157;	409·733;
T_r	r	2	4	6	8	10	12	14
	t, u	1, 2	1, 4	1, 6	1, 8	1, 10	1, 12	1, 14
	x, y	11, 13	47, 49	107, 109	191, 193	299, 301	431, 433	587, 589
	L	61;	13·109;	8389;	28753;	37·1993;	13·12157;	409·733;
	M	373;	3769;	13·1249;	13·3637;	61·1801;	61·3613;	13·37·829;

S_ρ	ρ	15	17	19	21	23	25
	ξ, η	15, 1	17, 1	19, 1	21, 1	23, 1	25, 1
	x, y	675, 2	867, 2	1083, 2	1323, 2	1587, 2	1875, 2
	L	13·37·829;	37·18061;	709·1489;	229·6949;	13·177601;	97·33457;
	M	520609;	13·193·337;	13·100237;	37·61·853;	1237·2221;	3808429;
T_r	r	16	18	20	22	24	26
	t, u	1, 16	1, 18	1, 20	1, 22	1, 24	1, 26
	x, y	767, 769	971, 973	1199, 1201	1451, 1453	1727, 1729	2027, 2029
	L	520609;	13·193·337;	13·100237;	37·61·853;	1237·2221;	3808429;
	M	37·18061;	709·1489;	229·6949;	13·177601;	97·33457;	4441477;

S_ρ	ρ	27	29	31
	ξ, η	27, 1	29, 1	31, 1
	x, y	2187, 2	2523, 2	2883, 2
	L	4441477;	5941321;	157·49633;
	M	5150713;	13·109·4813;	8865601;
T_r	r	28	30	32
	t, u	1, 28	1, 30	1, 32
	x, y	2351, 2353	2699, 2701	3071, 3073
	L	5150713;	13·109·4813;	8865601;
	M	5941321;	157·49633;	13·337·2293;

46. *Bin-Trin-Aurifeuillian Sextan.* It will now be shown that certain Sextans (N_{vi}) are expressible at once as Bin-Aurifeuillians (B), Trin-Aurifeuillians (T), and Trin-Aut-Aurifeuillians (T′).

By combining the conditions (84), (88a),

$$x = \xi^2, \quad y = 2\eta^2, \quad y^2 - x^2 = 3z^2 \quad \text{..............} \quad (102),$$

the Sextan (N_{vi}) will be found to be expressible at once in the four forms

$$\frac{x^6+y^6}{x^2+y^2} = \frac{(\xi^3)^4+4(\eta^3)^4}{\xi^4+4\eta^4} = \frac{(y^2)^3+(3z^2)^3}{y^2+3z^2} = \frac{(x^2)^3-(3z^2)^3}{x^2-3z^2} \quad \text{......} \quad (102a),$$

i.e. $\qquad N_{vi} \quad = \quad B \quad = \quad T \quad = \quad T'.$

The above conditions are included in the single condition

$$4\eta^4 - \xi^4 = 3z^2 \quad \text{..............................} \quad (102b).$$

If ξ_1, η_1, z_1 be one known solution of this last equation, a second solution (ξ_2, η_2, z_2) is given * by

$$\xi_2 = \xi_1(27z_1^4 \sim 4\xi_1^8), \quad \eta_2 = \eta_1(27z_1^4 \sim 64\eta_1^4), \quad z_2 = z_1\left\{324z_1^8 \sim 3(4\eta_1^4+\xi^4)^4\right\}$$
$$\text{.........} \quad (102c).$$

Similarly from (ξ_2, η_2, ζ_2) a third solution may be derived; and so on *ad inf.*, but the numbers ξ, η, ζ rise too rapidly to be of practical use.

46a. *Ex.* $\xi_1 = 1$, $\eta_1 = 1$, $z_1 = 1$ is an obvious solution. This leads to

$$\xi_2 = 23, \quad \eta_2 = 37, \quad z_2 = 1551\,; \quad x_2 = 529, \quad y_2 = 2738.$$

whence

$$N_2 = \frac{529^6+2738^6}{529^2+2738^2} = \frac{23^{12}+2^6.37^{12}}{23^4+2^2.37^4} = \frac{2738^6+3^3.1551^6}{2738^2+3.1551^2} = \frac{529^6 \sim 3^3.1551^6}{529^2 \sim 3.1551^2};$$

and the forms B, τ lead to the complete factorisation—

$$N = 13.61.4621 : 457.32353 = 61.32353 : 13.457.4621\,;$$

Another form of second solution † has been derived from the joint solution of

$$2\eta^2 - \xi^2 = \zeta'^2, \quad 2\eta^2 + \xi^2 = \zeta''^2, \quad z = \zeta', \zeta''\,;$$

but it is too long to quote here. It leads to the second solution in high numbers—

$$\xi_2 = 52487, \quad \eta_2 = 40573, \quad z_2 = 139.323.23183.$$

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* See Desboves's *Mémoire sur la résolution en nombres entiers de l'équation* $aX^m + bY^m = cZ^n$, Art. 16, Result (67); pub. in Nouv. Ann. de Mathém., 2e Sér., t. xviii, 1879.

† See Ed. Lucas's *Recherches sur l'Analyse Indéterminée*, &c., Moulin, 1873, § iv, p. 37.

47. *Four-factor Bin-Aurifeuillians*, (N).

Take

$$N = x^4 + 4y^4 = X^4 - x'^4 = PQRS \text{ .... } (103),$$

To construct this form N, take

$$\left. \begin{array}{c} x = 2\eta^4 - \xi^4, \quad y = 2\xi\eta^3, \quad X = 2\eta^4 + \xi^4, \quad x' = 2\xi^3\eta \\ r = X + 4\xi^2\eta^2, \quad s = x' + 2y \end{array} \right\} \text{......} (103a).$$

It will be found that this gives N of required form, and that

$$P = X - x', \quad Q = X + x', \quad R = r - s, \quad S = r + s \text{ ...... } (103b).$$

As x, x' are evidently *interchangeable* in the above formulæ, this solution provides also the set of conjugate numbers

$$\left. \begin{array}{ll} & N' = x'^4 + 4y'^4 = X^4 - x^4 = P'Q'R'S', \quad s' = x + 2y \\ \text{where} & P' = X - x, \quad Q' = X + x, \quad R' = r - s', \quad S' = r + s' \end{array} \right\} \text{......} (103c).$$

Ex. The Tables on pages 270, 271 give numerous Examples of the numbers N, wherein $\xi = 1$ throughout, completely factorised up to very high numbers.

47a. *Special factorisable Quartans.* Changing y into Y and x' into y in the above formula, they evidently provide for the construction of the following factorisable Quartans along with their Factors (L, M).

$$N = x^4 + y^4 = X^4 - 4Y^4 = LM \text{ .... } (103d).$$

Ex. The Table on page 128 gives numerous Examples of these numbers, along with their factorisation. The formulæ have been slightly modified, as shown at head of Table; the formulæ of the right-hand Table can be used with ξ, η both odd, as in Examples below—

ξ, η	x, y	X, Y	L : M	ξ, η	x, y	X, Y	L : M
3, 1	79, 54	83, 6	17.401 : 6961;	1, 3	161, 6	163, 54	89.233 : 32401;

CHAP. VI. *Dimorphs, &c., Polymorphs.*

48. *Dimorphs, &c.* A number N which is expressed in two *different ways*—*i.e.* for different values of x, y—in the *same* (*algebraic*) *form*, as

$$N = f(x_1, y_1) = f(x_2, y_2) \quad \ldots\ldots\ldots\ldots \quad (104)$$

is said to be *Dimorph* (in that form). If expressed in three or more *different ways* in the same form, it is said to be *Trimorph*, &c., or *Polymorph.*

[But note that Automorphs of the 2-ic form ($t^2 - Du^2$),—(see Art. 7),—and also mere *variant* (algebraically interconvertible) forms such as Duans and Half-Duans (Art. 10) and the triple variants of Cubans and Trito-Cubans (Art. 13) are *not considered different ways* of expression in those forms.]

48a. *Dimorph Forms, Table of.* A Table of 2-ic, 3-ic, 4-tan, 4-tic, and 6-tan (algebraic) Dimorph Forms is given on p. 226, with conditions and references to the articles in the Text, and to the Tables of Examples.

49. *Polymorph Duans and Cubans.* These are the only kinds of n-ans in which Polymorphs are known to exist. Being of the 2-ic forms

$$N_{ii} = a^2 + b^2, \quad N_{iii} \text{ and } N'_{iii} = A^2 + 3B^2 \quad \ldots\ldots\ldots \quad (105),$$

they are expressible in 2^{r-1} *different ways* (Art. 8a) in those forms when—(and only when)—they are product of r *different* odd primes, or prime-powers, and by quite simple Rules.

49a. *Polymorph Duans.* If $N_{ii} = (x_1^2 + y_1^2)(x_2^2 + y_2^2)$, then

$$N_{ii} = (x_1 x_2 \sim y_1 y_2)^2 + (x_1 y_2 + x_2 y_1)^2 = (x_1 x_2 + y_1 y_2)^2 + (x_1 y_2 \sim x_2 y_1)^2$$
$$= X_1^2 + Y_1^2 \ldots\ldots\ldots = X_2^2 + Y_2^2, \text{ [a } \textit{Dimorph}\text{]} \quad \ldots\ldots\ldots\ldots \quad (106).$$

By similar conformal multiplication by a third factor $(x_3^2 + y_3^2)$, *each* of the above expressions yields two new ones, thus giving a *Tetramorph*, and so on.

Ex. The smallest *Dimorph-Duan* is

$$N_{ii} = 65 = 5.13 = (1^2+2^2)(3^2+2^2) = 1^2+8^2 = 7^2+4^2,$$

and the smallest *Tetramorph* is

$$N_{ii} = 1105 = 5.13.17 = (1^2+2^2)(3^2+2^2)(1^2+4^2)$$
$$= 9^2+32^2 = 23^2+24^2 = 31^2+12^2 = 33^2+4^2.$$

49b. *Polymorph Cubans.* The simplest mode of forming Polymorph Cubans is by aid of their 2-ic form (A^2+3B^2). Thus, taking

$$N_{iii} = P_1.P_2 = (A_1^2+3B_1^2)(A_2^2+3B_2^2),\ [P_1, P_2 \text{ mutually prime}] \ldots \quad (107).$$

Then

$$N_{iii} = (A_1A_2 \sim 3B_1B_2)^2 + 3(A_1B_2+A_2B_1)^2 = (A_1A_2+3B_1B_2)^2+3(A_1B_2 \sim A_2B_1)^2$$
$$= \mathbf{A}_1^2+3\mathbf{B}_1^2 \qquad\qquad = \mathbf{A}_2^2+3\mathbf{B}_2^2 \text{ [a Dimorph 2-ic form]}$$
$$\ldots\ldots\ldots \quad (107a).$$

Each of these latter 2-ic forms may now be converted into its (triple) Cuban Form by the formulæ of Art. 13. Hereby N_{iii} is expressed as a *Dimorph* Cuban.

By *conformal* multiplication by a third factor $P_3 = A_3^2+3B_3^2$, (prime to both P_1, P_2), *each* of the above 2-ic forms $(\mathbf{A}_1, \mathbf{B}_1)$, $(\mathbf{A}_2, \mathbf{B}_2)$ yields *two* new 2-ic forms, giving a new N_{iii}

$$N_{iii} = \mathbf{A}_1'^2+3\mathbf{B}_1'^2 = \mathbf{A}_2'^2+3\mathbf{B}_2'^2 = \mathbf{A}_3'^2+3\mathbf{B}_3'^2 = A_4'^2+3B_4'^2$$
$$\text{[a Tetramorph 2-ic form]} \ldots \quad (107b],$$

and *each* of these last 2-ic forms may be converted into the (triple) Cuban form by the formulæ of Art. 13. Hereby a *Tetramorph* Cuban N_{iii} is obtained.

49c. *Ex.* The *smallest* Dimorph-Cuban is

$$N_{iii} = 91 = 7.13 = (2^2+3.1^2)(1^2+3.2^2) = 4^2+3.5^2 = 8^2+3.3^2,$$

giving

$$N_{iii} = \left(\frac{9^3-1^3}{9-1} = \frac{10^3+1^3}{10+1} = \frac{10^3+9^3}{10+9}\right) = \left(\frac{6^3-5^3}{6-5} = \frac{11^3+5^3}{11+5} = \frac{11^3+6^3}{11+6}\right).$$

The *triple* Cuban forms of this Dimorph are here shown.

And the *smallest* Tetramorph-Cuban is

$$N_{iii} = 1729 = 7.13.19 = 1^2+3.24^2 = 31^2+3.16^2 = 23^2+3.20^2 = 41^2+3.4^2$$
$$= \frac{25^3-23^3}{25-23} = \frac{32^3-15^3}{32-15} = \frac{40^3-3^3}{40-3} = \frac{37^3-8^3}{37-8}.$$

Here *only one* of the triple Cuban forms of this Tetramorph is shown.

50. *Dimorph Cubics.* An interesting Problem is to form the Dimorph Cubics

$$N = x^3 + y^3 = x'^3 + y'^3 \quad \ldots\ldots\ldots\ldots \quad (108).$$

Write $\quad x + y = \lambda, \quad x' + y' = \lambda',$

and $\quad x = \frac{1}{2}(\lambda + 2l), \quad y = \frac{1}{2}(\lambda - 2l), \quad x' = \frac{1}{2}(\lambda' + 2l'), \quad y' = \frac{1}{2}(\lambda' - 2l').$

These lead to $\quad \lambda l^2 - \lambda' l'^2 = \frac{1}{12}(\lambda'^3 - \lambda^3) \ldots\ldots\ldots\ldots \quad (108a).$

If now numerical values be assigned to λ, λ', this last becomes a 2-ic Diophantine equation with l, l' as unknowns. If any solution (l, l') can be found for this, then—by conformal multiplication by the "unit form" $\tau^2 - \lambda\lambda'.v^2 = +1$—an infinite number of solutions (l_1, l_1'), (l_2, l_2'), &c., arise, and hence an infinite series of x, y, x', y'. The Dimorphs N all have the L.C.M. of $\lambda\lambda'$ as a factor.

Ex. (Page 221.) Take $\lambda = 1$, $\lambda' = 7$; this gives $(2l)^2 - 7.(2l')^2 = +114$. The least solution is $11^2 - 7.1^2 = 114$, whence $2l_1 = 11$, $2l_1' = 1$. By help of the "unit-form" $8^2 - 7.3^2 = +1$, the series of solutions on page 221 is found, and from them the x, y, x', y'.

50a. *Problem.* Given a solution (x, y, x', y') of $N = x^3 + y^3 = x'^3 + y'^3$, to find a new Dimorph $\mathbf{N} = X^3 + Y^3 = X'^3 + Y'^3$ such that ...... (109)

$$X = m + x, \quad Y = m + y, \quad X' = mx', \quad Y' = m + y' \quad \ldots \quad (109a).$$

The notation at head of page 222 leads to

$$m(x + y) + x^2 + y^2 = m(x' + y') + x'^2 + y'^2.$$

From this may be deduced

$$m + \tfrac{1}{3}(\lambda + \lambda') = \tfrac{2}{3}Z \div \lambda' = \tfrac{2}{3}Z' \div \lambda = \tfrac{2}{3}N \div \lambda\lambda' \quad \ldots\ldots\ldots \quad (109b).$$

The Table on page 222 contains numerous examples. The original N in each line may be found as above from the assumed λ, λ' in the middle column.

50b. *Trimorph Cubics.* The Table at foot of page 221 gives a number of Examples of the *Trimorph*

$$N = x^3 + y^3 = x'^3 + y'^3 = x''^3 + y''^3 = K.M \quad \ldots\ldots\ldots \quad (110),$$

formed by a process similar to that of last Article.

51. *Factorisation of 2-ic Dimorphs.* A number (N) given in two *different ways* (Art. 7) in the *same 2-ic form*—[*i.e.* with same determinant $(\pm D)$]—may be readily factorised, as follows. Let

$$N = T_1^2 \mp DU_1^2 = T_2^2 \mp DU_2^2, \quad [T_2 > T_1] \;\ldots\ldots\ldots\ldots \quad (111).$$

This leads to

$$\begin{aligned} N &= \{(T_1U_2)^2 - (T_2U_1)^2\} \div (U_2^2 - U_1^2) \\ &= (T_1U_2 - T_2U_1)(T_1U_2 + T_2U_1) \div (U_2 - U_1)(U_2 + U_1) \quad \ldots \; (111a), \end{aligned}$$

whereby,—after cancelling out all the factors of the denominator,—N is (usually) resolved into two co-factors.

51a. *2-ic forms of above factors.* When the two co-factors of N are both expressible in the same 2-ic form as N itself, their forms may be found as follows :—Let

$$N = T_1^2 \mp DU_1^2 = T_2^2 \mp DU_2^2 = L.M, \quad [T_2 > T_1],$$

where $$L = t_1^2 \mp Du_1^2, \quad M = t_2^2 \mp Du_2^2.$$

Then, it will be found that—

$$\frac{t_1}{u_1} = \frac{T_2 + T_1}{U_2 - U_1} = \frac{U_2 + U_1}{T_2 - T_1}.D, \quad \frac{t_2}{u_2} = \frac{T_2 + T_1}{U_2 + U_1} = \frac{U_2 - U_1}{T_2 - T_1}.D \ldots \; (111b).$$

These formulæ give *apparently* only the *ratios* of the required 2-ic parts (t_1, u_1), (t_2, u_2) : but, on reducing the fractions to their *lowest terms*, the numerators give t_1 and t_2, and the denominators give u_1 and u_2.

[The uncertainty as to the $(\pm)$ signs of the given T_1, U_1, T_2, U_2—(see Art. 6c)—causes some uncertainty in applying these formulæ ; the results obtained (111b) should be verified by forming the product $LM = N$. These formulæ are more easily computed than the preceding one (111a), which requires forming the *products* T_1U_2, T_2U_1.]

52. *Dimorph Quartans.* One of the most interesting cases of *dimorphism* is that of a *Dimorph Quartan* or *Half-Quartan.*

$$N_{iv} = x^4 + y^4 = x'^4 + y'^4 \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (112).$$

Euler has treated of the mode of constructing these numbers in three *Memoirs (1772–1780), and in the 2nd Memoir

* Euler's *Comment. Arithm.*, Petropol., 1849 ; t. i, pp. 473–476 ; t. ii, pp. 281–293 and 450–456.

gives* explicit general formulæ for the sums $(x \pm x')$, $(y \pm y')$ from which those of x, y, x', y' can be at once written down. Mr. Desboves has also treated† this question independently (1880), and has given formulæ for x, y, x', y' identical with Euler's. These are—(in changed notation)—

$$x = \lambda . \{(t^2+u^2)(2t^5-t^4u+2t^3u^2+18t^2u^3-u^5) \quad +8tu^2(2t^4+u^4)\} \ldots \text{(112a)},$$

$$y = \lambda . \{(t^2+u^2)(t^5-18t^3u^2+2t^2u^3+tu^4+2u^5) \quad +8t^2u(t^4+2u^4)\} \ldots \text{(112b)},$$

$$x' = \lambda . \{(t^2+u^2)(2t^5+t^4u+2t^3u^2-18t^2u^3+u^5) \quad +8tu^2(2t^4+u^4)\} \ldots \text{(112c)},$$

$$y' = \lambda . \{(t^2+u^2)(-t^5+18t^3u^2+2t^2u^3-tu^4+2u^5)+8t^2u(t^4+2u^4)\} \ldots \text{(112d)},$$

where t, u are any two (mutually prime) integers

and $\lambda = 1$, if $tu = \epsilon$; $\lambda = \frac{1}{64}$, if $tu = \omega$ .... (112e).

Note that—If $x = \phi(t, u)$, then

$$y = \phi(u, -t), \quad x' = \phi(t, -u), \quad y' = \phi(u, t) \ldots\ldots\ldots \text{(112f)}.$$

These formulæ are of the 7th degree, so that x, y, x', y' rise in magnitude rapidly with t, u: they always yield one of x, y and one of x', y' even, so that N is always *odd*.

Euler's first ‡Memoir gives a different general method of constructing these numbers: the formulæ are of the 17th degree § in the parameter (Euler's b) involved; so it has not been thought worth while to detail them here; they lead to *odd* values of x, y, x', y', and therefore to *Half-Quartans*, (since N is *even*).

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* *Op. cit.*, t. ii, pp. 288, 289. Euler's p, q, r, s are the present x, x', y, y'; his f, g are the present t, u.

† Desboves's *Sur la résolution en nombres entiers ou complexes de l'équation* $U^n \pm V^n = S^n + W^n$, (Assocn. Franc. pour l'Avancemt. des Sciences, 1880). His U, V, S, W are the present x, y, x', y'; his x, y are the present t, u; but his formula for W requires correction (as above). His first Example contains a misprint (4176 should be 1176).

‡ *Op. cit.*, t. i, pp. 473–476.

§ This does not necessarily involve very high numbers in the result; as the x, y, x', y' given by the formulæ sometimes contain a high common factor which may be cancelled out. *E.g.* Euler's 2nd Example, pp. 475, 476 is incorrectly worked (the last on p. 475—and along with it all that follows—is incorrect): the corrected values (given in t. ii, p. 456) are $\ngtr$ 12231; the original incorrect figures are reprinted in M. Desboves's Memoir (above quoted) with a new misprint.

52a. *Dimorph Quartans, Arithmetical Factorisation.* From the symmetry of the quartan expressions in x, y, and in x', y', the method of Art. 51 may be applied in two ways, so that

$$\mathrm{N} = \frac{(xy')^4-(yx')^4}{(y'^4-y^4)=(x^4-x'^4)} = \frac{(xx')^4-(yy')^4}{(x'^4-y^4)=(x^4-y'^4)} = \mathrm{L.M} \quad \ldots \quad (113).$$

These apparently break up into three factors each, for

$$\mathrm{N} = \frac{(xy'-yx')(xy'+yx')(x^2y'^2+y^2x'^2)}{(y'-y)(y'+y)(y'^2+y^2)} = \frac{(xx'-yy')(xx'+yy')(x^2x'^2+y^2y'^2)}{(x'-y)(x'+y)(x'^2+y^2)}$$
$$\ldots\ldots\ldots \quad (113a),$$

where the denominators may be changed respectively to

$$(x-x')(x+x')(x^2+x'^2), \quad \text{or} \quad (x-y')(x+y')(x^2+y'^2) \ldots\ldots \quad (113b).$$

But two sets of L, M may be readily formed from the Dimorph (a, b), (a′ b′) 2-ic partitions of N. Thus if

$$\mathrm{L} = \alpha_1^2+\beta_1^2,\ \mathrm{M} = \alpha_2^2+\beta_2^2;\quad \mathrm{L}' = \alpha_1'^2+\beta_1'^2,\ \mathrm{M}' = \alpha_2'^2+\beta_2'^2 \ \ldots \quad (113c).$$

Then, by the formulæ of Art. 51a, the *ratios* of the several α, β are given by

$$\frac{\alpha_1}{\beta_1} = \frac{x'^2+x^2}{y^2-y'^2} = \frac{y^2+y'^2}{x'^2-x^2}, \qquad \frac{\alpha_2}{\beta_2} = \frac{x'^2+x^2}{y^2+y'^2} = \frac{y^2-y'^2}{x'^2-x^2} \ \ldots\ldots \quad (113d),$$

$$\frac{\alpha_1'}{\beta_1'} = \frac{x'^2+y^2}{x^2-y'^2} = \frac{x^2+y'^2}{x'^2-y^2}, \qquad \frac{\alpha_2'}{\beta_2'} = \frac{x'^2+y^2}{x^2+y'^2} = \frac{x^2-y'^2}{x'^2-y^2} \ \ldots\ldots \quad (113e),$$

and the actual values of the several α, β are formed by reducing the above fractions to their lowest terms.

As N is hereby resolved into two co-factors L, M in two different ways, it is evident that these L, M may be usually further resolved (into two co-factors each).

52b. *Odd Dimorph Quartans, Algebraic Factorisation.* In the case of odd Dimorph Quartans ($xy = \epsilon$, $x'y' = \epsilon$, $\mathrm{N}_{\mathrm{iv}} = \omega$), where x, y, x', y' are given by (112a–d), the late Mr. C. E. Bickmore discovered* the algebraic resolution of each of L, M into two co-factors, and therefore of N into four co-factors, say—

$$\mathrm{L} = \mathrm{L}'\mathrm{L}'',\ \mathrm{M} = \mathrm{M}'\mathrm{M}'';\quad \mathrm{N}_{\mathrm{iv}} = \mathrm{L.M} = (\mathrm{L}'\mathrm{L}'')(\mathrm{M}'\mathrm{M}'') \ldots \quad (114),$$

* This was communicated to the writer in 1898.

and has presented them in their (a, b), (c, d) partitions as follows—

$$L' = \lambda' . (t^4+u^4) \ldots\ldots \text{(114a)},$$

$$L'' = \lambda'' . \{(t^4+14t^2u^2+u^4)^2+4^2(t^4-u^4)^2\} \ldots\ldots \text{(114b)}$$

$$= \lambda'' . \{(3t^4+10t^2u^2+3u^4)^2+2(2t^4-4t^2u^2+2u^4)^2\} \ldots\ldots \text{(114b')},$$

$$M' = \mu' . \{(t^4-10t^2u^2+u^4)^2+(4tu)^2(t^2+u^2)^2\} \ldots\ldots \text{(114c)}$$

$$= \mu' . \{(t^4-2t^2u^2+u^4)^2+2(8t^2u^2)^2\} \ldots\ldots \text{(114c')},$$

$$M'' = \mu'' . \{(t^4-2t^2u^2+u^4)^2+(8tu)^2(t^2+u^2)^2\} \ldots\ldots \text{(114d)}$$

$$= \mu'' . \{(t^4+14t^2u^2+u^4)^2+2(4tu)^2(t^2-u^2)^2\} \ldots\ldots \text{(114d')},$$

[t, u are two (mutually prime) integers]

where $\left.\begin{array}{ll} \lambda' = \lambda'' = \mu' = \mu'' = 1, & \text{when } tu = \epsilon \\ \lambda' = \frac{1}{2},\ \lambda'' = \frac{1}{256} = \mu'',\ \mu' = \frac{1}{128}, & \text{when } tu = \omega \end{array}\right\} \ldots$ (114e).

The following remarkable relation holds between the Residues of L'', M', M'' modulo L'

$$L'' \equiv M' \equiv M'' \equiv 132\,(tu)^4 \pmod{L'}, \quad \text{when } tu = \epsilon \quad \text{(114f)},$$

$$L''/\lambda'' \equiv M'/\mu' \equiv M''/\mu'' \equiv 132\,(tu)^4 \pmod{L'/\lambda'}, \quad \text{when } tu = \omega \quad \text{(114g)}.$$

52c. *Odd Dimorph Quartans, Complex Factors.* The late Mr. Bickmore discovered also the complex 4-tic factors* of the above four algebraic factors L', L'', M', M''; viz.

If ρ be an imaginary root of $\rho^4+1=0$,

and $N\phi(\rho)$ denote the "Norm of $\phi(\rho)$,"

where $\phi(\rho)$ denotes a 4-tic complex number; then

$$L' = N(t+u\rho), \quad L'' = N\{t^2-u^2)+2(t^2+u^2)\rho+4(t^2+u^2)\rho^2\} \ldots \text{(115a)},$$

$$M' = N\{(t^2-u^2)+2tu(\rho+\rho^2)\}, \quad M'' = N\{(t+u)^2+(t^2-u^2)\rho+(t-u)^2\rho^2\} \ldots\ldots \text{(115b)}.$$

52d. *Properties of the small algebraic factor* (L'). The *small* algebraic factor (L') of the odd Dimorph Quartan N_{iv} of Art. 52b being itself a Quartan or Half-Quartan [$L' = t^4+u^4$, or $= \lambda(t^4+u^4)$], it follows that every Quartan and Half-Quartan (L') will generate a large odd Dimorph Quartan, whereof it is the least algebraic factor.

Hence also every Dimorph Quartan, say

$$L_1 = t_1^4+u_1^4 = t_2^4+u_2^4 = L_2$$

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* See note on page lxiii.

will generate *two* new Dimorph Quartans, say

$$N_1 = x_1^4 + y_1^4 = x_1'^4 + y_1'^4, \quad N_2 = x_2^4 + y_2^4 = x_2'^4 + y_2'^4 \;\ldots\ldots\; (115),$$

whose least algebraic factors are the equal Quartans $L_1 = L_2$.

52e. *Dimorph Quartans, Examples.* The Table on p. 127 gives the details* of 16 Examples of these Dimorphs.

Ex. 1 to 13 are examples of formulæ (112a–f) of *odd* Dimorphs up to 24 figures with the four algebraic factors L′, L″, M′, M″ completely factorised.

Ex. 14, 15 are Euler's own examples of his first Method.

Ex. 16 is Euler's own example of his third Method (too large to completely factorise).

53. *Allied Quartic Dimorphs.* The values of x, y, x', y' which yield Dimorph Quartans (Art. 52) give rise obviously to two solutions of the following† Quartic Dimorphs.

$$1^\circ.\quad x^4 - x'^4 = y'^4 - y^4; \qquad 2^\circ.\quad x^4 - y'^4 = x'^4 - y^4 \;\ldots\ldots\; (116).$$

Also, writing

$$1^\circ.\quad x = \xi + \xi',\quad x' = \xi - \xi',\quad y' = \eta' + \eta,\quad y = \eta' - \eta;$$

$$2^\circ.\quad x = \xi + \eta',\quad y' = \xi - \eta',\quad x' = \xi' + \eta,\quad y = \xi' - \eta,$$

the same values of x, y, x', y' yield two solutions of the Dimorphs

$$1^\circ.\quad \xi\xi'(\xi^2 + \xi'^2) = \eta\eta'(\eta^2 + \eta'^2); \qquad 2^\circ.\quad \xi\eta'(\xi^2 + \eta'^2) = \xi'\eta(\xi'^2 + \eta^2) \;\ldots\; (116a).$$

54. *Dimorph Sextans.*

By subjecting the Sextan (N_{vi}) to the Pythagorean condition

$$x^2 = y^2 + z^2, \quad [x \text{ and } y \textit{ odd}; \; z \textit{ even}] \;\ldots\ldots\ldots\ldots\; (117),$$

the Sextan becomes Dimorph as below, and its factorisation is obvious. Its duplicate 2-ic forms (a, b), (A, B), (A′, B′) are also shown.

$$N_{vi} = \frac{x^6 + y^6}{x^2 + y^2} = \frac{x^6 + z^6}{x^2 + z^2} = \frac{y^6 \sim z^6}{y^2 \sim z^2} \;\ldots\ldots\ldots\ldots\; (117a)$$

$$= x^4 - x^2y^2 + y^4 = x^4 - x^2z^2 + z^4 = y^4 + y^2z^2 + z^4 \;\ldots\ldots\; (117b)$$

$$= z^4 + (xy)^2 = y^4 + (xz)^2 = x^4 - (yz)^2 = L.M \;\ldots\; (117c)$$

$$= (y^2 + \tfrac{1}{2}z^2)^2 + 3(\tfrac{1}{2}z^2)^2 = (y^2 - z^2)^2 + 3(yz)^2 \;\ldots\; (117d)$$

$$= (x^2 + z^2)^2 - 3(xz)^2 = (x^2 + y^2)^2 - 3(xy)^2 \;\ldots\; (117e)$$

* This work has been verified by Miss B. E. Haselden.

† This solution is given by Euler in connexion with that of the Dimorph Quartan; *op. cit.*, t. ii, pp. 282, 287–293.

and the two co-factors (L, M) are the twin Cubans (N'_{iii}, N_{iii}) given below along with their three 2-ic partitions.

$L = N'_{iii}$	$M = N_{iii}$
$= (y^3+z^3) \div (y+z)$	$= (y^3 \sim z^3) \div (y \sim z)$ .... (118)
$= y^2 - yz + z^2$	$= y^2 + yz + z^2$ .... (118a)
$= x^2 - yz$	$= x^2 + yz$ .... (118b)
$= \left(\frac{x+y-z}{2}\right)^2 + \left(\frac{x-y+z}{2}\right)^2$	$= \left(\frac{x+y+z}{2}\right)^2 + \left(\frac{y+z-x}{2}\right)^2$ .... (118c)
$= (y \sim \frac{1}{2}z)^2 + 3(\frac{1}{2}z)^2$	$= (y + \frac{1}{2}z)^2 + 3(\frac{1}{2}z)^2$ .... (118d)
$= \left(\frac{3x-y-z}{2}\right)^2 - 3\left(\frac{y+z-x}{2}\right)^2$	$= \left(\frac{3x-(y\sim z)}{2}\right)^2 - 3\left(\frac{x-(y\sim z)}{2}\right)^2$ ... . (118e).

Note that $N_{vi} = \frac{1}{2}(x^4+y^4+z^4) = (xy)^2 + (xz)^2 - (yz)^2$ .... (118f).

54a. *Factorisation of Dimorph Sextans.* The Table on pages 190–194 gives the elements (x, y, z) of the Pythagorean $x^2 = y^2 + z^2$, and the complete factorisation of the twin factors (L, M) of all the Dimorph Sextans (N_{vi}) thence arising up to the limit $x = 2441$; and thence up to $x = 2917$ gives only those cases in which L and M $\not>$ *9.10^6.

54b. Allied with the Dimorph Sextan (N_{vi}) is the group of four equalities

$$(\xi^4+\eta^4+\zeta^4)^2 = (x'^2+y'^2+z'^2) = 2(x'^4+y'^4+z'^4) = 2(\xi^8+\eta^8+\zeta^8) \quad (119).$$

Here $N_{vi} = (\zeta^6+\xi^6) \div (\zeta^2+\xi^2) = (\zeta^6+\eta^6) \div (\zeta^2+\eta^2) = \frac{1}{2}(\xi^4+\eta^4+\zeta^4)$ (119a).

The Table at foot of page 129 gives numerous solutions $(\xi, \eta, \zeta, x', y', z')$ of (119) with Rules for their formation.

55. *Dimorph Sextans in Chains.* Dimorph Sextans do not appear to form Chains among each other, nor yet in combination with either Trin-Aurifeuillian Sextans or Sext-Aurifeuillians. But Dimorph Sextans do form Chains in combination with Bin-Aurifeuillian Sextans, alternate Links being taken from each form: these forms will be denoted by B, D.

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* Being the limit of the large Factor-Tables existing up to the time of publication of Lehmer's large Tables.

56. *Simple Bin-Aurifeuillian* (B) *cum Dimorph* (D) *Sextan Chain.*

$B = (y_r'^6 + 1^6) \div (y_r'^2 + 1^2) = L_r'.M_r'$, where $y_r' = 2\eta_r^2 = 2r^2$, $[\eta_r = r]$ ... (120a),

$D = (x_r^6 + y_r^6) \div (x_r^2 + y_r^2) = L_r.M_r$,

where $x = (r+1)^2 + r^2$, $y = 2r+1$, $z = x-1$ ... (120b).

For the formulæ for L_r', M_r' see Art. 42a, and for those for L_r, M_r see Art. 54. These give $M_r' = L_r$, $M_r = L_{r+1}'$, showing the Chain.

56a. *Ex.* (Page 198.) The Table gives only the elements (r, x, y) required for forming the co-factors L_r', M_r', L_r, M_r of B and D. The Table below gives fuller detail for a few values of r. The actual values of L_r', M_r' are taken from the Table on pages 172, 173; those of L_r, M_r are taken from the Table on pages 190–194.

	$r =$	1	2	3	4	5	...
B	$y_r' = 2r^2$	2	8	18	32	50	
	L_r', M_r'	1:13;	37:109;	229:457;	13.61:1321;	13.157:3061;	...
D	x_r, y_r, z_r	5, 3, 4	13, 5, 12	25, 7, 24	41, 9, 40	61, 11, 60	
	L_r, M_r	13:37;	109:229;	457:13.61;	1321:13.157;	3061:13.357;	...

	$r =$	...	35	36	37	...
B	$y_r' = 2r^2$		2450	2592	2738	
	L_r', M_r		313.18637: 13.37.12841;	6534361:6907753;	7296697: 7702069;	
D	x_r, y_r, z_r		2521, 71, 2520	2665, 73, 2664	2813, 75, 2812	
	L_r, M_r	...	13.37.12841: 6534361;	6907753:7296697;	7702069: 13.241.2593;	...

This Table stops at $r = 37$, giving $x = 2813$, being the limit for which complete factorisation of the L_r, M_r of D is available from the Table of D on pages 190–194.

The elements (r, x, y) connecting the B, D are, however, shown on page 198 without a break up to $r = 128$, $x = 33025$. It is clear that the L_r, M_r of D up to this high limit may be taken from the Table of B on pages 172, 173 (from the connexion $M_r' = L_r$, $M_r = L_{r+1}'$).

56b. *General Bin-Aurifeuillian* (B) *cum Dimorph* (D) *Sextan Chain.*

Here

$B = (x_\rho^6 + y_\rho^6) \div (x_\rho^2 + y_\rho^2) = L_\rho.M_\rho$; $x_\rho = \xi_\rho^2$, $y_\rho = 2\eta_\rho^2$; $\rho = \omega$ ... (121a),

$D = (x_r^6 + y_r^6) \div (x_r^2 + y_r^2) = L_r.M_r$; $x_r^2 = y_r^2 + z_r^2$, $r = \rho + 1 = \epsilon$ ... (121b).

Here two classes arise

CLASS 1°. In B; $x_\rho = \xi_\rho^2 = \xi^2\ (const.) = \omega$ .... (121c).

In D; $u_r = u = \xi\ (const.)$; $t_{r+2} = t_r + 2u = \omega$; $t_r, u_r = \omega$.

$x_r = \frac{1}{2}(t_r^2 + u_r^2) = \omega,\ y_r = \frac{1}{2}(t_r^2 - u_r^2) = \epsilon,\ z_r = t_r u_r = \omega$ .... (121d).

CLASS 2°. In B; $y_\rho = 2\eta_\rho^2 = 2\eta^2\ (const.),\ \xi_{\rho+2} = \xi_\rho + 2\eta = \omega$...... (121e).

In D; $u_r = u = \eta\ (const.)$; $t_{r+2} = t_r + 2u$;

t_r, u_r are one *odd*, one *even*.

$x_r = t_r^2 + u_r^2 = \omega,\ y_r = t_r^2 - u_r^2 = \omega,\ z_r = 2t_r u_r$ ... (121f).

Hereby in both classes a Chain is established given by

$$M'_\rho = L_r, \quad M_r = L'_{\rho+2}.$$

56c. *Ex.* The Tables on pages 199, 200 give the elements (ρ, ξ_ρ, t_r, x_r) of a number of these Chains, (Class 1° on page 199, Class 2° on page 200), from which the co-factors L'_ρ, M'_ρ of B and L_r, M_r of D may be computed, with the help of the other elements ($\eta_\rho, x_\rho, y_\rho, u_r, y_r, z_r$), as above, from the formulæ of Arts. 42a, 54.

The factorisation of L'_ρ, M'_ρ may be taken from the Table on pages 174–179, and that of L_r, M_r from the Table on pages 190-194. The Chain-elements on pages 199, 200 are given up to the limits for which those factorisations are available.

Detail Ex. Fuller details of the first few Links of the first Chain of each Class 1°, 2° are given in the short Tables below, sufficient to show the Chain-property.

CLASS 1°. Page 199.

	ρ	1	3	5	7	9	...
B	ξ_ρ, η_ρ	3, 1	3, 4	3, 7	3, 10	3, 13	
B	x_ρ, y_ρ	9, 2	9, 32	9, 98	9, 200	9, 338	
B	L'_ρ, M'_ρ	37 : 13.13;	409 : 2377;	6073 : 15061;	13.37.61 : 54421;	73.1237 : 97.1489;	
D	t_r, u_r	5, 3	11, 3	17, 3	23, 3	29, 3	
D	x_r, y_r, z_r	17, 8, 15	65, 56, 33	149, 140, 51	269, 260, 69	425, 416, 87	
D	L_r, M_r	13.13 : 409;	2377 : 6073;	15061 : 13.37.61;	54421 : 73.1237;	97.1489 : 157.1381;	...

CLASS 2°. Page 200.

	ρ	1	3	5	7	9	...
B	ξ_ρ, η_ρ	1, 1	3, 1	5, 1	7, 1	9.1	
B	x_ρ, y_ρ	1, 2	9, 2	25, 2	49, 2	81, 2	
B	L'_ρ, M'_ρ	1 : 13;	37 : 13.13;	409 : 13.73;	1789 : 3217;	5233 : 8221;	
D	t_r, u_r	2, 1	4, 1	6, 1	8, 1	10, 1	
D	x_r, y_r, z_r	5, 3, 4	17, 15, 8	37, 35, 12	65, 63, 16	101, 99, 20	
D	L_r, M_r	13.37;	13.13 : 409;	13.73 : 1789;	3217 : 5233;	8221 : 13.937;	...

57. *Dimorph Sums of 4th Powers.* $\Sigma(x^4) = \Sigma(x'^4)$.

Tables of Dimorph Sums of *several* (3, 4, ... 7) fourth powers, with Rules for their formation, are given on pages 273 (at foot) and 274.

58. *Dimorph Trinomial Quartic Forms.* A List of such algebraic forms is given on page 226. Numerical Tables of three Forms, with Rules for their formation, and copious numerical Examples, are given on pages 223 to 225, as below. Complete factorisation is given in each case.

P. 223.

$N = x^4 - kx^2y^2 + y^4 = x'^4 - kx'^2y'^2 + y'^4$; $[k = \mathrm{a}^2 + \mathrm{b}^2,\ x = x']$ .... (122).

Least solutions for every $k = \mathrm{a}^2 + \mathrm{b}^2 \neq \square$, and $\not> 101$; [a prime to b].

P. 224.

$N = ax^4 - x^2y^2 + ay^4 = ax'^4 - x'^2y'^2 + ay'^4$; $[a = \alpha^2 + \beta^2,\ x = a\xi = x']$ (123).

Least solutions for every $a = \alpha^2 + \beta^2 \neq \square$, and $\not> 101$; [α prime to β].

P. 225.

$N = X^4 + KX^2Y^2 + Y^4 = X'^4 + X'^2Y'^2 + Y'^4$; $[(K+2)(2K-12) = \alpha^2 + \beta^2]$

$N = \frac{1}{16}(ax^4 - kx^2y^2 + ax^4) = \frac{1}{16}(ax'^4 - kx'^2y'^2 + ay'^4)$; .... (124).

$K + 2 = a$, $(2K - 12) = k$, $ak = \alpha^2 + \beta^2$, [a prime to k, α prime to β].

Several solutions for $a \not> 26$, $k \not> 36$, $K \not> 24$.

The three above Dimorphs are all symmetric functions in x, y. The only solutions obtained are those in which $x = x'$. It will be seen that solutions with $-k$ are easier than those with $+K$.

59. *Dimorph Sums and Diffces.* A Table of Dimorph Sums and Differences of two n-ans, viz. of N_{iv} and N_{vi}, of N_{viii} and N_{xii}, &c., with Rules for their formation, is given on page 276.

60. *Dimorph Bin-Aurifeuillians.* These may be found as follows

Let $N = x^4 + 4y^4 = x'^4 + 4y'^4 = N'$ .... (125).

Then $N = (x^2 - 2y^2)^2 + (2xy)^2 = (x'^2 - 2y'^2)^2 + (2x'y')^2$.

Now take $x^2 = x'^2 - 2y'^2$, $y^2 = x'y'$.

As x', y' are supposed *mutually prime*, this involves

$$x' = \tau^2, \quad y' = v^2, \quad y = \tau v,$$

whence $$\tau^4 - 2v^4 = x^2 \quad \text{.............................} \quad (125a).$$

If τ_1, v_1, x_1 be one known solution of this last equation, a second solution (τ_2, v_2, x_2) is given by

$$\tau_2 = \tau_1^4 + 2v_1^4, \quad v_1 = 2\tau_1 v_1 x_1, \quad \text{where } x_1 = \sqrt{\tau_1^4 - 2v_1^4} = t_1 \ldots \quad (125b).$$

A third solution may be derived in the same way from the second, and so on *ad inf.*, but the numbers rise too rapidly to be of practical use.

60a. *Ex.* A few solutions are shown in the Table below.

Nos. 1, 5, 7 are basic solutions (*i.e.* not derivable from a lower solution).

Nos. 2, 3, 4 are derived *in succession* from No. 1, and No. 6 is derived from No. 5.

No.	τ_1, v_1, t_1	τ_2, v_2, t_2	$x = t_2$, $y = \tau v$	x', y'
1	. , . , .	1, 1, 1	1, 1	1, 1
2	1, 1, 1	3, 2, 7	7, 6	9, 4
3	3, 2, 7	113, 84, 7967	7967, 9492	12769, 7056
4	113, 84, 7967	τ_2, v_2, t_2	t_2, y	x', y'
5	. , . , .	1, 13, 239	239, 13	1, 169
6	1, 13, 239	57123, 6214, t_2	t_2, y	x', y'
*7	. , . , .	1343, 1525, t_2	t_2, y	x', y'

The dimorphism aids greatly in factorisation; for $N = L.M = L'.M' = N'$ algebraically; and the factors common to L, M and L', M' can be found by the process of G.C.M. Thus

$N_2 = 5.37.41$; $N_3 = 5.277.389.733.1553.59513$; $N_5 = 5.37.41.277.1553$;

N_7 has $x = t_2 = 2750257$, $y = 2048075$, $x' = 1803649$, $y' = 2325625$,

giving $$N_7 = 389.733.59513.78767173.9547977?$$

[Character of the two large factors not known.]

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* Solution No. 7 is given in Lebesgue's Paper, *Résolutions des Equations Biquadratiques*, pub. in *Journal des Mathém. Pures et Appliquées*, t. 18, p. 83.

61. *Trin-Aurifn. Dimorphs.*

Let $\quad N = x^4 - 3x^2y^2 + 9y^4 = x'^4 - 3x'^2y'^2 + 9y'^4 = N' \;\ldots\ldots\ldots\; (126).$

Expressing N, N′ in two *different* ways in the form $(A^2 + 3B^2)$,

$$N = (x^2 - 3y^2)^2 + 3(xy)^2 = (x'^2 - \tfrac{3}{2}y'^2)^2 + 3(\tfrac{1}{2}y'^2)^2 = N', \quad [y' = \epsilon].$$

Assume $\quad x^2 - 3y^2 = x'^2 - \tfrac{3}{2}y'^2, \quad xy = \tfrac{3}{2}y'^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\; (126a).$

Hereby $\;(x + \tfrac{1}{2}y)^2 - 13(\tfrac{1}{2}y)^2 = x'^2, \; [x, x' \text{ both } \omega \text{ and } \neq 3\omega;\; y, y' \text{ both } \epsilon]$
$\ldots\ldots\ldots\; (126b).$

The general solution of this Diophantine is

$$x + \tfrac{1}{2}y = m^2 + 13n^2, \quad \tfrac{1}{2}y = 2mn, \quad x' = m^2 - 13n^2,$$
$[m, n \text{ are one} = \omega, \text{ one} = \epsilon] \ldots\; (126c).$

Also $\quad xy = \tfrac{3}{2}y'^2 \quad$ involves $\quad x = \xi^2, \; y = 6\eta^2, \; y' = 2\xi\eta, \; [\xi = \omega]$;

and $\quad y = 4mn \quad$ involves $\quad$ one of $m, n = 3\mu^2$, one $= 2\nu^2$.

Hereby $\quad y = 4mn = 6\eta^2 = 24\mu^2\nu^2, \quad x = 9\mu^2 + 52\nu^2 - 12\mu^2\nu^2 = \xi^2 \;\ldots\; (126d).$

One solution is given by $\mu = 1, \; \nu = 1$; giving

$$x = 49, \quad y = 24, \quad x' = 43, \quad y' = 28.$$

$$N_1 = L_1M_1 = 601.7657; \quad N' = 589.7813; \quad N = 13.19.31.601.$$

But, it does not seem easy to satisfy both (126d) in a general manner.

62. *Dimorph Aurifeuillian Factor* (L, M).

When a pair of successive Links (N_r, N_{r+1}) of any Aurifeuillian Chain are expressed in the form

$$N_r = L_rM_r = P_r^2 - Q_r^2, \quad N_{r+1} = L_{r+1}M_{r+1} = P_{r+1}^2 - Q_{r+1}^2$$

whereby $\quad M_r = P_r + Q_r, \quad$ and $\quad L_{r+1} = P_{r+1} - Q_{r+1}$,

and $\quad M_r = L_{r+1}, \quad$ [by the Chain-property (63)].

Also, if $\quad M_r = f(\xi_r, \eta_r), \quad$ then $\quad L_{r+1} = f(\xi_{r+1} - \eta_{r+1}), \quad$ [by (82c)].

Hereby $\;M_r = L_{r+1}\;$ is seen to be *Dimorph* (for all values of r) $\ldots\ldots\; (127).$

[This is a property of *all* Aurifeuillian Chains.]

62a. *Indeterminate 4-tic Equations.* The Dimorph Aurifeuillian Factors which arise from Sextans,—being expressions, each of 4th degree in the auxiliaries (ξ, η)—, the equation (127) is evidently an *indeterminate* equation of 4th degree in $\xi_r, \eta_r, \xi_{r+1}, \eta_{r+1}$ *with known* solution (ξ, η).

[The Tables of Aurifeuillian Chains arising from Sextans furnish numerous Examples.]

63. *Dimorph Products.* $\Pi(N) = \Pi(N')$.

By this term is meant a number (N) expressible in two *different ways* as a product of r numbers (N_1, N_2, ...) all of same functional form in their elements (x_r, y_r), so that

$$\mathbf{N} = \Pi(N_r) = N_1 N_2 N_3 \ldots N_r = N_1' N_2' N_3' \ldots N_r' = \Pi(N_r) \ \ldots \ (128),$$

where $$N_r = \phi(x_r, y_r), \quad N_r' = \phi(x_r', y_r').$$

63a. *Dimorph Bin-Aurifeuillian and Trin-Aurifeuillian Products.*

A number of Examples of these are given on the following pages :

Bin-Aurifns., pp. 104, 105 ; *Trin-Aurifns.*, p. 154 ;

along with the Rules for their formation.

They are presented in the following form

$$\frac{N_1 N_3 N_5 \ldots N_{2r+1}}{N_0 N_2 N_4 \ldots N_{2r}} = \frac{N_\alpha}{N_\beta} \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (129),$$

where, in the numbers $N_0 \ldots N_{2r+1}$ one of the x_r, y_r is *constant* throughout, and the whole set of the other element (y_r or x_r) are derived in succession from the starting y_0 or x_0 ; whilst the N_α, N_β have special elements.

63b. *Dimorph Quartan and Sextan Products.* $\Pi(N_r) = \Pi(N_r')$.

A number of Examples of each of these kinds is given, viz.

Quartans and *Half-Quartans* at foot of page 272 ;
Sextans at foot of page 275.

These suffice to prove the existence of these Dimorphs : but no Rule has been found for their formation.

Chap. VII. *Product-Forms.*

64. *Product-Forms.* $[\mathbf{N} = \Pi\,(N_r)]$.

Let
$$\left.\begin{array}{c} \mathbf{N} = \phi\,(X, Y)\,; \\ N_1 = \phi\,(x_1, y_1),\quad N_2 = \phi\,(x_2, y_2),\quad \ldots,\quad N_r = \phi\,(x_r, y_r) \end{array}\right\} \quad (130),$$
wherein $\mathbf{N}$ and $N_1, N_2, \ldots N_r$ are all functions of *same form* in their (x, y) elements,

and let
$$\mathbf{N} = N_1 . N_2 . N_3 \ldots N_r = \Pi\,(N_r) \quad (130a).$$

Then $\mathbf{N}$ is said to be a *Product-Form.*

65. *Product-Duans and -Cubans.* The salient property of these is common to all pure 2-ic forms $(T^2 \mp DU^2)$, viz.

Every Dimorph, and every Polymorph, of $(T^2 \mp DU^2)$ is also a *Product-Form* of same type .... (131).

This is evident from the mode of formation, as is explained with examples in Art. 49a, b in case of Duans and Cubans.

This property is peculiar to 2-ic forms.

65a. *Product-Cubics.* $\mathbf{N} = N_1 . N_2 . N_3 \ldots = \Pi\,(N_r)$.

Let $\quad \mathbf{N} = X^3 + Y^3\,;\quad N_1 = x_1^3 + y_1^3,\ \ldots,\ N_r = x_r^3 + y_r^3$ .... (132).

The Tables on p. 265 give a number of Examples with 2 or 3 factors (N_1, N_2, N_3) and one with 4 factors, with formulæ for their formation.

The elements (x_1, y_1) of N_1 are taken at random: but those of the successive factors are N_2, N_3, N_4 are taken such that—

$$x_2 - y_2 = x_3 - y_3 = x_4 - y_4 = 1.$$

It will be seen that the factors N_2, N_3, N_4 increase rapidly in magnitude.

66. *Product-Quartans, Half-Quartans, and Sextans.*

A few Examples of each of these will be found in the Tables as follows:—

For N_{iv} and $\frac{1}{2}N_{iv}$, at top of p. 272; For N_{vi}, at top of p. 275;

sufficient to show the existence of such Product-Forms; but no general Rules have been found for their formation.

67. *Product Bin-Aurifeuillians*, ($\mathbf{N} = N_1 N_2$).

Let $\mathbf{N} = x^4 + 4y^4 = L.M,\quad N_1 = x_1^4 + 4y_1^4,\quad N_2 = x_2^4 + 4y_2^4.$

To construct the product-form $\mathbf{N} = N_1 N_2$.

Assume $$L = N_1,\quad M = N_2 \ldots\ldots\ldots\ldots \quad (133).$$

Here $L = (x-y)^2 + y^2,\ M = (x+y)^2 + y^2$, (algebraically *unique*); whilst N_1, N_2 each have *two* (algebraic) (a, b) forms, viz.

$$N_1 = (x_1^2)^2 + (2y_1^2)^2 = (x_1^2 - 2y_1^2)^2 + (2x_1 y_1)^2 = N_1';$$
$$N_2 = (x_2^2)^2 + (2y_2^2)^2 = (x_2^2 - 2y_2^2)^2 + (2x_2 y_2)^2 = N_2'.$$

Hence *four* systems of equations arise

$$\left.\begin{array}{llll} \text{I.} & L = N_1,\ M = N_2; & \text{III.} & L = N_1,\ M = N_2' \\ \text{II.} & L = N_1',\ M = N_2; & \text{IV.} & L = N_1',\ M = N_2' \end{array}\right\} \ldots\ldots \quad (133a).$$

The 2-ic parts (a, b) of L, M are now to be equated to the 2-ic parts of N_1, N_1', N_2, N_2' respectively.

As the a, b are interchangeable, and may be either + or −, numerous Cases arise, (16 in each System): so it must suffice to consider *three* Cases of System I.

I. Here
$$(x-y)^2 + y^2 = (x_1^2)^2 + (2y_1^2)^2,\quad (x+y)^2 + y^2 = (x_2^2)^2 + (2y_2^2)^2 \ldots \quad (134).$$

The general solutions of the three Cases (i, ii, iii) are given below in terms of two arbitrary integers m, n (one *odd*, one *even*).

i. $x - y = x_1^2,\quad x + y = x_2^2,\quad y = 2y_1^2 = 2y_2^2,\quad x = \frac{1}{2}(x_1^2 + x_2^2),$

giving $$x_2^2 - x_1^2 = 2y = 4y_1^2 = 4y_2^2 \ldots\ldots\ldots\ldots \quad (134a),$$

whence $x_2 = m^2 + n^2,\quad x_1 = x_2 = m^2 - n^2,\quad y_1 = y_2 = mn;$

$x = m^4 + n^4,\ y = 2m^2n^2$; [$x, x_1, x_2$ all *odd*, y, y_1, y_2 all *even*].

ii. $x - y = 2y_1^2,\quad x + y = 2y_2^2,\quad y = x_1^2 = x_2^2,\quad x = y_1^2 + y_2^2,$

giving $$y_2^2 - y_1^2 = y = x_1^2 = x_2^2 \ldots\ldots\ldots\ldots \quad (134b),$$

whence $y_2 = m^2 + n^2,\quad x_1 = x_2 = m^2 - n^2,\quad y_1 = 2mn;$

$x = m^4 + 6m^2n^2 + n^4,\ y = (m^2 \sim n^2)^2;$

[x, y, x_1, x_2, y_2 all *odd*, y_1 *even*].

iii. $y - x = 2y_1^2,\quad y + x = 2y_2^2,\quad y = x_1^2 = x_2^2,\quad x = y_2^2 - y_1^2,$

giving $$y_2^2 + y_1^2 = y = x_1^2 = x_2^2 \ldots\ldots\ldots\ldots \quad (134c),$$

whence $x_1 = x_2 = m^2 + n^2,\quad y_1$ or $y_2 = m^2 \sim n^2,\quad y_2$ or $y_1 = 2mn;$

$x = m^4 - 6m^2n^2 + n^4,\quad y = (m^2 + n^2)^2;$

[x, y, x_1, x_2 all *odd*, y_1 or y_2 *even*].

67a. *Ex.* The Table below gives a number of fully factorised Examples of each Case with low values of m, n.

Case.	m, n	x_1, y_1	x_2, y_2	x, y	N_1	N_2
i	1, 2	3, 2	5, 2	17, 8	5 : 29;	13 : 53;
	1, 4	15, 4	17, 4	257, 32	137 : 13.29;	5.37 : 457;
	1, 6	35, 6	37, 6	1297, 72	877 : 17.101;	997 : 5.13.29;
	3, 2	5, 6	13, 6	97, 72	37 : 157;	5.17 : 397;
	3, 4	7, 12	25, 12	337, 288	13.13 : 5.101;	313 : 17.89;
	5, 2	21, 10	29, 10	641, 200	221 : 1061;	461 : 1621;
ii	1, 2	3, 4	3, 5	41, 9	17 : 5.13;	29 : 89;
	1, 4	15, 8	15, 17	353, 225	113 : 593;	293 : 13.101;
	3, 2	5, 12	5, 13	313, 25	193 : 433;	233 : 17.29;
	3, 4	7, 24	7, 25	1201, 49	5.173 : 29.53;	13.73 : 17.97;
	5, 2	21, 20	21, 29	1241, 41	401 : 2081;	5.181 : 13.257;
iii	1, 2	5, 4	5, 3	7, 25	13 : 73;	17 : 97;
	1, 4	17, 8	17, 15	161, 289	5.29 : 1537;	229 : 1249;
	3, 2	13, 12	13, 5	119, 169	5.29 : 769;	89 : 349;
	3, 4	25, 24	25, 7	527, 625	577 : 13.229;	373 : 29.37;
	5, 2	29, 20	29, 21	41, 841	13.37 : 2081;	5.101 : 17.173;

68. *Product-Bin-Aurifeuillians*, ($\mathbf{N} = N_1 N_2 N_3$).

Let $N_1 = x_1^4 + 4y_1^4$, $N_2 = x_2^4 + 4y_2^4$, $N_3 = x_3^4 + 4y_3^4$;

$$*N_{12} = x_{12}^4 + 4y_{12}^4, \quad *N_{123} = x_{123}^4 + 4y_{123}^4 \quad \ldots\ldots\ldots\ldots \quad (135).$$

STEP I. Construct the product-form $*N_{12} = N_1 . N_2$ by the formulæ of Cases i, ii, iii of Art. 67, writing x_{12}, y_{12} for the x, y of each of those formulæ.

Next, multiply together the $(x \sim y)$, $(x+y)$ of each of those Cases, giving

$$\text{i. } x_{12}^2 - y_{12}^2 = (x_1 x_2)^2; \quad \text{ii. } x_{12}^2 - y_{12}^2 = (2y_1 y_2)^2; \quad \text{iii. } y_{12}^2 - x_{12}^2 = (2y_1 y_2)^2 \quad \ldots\ldots\ldots \quad (136).$$

STEP II. To adapt the above to forming the product

$$N_{123} = *N_{12} . N_3,$$

* The subscripts 12, 123 are *not to be read arithmetically;* they are *symbolic*, the subscript 12 indicates that $N_{12} = N_1 . N_2$, and x_{12}, y_{12} are the elements of N_{12}; the subscript 123 indicates $N_{123} = N_{12} . N_3$, and x_{123}, y_{123} are its elements.

In i; take $x_3 = x_{12}$, $y_3 = x_1 x_2$, giving $x_3^2 = y_{12}^2 + y_3^2$ ... (136a).

In ii; take $x_3 = x_{12}$, $y_3 = 2y_1 y_2$, giving $x_3^2 = y_{12}^2 + y_3^2$ ... (136b).

In iii; take $x_3 = x_{12}$, $y_3 = 2y_1 y_2$, giving $x_3^2 = y_{12}^2 - y_3^2$ ... (136c).

Next, with N_{12} formed as by i, ii, iii, proceed to form *$N_{123} = N_{12}.N_3$. This may be done by each of Cases ii, iii by writing x_{123}, y_{123} for the x, y, and x_{12}, y_{12}, x_3, y_3 for the x_1, y_1, x_2, y_2 respectively in the formulæ of those Cases, giving finally

ii. $x_{123} = y_3^2 + y_{12}^2$, $y_{123} = y_3^2 - y_{12}^2$, provided $x_3^2 = y_3^2 \sim y_{12}^2$.

This condition is satisfied by Result (136c) above, (Case iii).

iii. $x_{123} = y_3^2 - y_{12}^2$, $y_{123} = y_3^2 + y_{12}^2$, provided $x_3^2 = y_3^2 + y_{12}^2$.

This condition is satisfied by both Results (136a, b) above, (Cases i, ii).

Hereby it is seen that—

When N_{12} is formed by Case i, then $N_{123} = N_{12}.N_3$ may be formed by Case iii.

When N_{12} is formed by Case ii or iii, then $N_{123} = N_{12}.N_3$ may be formed by Case iii or ii respectively.

68a. *Continued Product-Bin-Aurifns.*, ($N_{123\ldots r} = N_1 N_2 N_3 \ldots N_r$).

The process of the last Article can be continued *indefinitely*. For—

(1) Starting to form N_{12} by Case i, the process can be continued under Case iii.

(2) Starting to form N_{12} by Case ii or iii, the process can be continued under Case iii or ii respectively.

(3) The next step falls under Case ii or iii respectively; and may be continued *indefinitely*, using Cases ii, iii *alternately*.

Ex. The Table below gives three Examples up to the 4th order (N_{1234}) completely factorised: the first starting with Case i, the second with Case ii, the third with Case iii. The final N_{1234} has 8 algebraic factors, viz. the L, M of the four components N_1, N_2, N_3, N_4.

Case.	x_1, y_1	x_2, y_2	x_{12}, y_{12}	x_3, y_3	x_{123}, y_{123}	x_4, y_4	x_{1234}, y_{1234}
i	3, 2 5 : 29;	5, 2 13:53;	17, 8	17, 15 229 : 1249;	161, 289	161, 240 63841 : 218401;	141121, 25921
ii	3, 4 17:5·13;	3, 5 29:89;	41, 9	41, 40 1601 : 8161;	1519, 1681	1519, 720 1156801 : 5531521;	3544164, 2307361
iii	5, 3 13 : 73;	5, 4 17:97;	7, 25	7, 24 5·173:29·53;	1201, 49	1201, 1200 337·4273:7204801;	1437599, 1442401

* See foot-note on previous page.

69. *Product-Trin-Aurifeuillians*, ($\mathbf{N} = N_1 . N_2$).

Let
$$\left.\begin{array}{c} \mathbf{N} = x^4 - 3x^2y^2 + 9y^4 = L.M \\ N_1 = x_1^4 - 3x_1^2 y_1^2 + 9y_1^4, \quad N_2 = x_2^4 - 3x_2^2 y_2^2 + 9y_2^4 \end{array}\right\} \quad \ldots\ldots \quad (137).$$

To construct the product-form $\mathbf{N} = N_1 N_2$, assume
$$L = N_1, \quad M = N_2 \quad \ldots\ldots\ldots\ldots \quad (138).$$

Express L, M, N_1, N_2 in the form $(A^2 + 3B^2)$.

Here L, M each have three (A, B), according as $x = \epsilon$, $y = \epsilon$, $xy = \omega$.

And N_1, N_2 each have four (A, B) forms, one general, and three depending on whether their $x = \epsilon$, $y = \epsilon$, $xy = \omega$.

To solve $L = N_1$, $M = N_2$,—

> Any pair of the 2-ic parts (A) of L, N_1 may be equated, and any pair of the 2-ic parts (A) of M, N_2 may be equated: the corresponding 2-ic parts (B) of L, N_1 must then be equal, and those of M, N_2 must be equal. And each of the A and B numbers may be taken either $+$ or $-$.

A great number of separate Cases thus arise: it must suffice here to consider only *four* of these.

CASE i. y, y_1, y_2 all *even*.

$$L = (x - \tfrac{3}{2}y)^2 + 3(\tfrac{1}{2}y)^2 = (x_1^2 - \tfrac{3}{2}y_1^2)^2 + 3(\tfrac{3}{2}y_1^2)^2 = N_1 \quad \ldots\ldots \quad (139a).$$
$$M = (x + \tfrac{3}{2}y)^2 + 3(\tfrac{1}{2}y)^2 = (x_2^2 - \tfrac{3}{2}y_2^2)^2 + 3(\tfrac{3}{2}y_2^2)^2 = N_2 \quad \ldots\ldots \quad (139b).$$

CASE ii. y, x_1, x_2 all *even*.

$$L = (x - \tfrac{3}{2}y)^2 + 3(\tfrac{1}{2}y)^2 = (\tfrac{1}{2}x_1^2 - 3y_1^2)^2 + 3(\tfrac{1}{2}x_1^2)^2 = N_1 \quad \ldots\ldots \quad (139c).$$
$$M = (x + \tfrac{3}{2}y)^2 + 3(\tfrac{1}{2}y)^2 = (\tfrac{1}{2}x_2^2 - 3y_2^2)^2 + 3(\tfrac{1}{2}x_2^2)^2 = N_2 \quad \ldots\ldots \quad (139d).$$

CASE iii. y_1 *even*, the rest *odd*.

$$L = \{\tfrac{1}{2}(x - 3y)\}^2 + 3\{\tfrac{1}{2}(x - y)\}^2 = (x_1^2 - \tfrac{3}{2}y_1^2)^2 + 3(\tfrac{3}{2}y_1^2)^2 = N_1 \ldots \quad (139e).$$
$$M = \{\tfrac{1}{2}(x + 3y)\}^2 + 3\{\tfrac{1}{2}(x + y)\}^2 = \{\tfrac{1}{2}(x_2^2 + 3y_2^2)\}^2 + 3\{\tfrac{1}{2}(x_2^2 - 3y_2^2)\}^2 = N_2 \quad \ldots\ldots \quad (139f).$$

CASE iv. x_1 *even*, the rest *odd*.

$$L = \{\tfrac{1}{2}(x - 3y)\}^2 + 3\{\tfrac{1}{2}(x - y)\}^2 = (\tfrac{1}{2}x_1^2 - 3y_1^2)^2 + 3(\tfrac{1}{2}x_1^2)^2 = N_1 \ldots \quad (139g).$$
$$M = \{\tfrac{1}{2}(x + 3y)\}^2 + 3\{\tfrac{1}{2}(x + y)\}^2 = \{\tfrac{1}{2}(x_2^2 + 3y_2^2)\}^2 + 3\{\tfrac{1}{2}(x_2^2 - 3y_2^2)\}^2 = N_2 \quad \ldots\ldots \quad (139h).$$

The equations $L = N_1$, $M = N_2$ may now be solved by equating the *like* 2-ic parts in each (A to A, B to B). This involves in each Case one Diophantine condition of form $\gamma^2 = \alpha^2 + \beta^2$, the general solution of which is expressible in two arbitraries m, n, (one *odd*, one *even*).

CASE i. $x-\frac{3}{2}y = x_1^2-\frac{3}{2}y_1^2$, $x+\frac{3}{2}y = x_2^2-\frac{3}{2}y_2^2$, $y = 3y_1^2 = 3y_2^2$,
giving $x = \frac{1}{3}(2x_1^2+x_2^2)$, provided $3y = x_2^2-x_1^2 = (3y_1)^2 = (3y_2)^2$ ... (140a).
Hence $x_2 = m^2+n^2$, $x_1 = m^2\sim n^2$, $y_1 = y_2 = \frac{2}{3}mn$, $[mn = 3i]$.

$$x = m^4-\tfrac{2}{3}m^2n^2+n^4 = x_1^2+3y_1^2, \quad y = \tfrac{4}{3}m^2n^2.$$

CASE ii. $x-\frac{3}{2}y = \frac{1}{2}x_1^2-3y_1^2$, $x+\frac{3}{2}y = \frac{1}{2}x_2^2-3y_2^2$, $y = x_1^2 = x_2^2$,
giving $x = -(y_1^2+2y_2^2)$, provided $y_1^2-y_2^2 = x_1^2 = x_2^2$ .... (140b).
Hence $y_1 = m^2+n^2$, $y_2 = m^2\sim n^2$, $x_1 = x_2 = 2mn$, $[mn \neq 3i]$.

$$x = -(3m^4-2m^2n^2+3n^4) = -(x_2^2+3y_2^2), \quad y = 4m^2n^2.$$

CASE iii. $-(x-3y) = 2x_1^2-3y_1^2$, $x+3y = x_2^2+3y_2^2$, $x-y = 3y_1^2$,
$-(x+y) = x_2^2-3y_2^2$,
giving $x = y_1^2+2y_2^2$, $y = x_1^2 = x_2^2$, provided $y_2^2-y_1^2 = x_1^2 = x_2^2$ ... (140c).
Hence $y_2 = m^2+n^2$, $y_1 = 2mn$, $x_1 = x_2 = m^2\sim n^2$, $[mn = 3i]$.

$$x = m^4+10m^2n^2+n^4 = y_2^2+2y_1^2, \quad y = (m^2\sim n^2)^2.$$

CASE iv. $x-3y = x_1^2-6y_1^2$, $x+3y = x_2^2+3y_2^2$, $x-y = x_1^2$, $x+y = x_2^2-3y_2^2$,
giving $x = \frac{1}{3}(2x_1^2+x_2^2)$, $y = 3y_1^2 = 3y_2^2$, provided $x_2^2-x_1^2 = (3y_1)^2 = 3y_2^2$. (140d).
Hence $x_2 = m^2+n^2$, $x_1 = 2mn$, $y_1 = y_2 = \frac{1}{3}(m^2\sim n^2)$, $[m^2\sim n^2 = 3i]$.

$$x = \tfrac{1}{3}(m^4+10m^2n^2+n^4) = x_1^2+3y_1^2, \quad y = \tfrac{1}{3}(m^2\sim n^2)^2.$$

Ex. The Table at foot of page 154 gives *three* Examples of each Case (in slightly different notation): two in quite low numbers, and one in very high numbers. [The term *Compound Trin-Aurifeuillians* has been used at head of this Table.]

Chap. VIII. *Perfect Squares.*

70. *Perfect Squares.* The question of Perfect Square Forms is one of considerable interest. A Table of Impossible Squares, and a Table of Possible Squares of degrees 2, 3, and 4, are given on page 228, together with Rules for forming them. Squares of degree >4* are unknown.

71. *Square 2-ic Forms.* In Binary 2-ic Forms there is a complete reciprocity, thus:

1°. If $Z = x^2 \mp Dy^2$, then $Z^2 = X^2 \mp DY^2$, *always*,
where $X = x^2 \pm Dy^2$, $Y = 2xy$ .... (141).

2°. If $X^2 \mp DY^2 = Z^2$, then $Z = x^2 \mp Dy^2$, *always*,
where $x = \sqrt{\tfrac{1}{2}(Z+X)}$, $y = \sqrt{\dfrac{1}{2D}(Z \sim X)}$ .... (141a).

[Use upper sign throughout, or lower sign throughout, in the above.]

All Duans (N_{ii}), and Cubans (N_{iii} and N'_{iii}) obey the above Rules, both being pure 2-ic forms, viz.

$$N_{ii} = x^2 + y^2, \quad (D = -1);$$

$$N_{iii} \text{ or } N'_{iii} = (x^3 \mp y^3) \div (x \mp y) = A^2 + 3B^2, \quad (D = -3);$$

so that—

1°. The square of every N_{ii}, and that of every N_{iii}, or N'_{iii}, is itself a N_{ii}, or N_{iii} .... (141b).

2°. Every square N_{ii}, and every square N_{iii}, or N'_{iii}, is the square of a N_{ii}, or N_{iii} .... (141c).

The formulæ, and properties, of square Cubans are most conveniently treated of by means of their equivalent 2-ic form

$$(N_{iii}, N'_{iii} = A^2 + 3B^2).$$

71a. *Ex.* The *smallest* Square-Duans and Cubans are

$N_{ii} = 5 = 1^2 + 2^2$, $\quad N_{ii}^2 = 5^2 = 3^2 + 4^2$;

$N_{iii} = 7 = 2^2 + 3.1^2$, $\quad N_{iii}^2 = 7^2 = 1^2 + 3.4^2 = \dfrac{5^3 - 3^3}{5-3} = \dfrac{8^3+3^3}{8+3} = \dfrac{8^3+5^3}{8+5}$;

* Square Quintans exist,—(but these are of degree 4). These will be treated of in Vol. II.

and the smallest Square Duo-Cuban is $N_{ii}^2 = 13^2 = N_{iii}^2$;

$N_{ii} = 13 = 2^2+3^2, \quad N_{ii}^2 = 13^2 = 5^2+12^2$;

$$N_{iii} = 13 = 1^2+3.2^2,\ N_{iii}^2 = 13^2 = 11^2+3.4^2 = \frac{8^3-7^3}{8-7} = \frac{15^3+7^3}{15+7} = \frac{15^3+8^3}{15+8}.$$

For a Table of Square Duans $x^2 = y^2+z^2$, see pages 190–194; this is complete to $x = 2441$.

For a Table of Square Cuban elements $y^2 = x^2+3z^2$ see pages 185–189 and 194; this is complete to $y = 1591$.

72. *Square Cubic Forms.* Numerical Tables of the two forms below, with some Rules for forming them, are given on the pages named.

Page 229.

$$x^3-y^3 = z^2, \quad z = \lambda z_1 z_3; \quad \lambda = 1 \text{ or } 3, \quad x-y = z_1^2 \text{ or } 3z_1^2 \ldots \quad (142).$$

Numerous examples are given with $z = \omega$ and $z = \epsilon$.

Pages 234, 235. $\qquad x^3 \mp Cy^3 = \pm z^2.$

Solutions are given for both $\pm C$, up to $C = 100$. In many cases several solutions.

73. *Square Quartic Forms.* Two classes of these forms, viz.

$$\text{I.} \quad x^4 \pm Ky^4 = \pm z^2, \qquad \text{II.} \quad x^4+Kx^2y^2+y^4 = z^2 \quad \ldots\ldots\ldots\ldots (143)$$

have been much studied by *Euler*, *Ed. Lucas*, *Lebesgue*, and later by many writers (too numerous to mention) in *L'Intermédiaire des Mathématiciens*, and *Sphinx-Œdipe*.

As to Form I; successive solutions may be formed *ad inf.* when *one* solution (x_r, y_r, z_r) is known by the succession-formulæ—

If $\qquad x_r^4+Ky_r^4 = \pm z_r^2, \quad [K = \pm k],$

then $\qquad x_{r+1}^4+Ky_{r+1}^4 = +z_{r+1}^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (144),$

where $\quad x_{r+1} = x_r^4 \sim Ky_r^4, \quad y_{r+1} = 2x_r y_r z_r, \quad z_{r+1} = z_{r+1}^4+Kx_r^4 y_r^4 \ldots$ (144a).

There is here a striking similarity to the succession-formulæ of the 2-ic form $x_r^2+ky_r^2 = \pm z_r^2$. The successive solutions usually rise rapidly in magnitude.

73a. *Ex.* Numerical Tables of the above Forms are given on the pages quoted below.

Binomial Forms, (pages 230, 231, 236).

Pages 230, 236. $\quad x^4+ky^4 = +z^2$, and $x^4-ky^4 = \pm z^2$.

Least solutions (x, y, z) are given for many values of k, and in some cases several solutions, up to $k = 100$.

Page 231. $x^4+Ky^4=\pm z^2$, $[K=\pm 2^m]$.

I. $K=+2^m$ requires $m=4\mu+3$, $K=-2^m$ requires $m=4\mu+1$.

Several solutions are given; i. with $K=+8$; ii. with $K=-2$ and $\pm z^2$; and iii. with $K=-2$ and $-z^2$.

II. $K=\pm k^2$. Least solutions are given in the Tables at foot of the page for various k.

[The results on page 231 depend on Lucas's "Recherches," there quoted.]

74. *Trinomial Forms*, (pages 232, 233).

Least solutions are given for many values of k, up to $k \not> 200$; and in some cases several solutions, as below. [See Euler's "Comment. Arithm.," t. ii, p. 496.]

Page 232. $x^4-kx^2y^2+y^4=z^2$. *Page* 233. $x^4+kx^2y^2+y^4=z^2$.
.... (145).

Of each Form three algebraic solutions are also given as below:

Page 232.	*Page* 233.
i. $k=\kappa^2$, $x=1$, $y=\kappa$, $z=\pm 1$.	i. $k=\kappa^2-2$, $x=y$, $z=\pm\kappa x^2$.
ii. $x=1$, $k=y^2-K$, $z^2=1+Ky^2$.	ii. $x=1$, $k=K-y^2$, $z^2=1+Ky^2$.
iii. $k=\lambda y^2-2C$, $\lambda=(C^2-1)\div x^2$, $z=\pm(x^2-Cy^2)$.	iii. $k=\lambda y^2+2C$, $\lambda=(C^2-1)\div x^2$, $z=\pm(x^2+Cy^2)$.

75. *Chain-Factor Squares.* If $N_1, N_2, N_3, \ldots$ be a Chain-Series, and $N_r = L_r . M_r$, then as already noticed (Art. 31)

$$M_r . L_{r+1} = \square \text{ for all values of } r.$$

Ex. All the Aurifeuillian Sextan Chains give interesting Examples of this, as every Aurifeuillian Factor thereof is a 4-tic formation in ξ, η.

75a. *Circular Chain Squares.* Every Circular Chain (Art. 36–36d) gives — by the continued product of its Links — a *perfect square.* For examples of the squares produced by a Quartan-Nexus, see the Table at top of p. 129.

76. *Square Quartan Products.* The Table at top of page 273 gives a number of Examples of such Products

$$\Pi(N_{iv}), \quad \text{or} \quad \Pi(\tfrac{1}{2}N_{iv}) = \square.$$

but no general Rule is known for their formation.

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Chap. IX. *Diophantine Process.*

76. In this Chapter a Diophantine* process of factorisation of *quartic* functions (N) will be developed.

76a. In all the 4-tic functions (N),—whether Quartans, Sextans, Bin-Aurifeuillians, or Trin-Aurifeuillians—treated of in this volume one or more *algebraic* 2-ic partitions ($t^2 \pm Du^2$) are known. A method of great generality will now be developed whereby these functions (N) may be presented in the form of a *difference of squares* ($P^2 - Q^2$)—wherein factorisation is obvious—or else in one or more *arithmetical* 2-ic partitions ($t^2 \pm Du^2$) isomorph with, but *different from* the known algebraic partition, thereby securing the means of factorisation by the process described in Art. 51, 51a.

The general procedure is treated of in Art. 76a–81b. Its application to each of the above functions follows in Art. 82–86b.

The 4-tic functions indicated are all of type

$$N = x^4 + \beta x^2y^2 + \gamma y^4 \qquad (146),$$

which, by taking $\beta = 2C + \mu R$, $\gamma = C^2 + \mu S$,

may be presented in the form

$$N = (x^2 + Cy^2)^2 + \mu y^2 (Rx^2 + Sy^2) \qquad (146a),$$

which can be reduced to the pure 2-ic form

$$N = (x^2 + Cy^2)^2 + \mu (yz)^2 \qquad (146b),$$

provided only that values of x, y can be found to satisfy the Diophantine equation

$$Rx^2 + Sy^2 = z^2 \qquad (146c).$$

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* Much of this Chapter is contained in the Author's Papers—
High Quartans, Nos. (2), (3); High Sextans, Nos. (2), (3);
the full Titles, &c., of which are given in the foot-note to Art. 2, Chap. I.

By taking $\mu = -1, +1, +D, -D$,
N is presented in one of the forms required.

$$N = P^2 - Q^2, = a^2 + b^2, = t^2 + Du^2, = t^2 - Du^2 \quad \ldots\ldots (146d).$$

Each kind of n-an has its own kind of above 2-ic forms, as shown below :—

N =	*Quartan*	*Sextan.*	*Bin-Aurifn.*	*Trin-Aurifn.*
$\mu =$	$-1, +1, +2, -2$;	$-1, +1, +3, -3$;	$-1, +1$;	$-1, +3$

.... (146e).

76b. *Associate 2-ic forms.* Every n-an,—(prime or composite),—has one such set, viz. the *algebraic* set of 2-ic forms : every composite n-an has one more set, an *arithmetical* set of such forms, for every way in which it can be shown as a product of two factors $N = L.M$, [with L and M > 1, and $L \neq M$].

The complete set of such 2-ic forms arising from any one resolution $N = LM$ are styled *Associate 2-ic Forms.*

77. *Factorisants, Characteristics.* The four Diophantine Equations,—[obtained by taking $\mu = -1, +1, +D, -D$ in (146b),]—which lead to factorisable forms of N, and give also the means of their factorisation, will be styled *Factorisants*, and the quantity C which characterises each will be styled their *Characteristic.* The four Diophantines defined by $\mu = -1, +1, +D, -D$ will be described as of *Class* i, ii, iii, iv, and the Characteristics (C) involved in them will be symbolized by C', C'', C''', C^{iv}. Similarly the auxiliary z will be symbolized by z', z'', z''', z^{iv}.

77a. *2-ic Parts.* When a solution (x, y, z) of any of the factorisants (i, ii, iii, iv) has been obtained, the 2-ic "parts" (P, Q), (a, b), (t, u), (t', u') of the *arithmetic* partition required for factorisation (see Art. 76a) are given by

i. $P = x^2 + C'y^2, \quad Q = yz'$ .... (147a).

ii. a or $b = x^2 + C''y^2, \quad b$ or $a = yz''$ .... (147b).

iii. $t = x^2 + C'''y^2, \quad u = yz'''$ .... (147c).

iv. $t' = x^2 + C^{iv}y^2, \quad u' = yz^{iv}$ .... (147d).

77b. *Primary and Secondary Characteristics, &c.* The Characteristics and Factorisants arising from the *algebraic* set of 2-ic forms are styled *Primary:* those arising from any one arithmetical set of the 2-ic forms (Art. 76b) are styled Secondary.

77c. *Associate Characteristics and Factorisants.* The set of Characteristics and Factorisants obtained from a set of Associate 2-ic Forms (Art. 76b) are styled *Associate Characteristics* and *Factorisants.* By Art. 78 it is seen that,—owing to the double ($\pm$) sign of P_0, a_0, or b_0, t_0, t'_0 — the *number* of such Associates is

2 in each of Classes i, iii, iv; 4 in Class ii.

Further, if the Base n-an be symmetric in x, y, those numbers will be doubled by interchange of x, y in the formulæ for C. Thus the *Total number* of such Associates is

20 for Quartans and Sextans; 10 for Bin- and Trin-Aurifns.

78. *Suitable Characteristics* (C). To use this Diophantine process successfully it is necessary that the Factorisants should be *solvable.* A method will now be developed of determining Characteristics (C) which shall yield solvable Factorisants. These will be styled *Suitable* Characteristics. It will be shown that every n-an (N) of 4th degree of type (146) will yield several Suitable Characteristics (C) for each independent group of 2-ic partitions (P, Q), (a, b), (t, u), (t', u') in which it can be expressed. The n-an (N_0) used for this purpose will be styled the *Base n-an*, and will be denoted by N_0; and all the quantities connected with it, viz. x, y, z; P, Q, a, b, t, u, t', u' (except C) will take the subscript $_0$.

Let $$N_0 = x_0^4 + \beta x_0^2 y_0^2 + \gamma y_0^4 \quad\text{.......................}\quad (148),$$

and let $$N_0 = P_0^2 - Q_0^2 = a_0^2 + b_0^2 = t_0^2 + Du_0 = t_0'^2 - Du_0^2 \quad\text{........}\quad (148a)$$

be any one set of the 2-ic forms of which it is capable.

Then, by

i. $$\pm P_0 = x_0^2 + C'y_0^2, \quad Q = y_0 z_0';$$

whence $$C' = (\pm P_0 - x_0^2)/y_0^2 \quad\text{.........................}\quad (149a).$$

ii. $$\pm a_0 \text{ or } \pm b_0 = x_0^2 + C''y_0^2, \quad \pm b_0 \text{ or } \pm a_0 = y_0 z_0'';$$

whence $$C'' = (\pm a_0 - x_0^2)/y_0^2, \text{ or } = (\pm b_0 - x_0^2)/y_0^2 \quad\text{..........}\quad (149b).$$

iii. $$\pm t_0 = x_0^2 + C'''y_0^2, \quad u_0 = y_0 z_0''';$$

whence $$C''' = (\pm t_0 - x_0^2)/y_0^2 \quad\text{.......................}\quad (149c).$$

iv. $$\pm t_0' = x_0^2 + C^{\text{iv}}y_0^2, \quad u_0' = y_0 z_0^{\text{iv}};$$

whence $$C^{\text{iv}} = (\pm t_0' - x_0^2)/y_0^2 \quad\text{.......................}\quad (149d).$$

All the Factorisants arising from these Characteristics (C) will be certainly solvable, because one solution (x_0, y_0) is known.

78a. *Connected Characteristics and Factorisants.* The members of any one set of such Associate Characteristics and Factorisants are not always independent, but are connected by certain relations, thus

Two Characteristics C_1, C_2 of different Classes (Art. 77), and their dependent Factorisants,

$$R_1x^2 + S_1y^2 = z_1^2, \quad R_2x^2 + S_2y^2 = z_2^2 \ldots\ldots\ldots\ldots\ldots \quad (150),$$

will be said to be *Equivalent*, or *Reciprocal*, when they lead to the *same value* of N, viz.

Equivalent, with same x, y; *Reciprocal*, with interchanged x, y.

The necessary and sufficient conditions are—

Equivalent, (same x, y); $R_1 : R_2 = S_1 : S_2 = z_1^2 : z_2^2$ ... (150a).

Reciprocal, (interchanged x, y); $R_1 : S_1 = S_2 : R_1 = z_1^2 : z_2^2$ ... (150b).

Each of these relations will be found to lead to a *single* relation between C_1, C_2, as will appear hereafter.

78b. *Ineffective Factorisants.* Certain values of C (*e.g.* $C = 0$, ± 1) lead to Factorisants which simply reproduce the known algebraic 2-ic forms of the Base n-an, instead of yielding new *arithmetical* forms: thus these are *ineffective* (for factorisation purposes).

79. *Use of Factorisants.*

The Characteristics (C) of any one Associate set (Art. 77c) lead usually to *different* Factorisants,—(*i.e.* with the exception of the few *equivalent* and *reciprocal* cases of Art. 78c)—and each of these Factorisants has usually an infinite series of solutions (x, y, z)—and each such solution gives the *elements* (x, y) of a new factorisable n-an (N), together with the data (P, Q), (a, b), (t, u), (t', u') for its factorisation into two factors N = L.M.

80. *General Solutions.* The four Factorisants may (to some extent) be treated of together under the single type

$$Rx^2 + Sy^2 = z^2, \quad [R \text{ prime to } S] \ldots\ldots\ldots\ldots\ldots\ldots \quad (151).$$

Two *simple* Cases arise, viz. when 1°. $R = \rho^2$, or 2°. $S = \sigma^2$. In these Cases the factor ρ or σ may be absorbed into the symbol x or y respectively, giving the two simple general forms—

$$1^\circ.\ \ x^2 + Sy^2 = z^2; \quad 2^\circ.\ \ Rx^2 + y^2 = z^2 \ldots\ldots\ldots\ldots\ldots \quad (151a).$$

These admit of *general* solution in *two ways* each, in terms of two arbitrary—(but *mutually prime*)—integers (t, u), as follows

$$\begin{cases} 1^\circ \text{a.} & x = t^2 - Su^2, & y = 2tu, & z = t^2 + Su^2 \text{ ...... (151b).} \\ 1^\circ \text{b.} & x = \tfrac{1}{2}(t^2 - Su^2), & y = tu, & z = \tfrac{1}{2}(t^2 + Su^2) \text{ ... (151c).} \end{cases}$$

$$\begin{cases} 2^\circ \text{a.} & x = 2tu, & y = t^2 - Ru^2, & z = t^2 + Ru^2 \text{ ...... (151d).} \\ 2^\circ \text{b.} & x = tu, & y = \tfrac{1}{2}(t^2 - Ru^2), & z = \tfrac{1}{2}(t^2 + Ru^2) \text{ ... (151e).} \end{cases}$$

81. *Serial Solutions.* When any one solution (x_0, y_0, z_0) of a Factorisant is known, then writing the Factorisant in the three forms

1°. $z^2 - Sy^2 = Rx^2$	2°. $z^2 - Rx^2 = Sy^2$	3°. $Rx^2 + Sy^2 = z^2$
S being +, & $\neq \square$	R being +, & $\neq \square$	RS being −, & $\neq -\square$

.... (152),

two infinite series of solutions (x_r, y_r, z_r) may usually be obtained for the simple Cases of

$$1^\circ.\ x = x_0 \text{ (const.)};\quad 2^\circ.\ y = y_0 \text{ (const.)};\quad 3^\circ.\ z = z_0 \text{ (const.)} \dots \text{ (152a)}$$

respectively, when the conditions stated (as to R, S) are satisfied: but these conditions can only be satisfied in *two* of the three Cases (1°, 2°, 3°) at once. This is to be done by aid of the solution of the associated *Unit-form*—(here supposed known)—

$$1^\circ.\ \tau^2 - Sv^2 = +1;\quad 2^\circ.\ \tau^2 - Rv^2 = +1;\quad 3^\circ.\ \tau^2 + RSv^2 = +1 \dots \text{ (152b).}$$

The first application of—(*i.e.* multiplication by)—the Unit-form usually gives *two* distinct new solutions (x_1, y_1, z_1) in each possible Case of 1°, 2°, 3°—viz.

1°.	2°.	3°.
$z_1 = \tau z_0 \mp Svy_0$	$z_1 = \tau z_0 \mp Rvx_0$	$x_1 = \tau x_0 \pm Svy_0$
$y_1 = vz_0 \mp \tau y_0$	$x_1 = vz_0 \mp \tau x_0$	$y_1 = Rvx_0 \mp \tau y_0$
$x_1 = x_0$ (const.)	$y_1 = y_0$ (const.)	$z_1 = z_0$ (const.)

(152c),

wherein the *same signs* are to be used in z_1, y_1 of Case 1°, and in z_1, x_1 of Case 2°, and *opposite signs* in x_1, y_1 of Case 3°.

From the new solutions (x_1, y_1, z_1) just obtained an infinite series of solutions (x_r, y_r, z_r) may now be derived by *successive steps* by the "succession-formulæ"—

1°.	2°.	3°.
$z_{r+1} = \tau z_r + Svy_r$	$z_{r+1} = \tau z_r + Rvx_r$	$x_{r+1} = \tau x_r - Svy_r$
$y_{r+1} = vz_r + \tau vy_r$	$x_{r+1} = vz_r + \tau x_r$	$y_{r+1} = Rvx_r + \tau y_r$
$x_{r+1} = x_r = x_0$	$y_{r+1} = y_r = y_0$	$z_{r+1} = z_r = z_0$

(152d);

or, by the following, (which give either series of x_r, y_r, z_r *separately*)—

1°.	2°.	3°.
$z_{r+1} = 2\tau z_r - z_{r-1}$	$z_{r+1} = 2\tau z_r - z_{r-1}$	$x_{r+1} = 2\tau x_r - x_{r-1}$
$y_{r+1} = 2\tau y_r - y_{r-1}$	$x_{r+1} = 2\tau x_r - x_{r-1}$	$y_{r+1} = 2\tau y_r - y_{r-1}$

(152e).

Thus there are usually *two* infinite series of solutions—arising from the double ($\pm$) sign in the formulæ for (x_1, y_1, z_1) in each of the possible Cases of 1°, 2°, 3°. But, it occasionally happens that the two Series *coincide* into one Series.

Solutions, formed as above—(*i.e.* with x, y, or z *constant* throughout)—will be termed *Serial Solutions*, and the solution (x_0, y_0, z_0) from which they originate will be styled the *Base-solution.* These Series are interesting in that they are frequently *Chains*, (as on pages 135, 136, 138, 211, 212, 214).

81a. *Simple Serial Solutions.* This Series, obtained as above, with the condition

$$1°.\quad x_r = x_0 = 1\,;\quad \text{or,}\quad 2°.\quad y_r = y_0 = 1 \quad\ldots\ldots\ldots\quad (153),$$

giving *Simple Quartans* $N_r = (1+y_r^4)$ or (x_r^4+1) are among the most interesting: these will be styled *Simple Serial Solutions.*

81b. *Unit-forms.* The unit-forms (152b), required to aid in the serial solutions of the factorisants, always admit of solution; but, in some cases, the solution is known algebraically; much work may often be saved by observing this.

Form 1° of $\qquad \tau^2 - S\upsilon^2 = +1$; [$S$ is supposed +] ... (154).

Class i. Here $S = C'^2 - 1$; then $\tau = C'$, $\upsilon = 1$, $[C' > 1]$.

Class ii. Here $S = 1 - C''^2$; $[C'' < 1$ is fractional$]$.

In this latter case the factorisant, divided by C''^2, becomes

$$(z/C'')^2 - (1/C''^2 - 1)\,y^2 = -2x^2/C''^2 \quad\ldots\ldots\ldots\ldots\ldots\quad (154a),$$

whilst the unit-form becomes

$$\tau^2 - (1/C''^2 - 1)\,\upsilon^2 = +1 \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\quad (154b),$$

with solution $\qquad \tau = 1/C''$, $\quad \upsilon = 1$.

82. *Special Applications.* The preceding Articles 76a–81b give the *general development* of the Diophantine process to quartic functions of the general type. Its special applications to the 4-tic functions of this Volume will now be shown, as follows—

Quartans, Art. 83. *Sextans*, Art. 84.

Trinomial Forms ; of 4-*tans* and 6-*tans*, Art. 85.

Bin-Aurifns., Art. 86a. *Trin-Aurifns.*, Art. 86b.

A pretty full account will be given of its application to the first Case, of Quartans; a much shorter account will be given of the other Cases.

83. *Quartans.* The Quartan $N_{iv} = x^4 + y^4$ .... (155) has the four *Associate* 2-ic Forms (Art. 76a), defined by $\mu = -1, +1, +2, -2$ in the general formula (76a), here said to be of Classes i, ii, iii, iv, with Characteristics C', C'', C''', C^{iv}, as follows—

i. $N = (x^2 + C'y^2)^2 - y^2.\{2C'x^2 + (C'^2 - 1)y^2\}$
$= P^2 - Q^2$, $[\mu = -1]$ .... (155a),
if $2C'x^2 + (C'^2 - 1)y^2 = z'^2$ .... (155b).

ii. $N = (x^2 + C''y^2)^2 + y^2.\{-2C''x^2 - (C''^2 - 1)y^2\}$
$= a^2 + b^2$, $[\mu = +1]$ .... (155c),
if $-2C''x^2 - (C''^2 - 1)y^2 = z''^2$ .... (155d).

iii. $N = (x^2 + C'''y^2)^2 + 2y^2.\{-C'''x^2 - \frac{1}{2}(C'''^2 - 1)y^2\}$
$= c^2 + 2d^2$, $[\mu = +2]$ .... (155e),
if $-C'''x^2 - \frac{1}{2}(C'''^2 - 1)y^2 = z'''^2$ .... (155f).

iv. $N = (x^2 + C^{iv}y^2)^2 - 2y^2.\{C^{iv}x^2 + \frac{1}{2}(C^{iv2} - 1)y^2\}$
$= e^2 - 2f^2$, $[\mu = -2]$ .... (155g),
if $C^{iv}x^2 + \frac{1}{2}(C^{iv2} - 1)y^2 = z^{iv2}$ .... (155h).

The four indeterminate 2-ics in x, y, z are the set of Associate Factorisants of Classes i, ii, iii, iv.

When a solution (x, y, z) of any Factorisant has been obtained, the "2-ic parts" (P, Q), (a, b), (c, d), (e, f) of the *arithmetic* partitions required for factorisation of N are given by

i. $\pm P = (x^2 + C'y^2)$, $Q = yz'$ .... (156a).
ii. $\pm a$ or $\pm b = x^2C'' + y^2$, b or $a = yz''$ .... (156b).
iii. $\pm c = x^2 + C'''y^2$, $d = yz'''$ .... (156c).
iv. $\pm e = x^2 + C^{iv}y^2$, $f = yz^{iv}$ .... (156d).

and the *algebraic* partitions are

$$P = \tfrac{1}{2}(N + 1), \quad Q = \tfrac{1}{2}(N - 1),$$

(a, b), (c, d), (e, f) are as given in Art. 11 .... (157).

83a. *Equivalence and Reciprocity.* Denoting *any two* of the four Associate Characteristics (C', C'', C''', C^{iv}) by C_1, C_2, and comparing the above four Factorisants with the general form $Rx^2 + Sy^2 = z^2$ (Art. 80), and substituting in the general conditions of Equivalence and Reciprocity, (Art. 78a), these conditions will be found to reduce to

Equivalence, $\left(C_1 - \frac{1}{C_1}\right) = C_2 - \frac{1}{C_2}$, whence $C_1 C_2 = -1$ ... (158a).

Reciprocity, $\left(C_1 - \frac{1}{C_1}\right)\left(C_2 - \frac{1}{C_2}\right) = 4$, whence

$$\frac{C_2 + 1}{C_2 - 1} = C_1 \text{ or } -\frac{1}{C_1}, \quad \frac{C_1 + 1}{C_1 - 1} = C_2 \text{ or } -\frac{1}{C_2} \quad \text{...... (158b).}$$

83b. *Suitable Characteristics.* To obtain *suitable* Characteristics leading to *solvable* Factorisants (Art. 78), take as *Base-Quartan*

$$N_0 = x_0^4 + y_0^4 = h^4 + k^4, \quad \text{so that} \quad x_0 = h \text{ or } k, \;\; y_0 = k \text{ or } h \;\ldots\; (159).$$

Now, see the Table on p. 131—

The upper Table shows that there are always 6 Characteristics *ineffective* (for factorisation purposes); for, in the Factorisants

$$\left.\begin{array}{r} C'' = 0 \quad \text{gives} \quad y^2 = z^2; \\ C''' = -1, \quad \text{and} \quad C^{\text{v}} = +1, \quad \text{both give} \quad x^2 = z^2 \end{array}\right\} \;\ldots\; (159a),$$

thus merely *reproducing the original* Base-Quartan.

The *second* Table shows

In lines 1 and 2;	2 cases of C'', C''' *reciprocal.*
In lines 3 and 4;	2 cases of C'', C^{iv} *reciprocal.*
In line 5, [when $h = 1$];	1 case of C', C'' *equivalent.*

Thus, of the 20 *primary* Characteristics (Art. 77c) of every Base-Quartan, 6 are always *ineffective* (for factorising purposes); and there are always 2 reciprocal pairs, and [when $h = 1$] one equivalent pair.

The large Table at foot of page 131 shows in full detail the elements (x_0, y_0, z_0) and the Characteristics (C) of the 20 Factorisants of the Simple Base-Quartan $N_0 = 1^4 + k^4$, with the data x_0, y_0, P_0, Q_0, a_0, b_0, c_0, d_0, e_0, f_0.

[The letters E, I, R in the right column of this and later Tables mean *Equivalent*, *Ineffective*, *Reciprocal* respectively.]

On page 132; the upper Table shows the arithmetical values of the data and results of the Table at foot of page 131 applied to the two prime *Base-Quartans* $N_0 = 1^4 + 2^4 = 17$, $N_0 = 3^4 + 2^4 = 97$.

The Table at foot of page 132 shows the *Factorisants* of Classes i, ii for the Base-Quartan $N_0 = 1^4 + 2^4 = 17$ derived from the Characteristics (C) in the upper Table, omitting Nos. 1, 2, 7 of Class ii (Nos. 1 and 7 being *ineffective* and No. 2 being *reciprocal* to No. 1 of Class i). Those of Classes iii, iv are omitted as 4 are *ineffective* and 4 are *reciprocal* to Class ii.

On page 133; the upper Table shows two sets of Results z_0 and C for the *composite* Base-Quartan $N_0 = 5^4 + 6^4 = 1913 = 17.113$: the *primary* set arising from the *algebraic* 2-ic forms, the *secondary* set from the arithmetical set of 2-ic forms due to the factors 17.113; (Art. 77b).

The Tables on pages 134 (at foot), 135, 136 (at top) give worked Examples.

That on page 134 (at foot) starts from $N_0 = 1^4 + 1^4 = 2$.

Those on pages 135, 136 (at top) give the Results obtained from $N_0 = 1^4 + 2^4 = 17$ with *each* of the Factorisants of Table at foot of page 132.

83c. *Co-factors* (L, M), *2-ic Partitions.* It is sometimes possible to exhibit *algebraic* formulæ for the 2-ic Partitions of the Co-factors (L, M) of the Quartan Series,

$$L = \alpha^2+\beta^2 = \gamma^2+2\delta^2 = \epsilon^2-2\phi^2.$$

A number of Examples are given on pages 136, 137 for the Co-factors L, M of the Quartan Series of page 135 obtained from the Base $N_0 = 17$. The numerical values of (α, β), (γ, δ), (ϵ, ϕ) are given on page 136; the algebraic formulæ on page 137. The Examples given are those marked i—2, i—3, ii—3, ii—4, ii—5 on each of the pages 135, 136, 137. [But this subject is too wide to proceed further.]

84. *Sextans.* The Sextan

$$N_{vi} = x^4-x^2y^2+y^4 \quad \ldots\ldots\ldots\ldots \quad (160)$$

has the four *Associate* 2-ic forms i, ii, iii, iv, [Art. 76a, b].

$$\left.\begin{array}{llllllllll} & N_{vi} = & P^2-Q^2 & = & a^2+b^2 & = & A^2+3B^2 & = & A'^2-3B'^2 & \\ \text{with} & \mu = & -1 & = & +1 & = & +3 & = & -3 & [\text{Art. } 76b] \end{array}\right\} \quad (160a),$$

where i. $\pm P = \frac{1}{2}(N+1)$ .... (160b),

ii. $\pm a$ or $\pm b = x^2-y^2$, b or $a = xy$ .... (160c),

$$\left.\begin{array}{llll} \text{iii.} \quad \pm A = x^2-\frac{1}{2}y^2, & B = \frac{1}{2}y^2, & [y=\epsilon] \\ \pm A = \frac{1}{2}x^2-y^2, & B = \frac{1}{2}x^2, & [x=\epsilon] \\ \pm A = \frac{1}{2}(x^2+y^2), & B = \frac{1}{2}(x^2-y^2), & [xy=\omega]. \end{array}\right\} \ldots\ldots (160d),$$

iv. $\pm A' = x^2+y^2$, $B' = xy$ .... (160e).

The conditions of equivalence and reciprocity (Art. 78a) will be found to reduce to

$$\textit{Equivalence,} \quad C_2 = -\frac{C_1+2}{2C_1+1} \quad \ldots\ldots\ldots\ldots \quad (161a),$$

$$\textit{Reciprocity,} \quad C_2 = -\frac{C_1+2}{C_1-1} \quad \text{or} \quad -\frac{C_1}{C_1+1} \quad \ldots\ldots\ldots\ldots \quad (161b),$$

where C_1, C_2 mean any pair of C', C'', C''', C^{iv}, and C_1, C_2 are here *interchangeable.* Hence, to any given Characteristic (C_1) there correspond in general *one equivalent* and *two reciprocal* to it.

To find *suitable* Characteristics (C) and Factorisants, take as *Base-Sextan* (Art. 78)

$$N_0 = x_0^4-x_0^2y_0^2+y_0^4 = h^4-h^2k^2+k^4; \quad \text{so that} \quad x_0 = h \text{ or } k, \quad y_0 = k \text{ or } h.$$

Now, see the Tables on page 206.

The upper Table shows that there are 6 *ineffective* Characteristics; for in the Factorisants

$$\left.\begin{array}{l} C'' = -1 \text{ gives } x^2 = z^2, \quad C''' = -\tfrac{1}{2} \text{ gives } (\tfrac{1}{2}y^2)^2 = z^2 \\ \qquad C^{\mathrm{i}} \ = +1 \text{ gives } x^2 = z^2 \end{array}\right\} \ldots \ (162);$$

thus merely *reproducing the original* Base-Sextan.

The second Table shows

In lines 1 and 2; 2 cases of C'', C''' reciprocal.
In lines 3 and 4; 2 cases of C'', C^{iv} reciprocal.
In lines 5 and 6; 2 cases of C', C''' reciprocal.

Thus, of the 20 *primary* Characteristics (Art. 77c) of every Base-Sextan, 6 are always *ineffective* (for factorising), and there are always 2 *reciprocal* pairs, and also (when $h = 1$) 2 equivalent pairs.

The large Table on page 207 shows in full detail the elements (x_0, y_0, z_0) and the Characteristics (C) of the 20 Factorisants of the Simple Base-Sextan $\mathrm{N}_0 = 1^4 - 1^2 . k^2 + k^4$, with the data $x_0, y_0, \mathrm{P}_0, \mathrm{Q}_0, \mathrm{a}_0, \mathrm{b}_0, \mathrm{A}_0, \mathrm{B}_0, \mathrm{A}_0', \mathrm{B}_0'$. The last 4 lines at foot of page show the same details for an extra set of Class iv, when the 2-ic form $(\mathrm{A}_0'^2 - 3\mathrm{B}_0'^2)$ used is *not a Base-form* (Art. 7a).

[The letters *E*, *I*, *R* in the right column indicate *Equivalent*, *Ineffective*, *Reciprocal* respectively.]

The Table on page 208 shows the arithmetical values of the data and results of the Table of page 207 applied to the two prime Base-Sextans $\mathrm{N}_0 = 1^4 - 1^2 . 2^2 + 2^4 = 13$ and $\mathrm{N}_0 = 3^4 - 3^2 . 2^2 + 2^4 = 61$.

The Table on page 209 gives *two sets* of data and results (similar to those on page 208) for the *composite* Base-Sextan

$$\mathrm{N}_0 = 1^4 - 1^2 . 6^2 + 6^4 = 1261 = 13 . 97;$$

viz. the *primary* set arising from the *algebraic* 2-ic forms, and the *secondary* set from the *arithmetical* set of 2-ic forms due to the factors 13.97, (Art. 77b): (omitting however the "extra set" at foot of pages 207, 208).

The Table at foot of page 206 shows the Factorisants arising from a selected few of the Characteristics (C) of Classes i, ii, iv for the Base-Sextan $\mathrm{N}_0 = 13$, and of Class i for the Bases $\mathrm{N}_0 = 61, 73, 193, 481, 13.97, 13.157$.

[The column of "Ref. No." indicates the formulæ used from page 207: the column of "Serial" shows the sort of "Serial solutions" obtainable from each Factorisant (Art. 81).]

The Tables on pages 210–212 give worked Examples arising from the Factorisants in the Table at foot of page 206 for the Base-Sextans $\mathrm{N}_0 = 13$ and 61. The marks i—2, i—3, i—4; ii—2, ii—3, ii—5, ii—8; iv—4, iv—8 give the References connecting the two Tables.

The Table on page 213 gives the *arithmetical* values of the 2-ic parts (a, b), (A, B), (A′, B′) of the Co-factors (L, M) obtained in the Table on page 212.

85. *Trinomial forms of 4-tans and 6-tans.* The Diophantine process can be conveniently applied to the Trinomial forms of both Quartans (Art. 28) and Sextans (Art. 29), viz.

$$\textit{Quartans},\ N_{iv} = x^4 + 6x^2y^2 + y^4 \quad \ldots\ldots\ldots\ldots\ (163a).$$

$$\textit{Sextans},\ N_{vi} = x^4 + 14x^2y^2 + y^4 \quad \ldots\ldots\ldots\ldots\ (163b).$$

The Tables on pages 138, 214 show the details of the application to both.

The formula for the Factorisant of Class i, with Characteristic (C'), is placed at head of the page. The upper Table shows the Factorisants obtained from a number of small Base-Quartans and -Sextans. The lower Tables give a number of worked Examples from those Factorisants.

The 2-ic partitions (a, b), (A, B), (A′, B′) of a number of the L, M of the Sextans on pages 210, 211 are given on p. 213.

[Note that Factorisants exist in all the four Classes i, ii, iii, iv as previously obtained (Art. 83, 84), with the properties of Equivalence, Reciprocity, and Ineffective cases similar to those previously obtained; but it is thought not worth while to develop this further here.]

86. *Aurifeuillians, Diophantine process.* All Aurifeuillians (N) possess one *algebraic* resolution (Art. 37a) into the twin Aurifeuillian Factors N = L.M. The Diophantine process yields generally a (different) *arithmetical* resolution, say N = L′.M′. This serves to provide very large Aurifeuillians (of 4th degree in x, y) with factorisation into 4 factors.

These are treated of as follows

Bin-Aurifeuillians, Art. 86a. *Trin-Aurifeuillians*, Art. 86b.

86a. *Bin-Aurifeuillians*

$$N = x^4 + 4y^4 \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (164).$$

This has only two Associate 2-ic forms, (Art. 76a) and two Classes (i, ii).

$$N = P^2 - Q^2,\ \text{with}\ \mu = -1;\ \ N = a^2 + b^2,\ \text{with}\ \mu = +1 \ldots\ (164a),$$

where i. $\pm P = \frac{1}{2}(N+1)$, $\pm Q = \frac{1}{2}(N-1)$, ... with C', $C_{\prime}$ .... (164b),

ii. $\pm a$ or $\pm b = x^2$, $\pm b$ or $\pm a = 2y^2$, ... with C'', $C_{\prime\prime}$ ... (164c).

As N is not symmetric in x, y the factorisants differ according as the Characteristic (C) is attached to x or to y: hence each Class has *two* Characteristics, say C', $C_{\prime}$ of Class i, and C'', $C_{\prime\prime}$ of Class ii.

The conditions of equivalence and reciprocity (Art. 78a) reduce to

Equivalence. $C'C'' = -4, \quad C^{\backprime}C^{\backprime\backprime} = -4$ .... (165a).

Reciprocity. $4C'C'' = (C'^2-4)(C''^2-4), \quad 16C^{\backprime}C^{\backprime\backprime} = (C^{\backprime 2}-1)(C^{\backprime\backprime 2}-1)$ .... (165b).

Thus the characteristics exist in *equivalent pairs* (C', C''), $(C^{\backprime}, C^{\backprime\backprime})$, and reciprocity exists only when the roots of (165b) are real.

The Table at top of p. 147 shows the two Characteristics $(C', C^{\backprime})$, and the Factorisants arising from them for the Base Bin-Aurifeuillians $N_0 = 5, 65, 5.13, 5.17$, and also the kind of *Serial solution* (x, y, or z, Art. 81) obtainable from each.

The Table at foot of p. 147 shows the x-Chains obtained from Ex. I—2, ($N_0 = 65$), and I–3, ($N_0 = 65$), of the Table at top of page.

The Table on page 148 shows the two y-Chains obtained from Ex. II—2, ($N_0 = 5$) of the Table at top of page 147.

The numbers factorised $N_r = L_r . M_r = L'_r . M'_r$ run very large (up to 28 figures). The factor M'_r alone is shown; here of course $L'_r = M'_{r-1}$.

86b. *Trin-Aurifeuillians.*

$$N = x^4 - 3x^2y^2 + 9y^4 \quad \text{........................} \quad (166).$$

This has only two Associate 2-ic forms, (Art. 76a), and two Classes (i, iii).

$N = P^2 - Q^2$, with $\mu = -1$; $N = A^2 + 3B^2$, with $\mu = +3$ ... (166a),

where i. $\pm P = \frac{1}{2}(N+1), \quad \pm Q = \frac{1}{2}(N-1)$ ... with $C', C^{\backprime}$ ...... (166b),

iii. $\pm A = x^2 - 3y^2, \quad B = xy$ ... with $C''', C^{\backprime\backprime\backprime}$ .... (166c).

As N is not symmetric in x, y the factorisants differ according as the Characteristic (C) is attached to x or to y: hence each Class has *two* Characteristics, say C', $C^{\backprime}$ of Class i, and C''', $C^{\backprime\backprime\backprime}$ of Class iii.

The conditions of Equivalence and Reciprocity are too complicated to be readily satisfied.

The Table at foot of page 155 shows the two Characteristics $(C', C^{\backprime})$ of Class i, and the Factorisants arising from them for the Base Trin-Aurifeuillians $N_0 = 7, 13, 133, 7.19$, and also the kind of *Serial Solutions* (x, y, or z, Art. 81) obtainable from each.

The Tables on page 156 show worked examples of various sorts from the factorisants on page 155.

Ex. I—2 has *general* (non-serial) solutions from $C' = -5$, $N_0 = 7$, $x_0 = 1$.

Ex. I—5 shows the two x-Series from $C' = -7/2$, $N_0 = 7.19$.

Ex. II—2 shows the two y-Chains from $C' = -7$, $N_0 = 7$, $y_0 = 1$.

87. *Limitations of Diophantine process.* It should be clear from the developments in this Chapter that this process is of no help in the factorisation of *given numbers.* It only provides numbers which are certainly resolvable into two Co-factors, along with the data for such resolution.

INDEX.

[The figures refer to the *numbered Articles* in the Introduction.]

ADDENDA.

Page.

128 Left Table *Add* | ξ, η | x, y | L | M |
|---|---|---|---|
| 1, 16 | 32767, 4096 | 1073806849? | 17.89.457.1553; |

143 At side of Table II *For* < 32350 *Read* < 100000

ADDITIONAL CORRIGENDA.

Page.	*Line.*	*Col.*	*For.*	*Read.*
xlii	2	—	(84)	(85)
17	21	y, y	18679, 21030	18179, 22430
131	Last of Table II	C'	$\frac{1}{2}k^2$	$\frac{1}{2}k$
167	$x = 39$, $y = 52$	N	13.13.13.193	Cancel the line
172	$\eta = 37$	L	7296769;	7296697;
203	7	—	$\rho + 1 = 2$	$\rho + 1 = \epsilon$
204	6	—	$t_r u_r$	$2t_r u_r$
204	7	—	$\rho + 1 = 2$	$\rho + 1 = \epsilon$
210	3	—	$M_0 = 13$	Cancel
210	13	$r = 4$	1, 1697245	1, 1650245
219	$p = 1601$	$y^{32} + 1$	674	677
219	$p = 4673$	$y^{32} + 1$	2204	*48
226	Head	—	(x', y')	$f(x', y')$
231	3	—	z_r^1	z_r^2
231	23	—	$+ c^2$	$+ a^2$

p	y	y	p	y	y	p	y	y	p	y	y
5	2,	3	433	179,	254	1 009	469,	540	1 609	523,	1086
13	5,	8	449	67,	382	1 013	45,	968	1 613	127,	1486
17	4,	13	457	109,	348	1 021	374,	647	1 621	166,	1455
29	12,	17	461	48,	413	1 033	355,	678	1 637	316,	1321
37	6,	31	509	208,	301	1 049	426,	623	1 657	783,	874
41	9,	32	521	235,	286	1 061	103,	958	1 669	220,	1449
53	23,	30	541	52,	489	1 069	249,	820	1 693	92,	1601
61	11,	50	557	118,	439	1 093	530,	563	1 697	414,	1283
73	27,	46	569	86,	483	1 097	341,	756	1 709	390,	1319
89	34,	55	577	24,	553	1 109	354,	755	1 721	473,	1248
97	22,	75	593	77,	516	1 117	214,	903	1 733	410,	1323
101	10,	91	601	125,	476	1 129	168,	961	1 741	59,	1682
109	33,	76	613	35,	578	1 153	140,	1013	1 753	713,	1040
113	15,	98	617	194,	423	1 181	243,	938	1 777	775,	1002
137	37,	100	641	154,	487	1 193	186,	1007	1 789	724,	1065
149	44,	105	653	149,	504	1 201	49,	1152	1 801	824,	977
157	28,	129	661	106,	555	1 213	495,	718	1 861	61,	1800
173	80,	93	673	58,	615	1 217	78,	1139	1 873	737,	1136
181	19,	162	677	26,	651	1 229	597,	632	1 877	137,	1740
193	81,	112	701	135,	566	1 237	546,	691	1 889	331,	1558
197	14,	183	709	96,	613	1 249	585,	664	1 901	218,	1683
229	107,	122	733	353,	380	1 277	113,	1164	1 913	712,	1201
233	89,	144	757	87,	670	1 289	479,	810	1 933	598,	1335
241	64,	177	761	39,	722	1 297	36,	1261	1 949	589,	1360
257	16,	241	769	62,	707	1 301	51,	1250	1 973	259,	1714
269	82,	187	773	317,	456	1 321	257,	1064	1 993	834,	1159
277	60,	217	797	215,	582	1 361	614,	747	1 997	412,	1585
281	53,	228	809	318,	491	1 373	668,	705	2 017	229,	1788
293	138,	155	821	295,	526	1 381	366,	1015	2 029	992,	1037
313	25,	288	829	246,	583	1 409	452,	957	2 053	244,	1809
317	114,	203	853	333,	520	1 429	620,	809	2 069	164,	1905
337	148,	189	857	207,	650	1 433	542,	891	2 081	102,	1979
349	136,	213	877	151,	726	1 453	497,	956	2 089	789,	1300
353	42,	311	881	387,	494	1 481	465,	1016	2 113	65,	2048
373	104,	269	929	324,	605	1 489	225,	1264	2 129	372,	1757
389	115,	274	937	196,	741	1 493	432,	1061	2 137	296,	1841
397	63,	334	941	97,	844	1 549	88,	1461	2 141	419,	1722
401	20,	381	953	442,	511	1 553	339,	1214	2 153	232,	1921
409	143,	266	977	252,	725	1 597	610,	987	2 161	147,	2014
421	29,	392	997	161,	836	1 601	40,	1561	2 213	1083,	1130

p	y y	p	y y	p	y y	p	y y
2 221	790, 1431	2 801	1258, 1543	3 517	596, 2921	4 177	457, 3720
2 237	1021, 1216	2 833	1357, 1476	3 529	808, 2721	4 201	1154, 3047
2 269	982, 1287	2 837	416, 2421	3 533	548, 2985	4 217	1911, 2306
2 273	290, 1983	2 857	896, 1961	3 541	852, 2689	4 229	2082, 2147
2 281	710, 1571	2 861	1202, 1659	3 557	943, 2614	4 241	1044, 3197
2 293	600, 1693	2 897	1120, 1777	3 581	364, 3217	4 253	561, 3692
2 297	365, 1932	2 909	878, 2031	3 593	1153, 2440	4 261	721, 3540
2 309	688, 1621	2 917	54, 2863	3 613	85, 3528	4 273	1200, 3073
2 333	108, 2225	2 953	1226, 1727	3 617	1234, 2383	4 289	528, 3761
2 341	153, 2188	2 957	1222, 1735	3 637	1027, 2610	4 297	1972, 2325
2 357	633, 1724	2 969	964, 2005	3 673	994, 2679	4 337	886, 3451
2 377	1134, 1243	3 001	1353, 1648	3 677	1309, 2368	4 349	608, 3741
2 381	69, 2312	3 037	281, 2756	3 697	1131, 2566	4 357	66, 4291
2 389	285, 2104	3 041	774, 2267	3 701	1279, 2422	4 373	1904, 2469
2 393	971, 1422	3 049	475, 2574	3 709	1609, 2100	4 397	505, 3892
2 417	592, 1825	3 061	501, 2560	3 733	851, 2882	4 409	332, 4077
2 437	398, 2039	3 089	393, 2696	3 761	604, 3157	4 421	952, 3469
2 441	672, 1769	3 109	727, 2382	3 769	1445, 2324	4 441	2146, 2295
2 473	567, 1906	3 121	79, 3042	3 793	803, 2990	4 457	1880, 2577
2 477	915, 1562	3 137	56, 3081	3 797	742, 3055	4 481	276, 4205
2 521	71, 2450	3 169	1325, 1844	3 821	376, 3445	4 493	2213, 2280
2 549	357, 2192	3 181	282, 2899	3 833	361, 3472	4 513	95, 4418
2 557	611, 1946	3 209	484, 2725	3 853	1305, 2548	4 517	1474, 3043
2 593	918, 1675	3 217	1436, 1781	3 877	502, 3375	4 549	1260, 3289
2 609	389, 2220	3 221	234, 2987	3 881	197, 3684	4 561	2205, 2356
2 617	667, 1950	3 229	839, 2390	3 889	454, 3435	4 597	2129, 2468
2 621	472, 2149	3 253	1598, 1655	3 917	835, 3082	4 621	152, 4469
2 633	1224, 1409	3 257	291, 2966	3 929	226, 3703	4 637	2044, 2593
2 657	163, 2494	3 301	1212, 2089	3 989	481, 3508	4 649	1846, 2803
2 677	550, 2127	3 313	407, 2906	4 001	899, 3102	4 657	1912, 2745
2 689	1142, 1547	3 329	1600, 1729	4 013	1230, 2783	4 673	1993, 2680
2 693	859, 1834	3 361	900, 2461	4 021	723, 3298	4 721	1697, 3024
2 713	887, 1826	3 373	1105, 2268	4 049	884, 3165	4 729	1365, 3364
2 729	1102, 1627	3 389	1344, 2045	4 057	1857, 2200	4 733	897, 3836
2 741	656, 2085	3 413	1471, 1942	4 073	549, 3524	4 789	1481, 3308
2 749	640, 2109	3 433	1651, 1782	4 093	1059, 3034	4 793	1480, 3313
2 753	794, 1959	3 449	1122, 2327	4 129	895, 3234	4 801	1403, 3398
2 777	190, 2587	3 457	708, 2749	4 133	733, 3400	4 813	1868, 2945
2 789	167, 2622	3 461	1453, 2008	4 153	1643, 2510	4 817	1291, 3526
2 797	603, 2194	3 469	1003, 2466	4 157	1761, 2396	4 861	493, 4368

p	y	y	p	y	y	p	y	y	p	y	y
4 877	719,	4158	5 581	1437,	4144	6 301	2184,	4117	6 977	2063,	4914
4 889	730,	4159	5 641	1429,	4212	6 317	1963,	4354	6 997	1796,	5201
4 909	1613,	3296	5 653	310,	5343	6 329	2219,	4110	7 001	1198,	5803
4 933	1194,	3739	5 657	1670,	3987	6 337	178,	6159	7 013	2480,	4533
4 937	849,	4088	5 669	1046,	4623	6 353	1392,	4961	7 057	84,	6973
4 957	359,	4598	5 689	2124,	3565	6 361	1751,	4610	7 069	188,	6881
4 969	1076,	3893	5 693	1193,	4500	6 373	1879,	4494	7 109	304,	6805
4 973	223,	4750	5 701	385,	5316	6 389	2092,	4297	7 121	778,	6343
4 993	158,	4835	5 717	2416,	3301	6 397	1302,	5095	7 129	267,	6862
5 009	539,	4470	5 737	1126,	4611	6 421	825,	5596	7 177	1965,	5212
5 021	1363,	3658	5 741	2378,	3363	6 449	1854,	4595	7 193	967,	6226
5 077	858,	4219	5 749	806,	4943	6 469	2977,	3492	7 213	1999,	5214
5 081	2412,	2669	5 801	1145,	4656	6 473	1808,	4665	7 229	3572,	3657
5 101	101,	5000	5 813	796,	5017	6 481	729,	5752	7 237	2502,	4735
5 113	2025,	3088	5 821	1242,	4579	6 521	2364,	4157	7 253	2211,	5042
5 153	227,	4926	5 849	2839,	3010	6 529	2311,	4218	7 297	3553,	3744
5 189	2446,	2743	5 857	1310,	4547	6 553	3186,	3367	7 309	2717,	4592
5 197	1969,	3228	5 861	754,	5107	6 569	3038,	3531	7 321	121,	7200
5 209	2098,	3111	5 869	1042,	4827	6 577	1624,	4953	7 333	2909,	4424
5 233	2253,	2980	5 881	1098,	4783	6 581	2727,	3854	7 349	2061,	5288
5 237	369,	4868	5 897	543,	5354	6 637	2828,	3809	7 369	607,	6762
5 261	827,	4434	5 953	2403,	3550	6 653	752,	5901	7 393	2361,	5032
5 273	944,	4329	5 981	1317,	4664	6 661	658,	6003	7 417	2737,	4680
5 281	1673,	3608	6 029	1801,	4228	6 673	2437,	4236	7 433	983,	6450
5 297	2313,	2984	6 037	2652,	3385	6 689	2759,	3930	7 457	1275,	6182
5 309	1804,	3505	6 053	2832,	3221	6 701	1721,	4980	7 477	1652,	5825
5 333	2630,	2703	6 073	2524,	3549	6 709	2150,	4559	7 481	1408,	6073
5 381	1739,	3642	6 089	455,	5634	6 733	2217,	4516	7 489	1591,	5898
5 393	665,	4728	6 101	247,	5854	6 737	2393,	4344	7 517	3409,	4108
5 413	429,	4984	6 113	1089,	5024	6 761	1775,	4986	7 529	2445,	5084
5 417	368,	5049	6 121	2583,	3538	6 781	995,	5786	7 537	1049,	6488
5 437	630,	4807	6 133	865,	5268	6 793	709,	6084	7 541	2867,	4674
5 441	2452,	2989	6 173	2447,	3726	6 829	1596,	5233	7 549	2931,	4618
5 449	635,	4814	6 197	2007,	4190	6 833	1307,	5526	7 561	2923,	4638
5 477	74,	5403	6 217	2372,	3845	6 841	1625,	5216	7 573	3743,	3830
5 501	1115,	4386	6 221	1121,	5100	6 857	1348,	5509	7 577	1540,	6037
5 521	765,	4756	6 229	1451,	4778	6 869	998,	5871	7 589	3270,	4319
5 557	2478,	3079	6 257	1584,	4673	6 917	263,	6654	7 621	2038,	5583
5 569	973,	4596	6 269	1523,	4746	6 949	932,	6017	7 649	2363,	5286
5 573	2017,	3556	6 277	1033,	5244	6 961	344,	6617	7 669	2292,	5377

p	y	y	p	y	y	p	y	y
7 673	277,	7396	8 369	666,	7703	9 133	3732,	5401
7 681	3383,	4298	8 377	330,	8047	9 137	1286,	7851
7 717	2953,	4764	8 389	3449,	4940	9 157	2203,	6954
7 741	3199,	4542	8 429	2190,	6239	9 161	3125,	6036
7 753	2555,	5198	8 461	1786,	6675	9 173	2514,	6659
7 757	812,	6945	8 501	4020,	4481	9 181	303,	8878
7 789	3378,	4411	8 513	1203,	7310	9 209	346,	8863
7 793	2214,	5579	8 521	2606,	5915	9 221	3300,	5921
7 817	2564,	5253	8 537	1595,	6942	9 241	1829,	7412
7 829	2037,	5792	8 573	2195,	6378	9 257	1097,	8160
7 841	198,	7643	8 581	131,	8450	9 277	888,	8389
7 853	1759,	6094	8 597	2318,	6279	9 281	586,	8695
7 873	3590,	4283	8 609	1830,	6779	9 293	482,	8811
7 877	320,	7557	8 629	4123,	4506	9 337	3404,	5933
7 901	3346,	4555	8 641	1583,	7058	9 341	2638,	6703
7 933	2950,	4983	8 669	3876,	4793	9 349	3641,	5708
7 937	1962,	5975	8 677	3963,	4714	9 377	2848,	6529
7 949	679,	7270	8 681	3911,	4770	9 397	1852,	7545
7 993	2110,	5883	8 689	4061,	4628	9 413	4658,	4755
8 009	283,	7726	8 693	4048,	4645	9 421	2301,	7120
8 017	1813,	6204	8 713	3279,	5434	9 433	1014,	8419
8 053	370,	7683	8 737	4264,	4473	9 437	830,	8607
8 069	2732,	5337	8 741	3320,	5421	9 461	1510,	7951
8 081	3940,	4141	8 753	2569,	6184	9 473	1172,	8301
8 089	2293,	5796	8 761	468,	8293	9 497	624,	8873
8 093	3060,	5033	8 821	297,	8524	9 521	2140,	7381
8 101	90,	8011	8 837	94,	8743	9 533	1977,	7556
8 117	1733,	6384	8 849	2994,	5855	9 601	404,	9197
8 161	202,	7959	8 861	1791,	7070	9 613	3237,	6376
8 209	1939,	6270	8 893	2851,	6042	9 629	3832,	5797
8 221	3745,	4476	8 929	3424,	5505	9 649	845,	8804
8 233	856,	7377	8 933	762,	8171	9 661	139,	9522
8 237	287,	7950	8 941	3080,	5861	9 677	1338,	8339
8 269	643,	7626	8 969	510,	8459	9 689	2212,	7477
8 273	2162,	6111	9 001	1237,	7764	9 697	1557,	8140
8 293	531,	7762	9 013	1658,	7355	9 721	4406,	5315
8 297	2097,	6200	9 029	4467,	4562	9 733	2709,	7024
8 317	1371,	6946	9 041	2284,	6757	9 749	356,	9393
8 329	1443,	6886	9 049	1362,	7687	9 769	4774,	4995
8 353	2688,	5665	9 109	1986,	7123	9 781	1195,	8586

p	y	y
9 817	2479,	7338
9 829	1304,	8525
9 833	534,	9299
9 857	222,	9635
9 901	1000,	8901
9 929	2102,	7827
9 941	141,	9800
9 949	2543,	7406
9 973	2798,	7175

p^κ	y	y
5^2	7,	18
5^3	57,	68
13^2	70,	99
17^2	38,	251
5^4	182,	443
29^2	41,	800
37^2	117,	1252
41^2	378,	1303
13^3	239,	1958
53^2	500,	2309
5^5	1068,	2057
61^2	682,	3039
17^3	1985,	2928
73^2	776,	4553
89^2	3861,	4060
97^2	4052,	5357

p	y	y	p	y	y	p	y	y
10 009	3303,	6706	10 837	3410,	7427	11 677	551,	11126
10 037	3271,	6766	10 853	2573,	8280	11 681	4556,	7125
10 061	4602,	5459	10 861	2657,	8204	11 689	4654,	7035
10 069	4630,	5439	10 889	5037,	5852	11 701	446,	11255
10 093	2388,	7705	10 909	2060,	8849	11 717	2374,	9343
10 133	1758,	8375	10 937	1998,	8939	11 777	5322,	6455
10 141	1313,	8828	10 949	3873,	7076	11 789	4546,	7243
10 169	2339,	7830	10 957	3542,	7415	11 801	5608,	6193
10 177	3286,	6891	10 973	5039,	5934	11 813	3014,	8799
10 181	1500,	8681	10 993	2123,	8870	11 821	4071,	7750
10 193	4729,	5464	11 057	3355,	7702	11 833	2739,	9094
10 253	1236,	9017	11 069	4819,	6250	11 897	2947,	8950
10 273	591,	9682	11 093	1508,	9585	11 909	4051,	7858
10 289	4836,	5453	11 113	4475,	6638	11 933	3792,	8141
10 301	1020,	9281	11 117	4008,	7109	11 941	5530,	6411
10 313	1681,	8632	11 149	1559,	9590	11 953	2103,	9850
10 321	3151,	7170	11 161	4045,	7116	11 969	3129,	8840
10 333	3058,	7275	11 173	3456,	7717	11 981	1209,	10772
10 337	2747,	7590	11 177	1182,	9995	12 037	3417,	8620
10 357	4517,	5840	11 197	3938,	7259	12 041	347,	11694
10 369	4278,	6091	11 213	1505,	9708	12 049	2639,	9410
10 429	4192,	6237	11 257	541,	10716	12 073	2181,	9892
10 433	323,	10110	11 261	2231,	9030	12 097	1873,	10224
10 453	2972,	7481	11 273	4043,	7230	12 101	110,	11991
10 457	1967,	8490	11 317	5018,	6299	12 109	4073,	8036
10 477	2011,	8466	11 321	532,	10789	12 113	3496,	8617
10 501	1284,	9217	11 329	238,	11091	12 149	5191,	6958
10 513	145,	10368	11 353	4150,	7203	12 157	706,	11451
10 529	1369,	9160	11 369	3070,	8299	12 161	4993,	7168
10 589	2740,	7849	11 393	1217,	10176	12 197	4718,	7479
10 597	804,	9793	11 437	4259,	7178	12 241	892,	11349
10 601	525,	10076	11 489	625,	10864	12 253	2282,	9971
10 613	5255,	5358	11 497	1594,	9903	12 269	1896,	10373
10 657	2499,	8158	11 549	3454,	8095	12 277	4277,	8000
10 709	3223,	7486	11 593	975,	10618	12 281	5521,	6760
10 729	3970,	6759	11 597	4267,	7330	12 289	1479,	10810
10 733	518,	10215	11 617	5688,	5929	12 301	248,	12053
10 753	4489,	6264	11 621	5541,	6080	12 329	4162,	8167
10 781	2370,	8411	11 633	3275,	8358	12 373	3243,	9130
10 789	4883,	5906	11 657	4820,	6837	12 377	3673,	8704

p	y	y	p	y	y	p	y	y
12 401	6076,	6325	13 037	1914,	11123	13 829	424,	13405
12 409	1897,	10512	13 049	976,	12073	13 841	2263,	11578
12 413	4686,	7727	13 093	5098,	7995	13 873	2954,	10919
12 421	1231,	11190	13 109	4455,	8654	13 877	6644,	7233
12 433	2567,	9866	13 121	6354,	6767	13 901	3747,	10154
12 437	3327,	9110	13 177	363,	12814	13 913	373,	13540
12 457	5714,	6743	13 217	2160,	11057	13 921	3135,	10786
12 473	1166,	11307	13 229	6557,	6672	13 933	4605,	9328
12 497	3965,	8532	13 241	3339,	9902	13 997	5779,	8218
12 517	902,	11615	13 249	928,	12321	14 009	4507,	9502
12 541	2394,	10147	13 297	2692,	10605	14 029	1092,	12937
12 553	4147,	8406	13 309	1878,	11431	14 033	1446,	12587
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17 137	7318,	9819	17 981	3623,	14358	18 637	3766,	14871
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17 209	6167,	11042	18 013	5011,	13002	18 701	2276,	16425
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17 393	769,	16624	18 149	7537,	10612	18 917	1835,	17082
17 401	1544,	15857	18 169	486,	17683	18 973	308,	18665
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17 449	7006,	10443	18 217	5810,	12407	19 009	7463,	11546
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26 993	8647,	18346	27 941	8165,	19776	28 793	7977,	20816
27 017	2471,	24546	27 953	3515,	24438	28 813	5885,	22928
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27 241	6769,	20472	28 081	3679,	24402	28 933	916,	28017
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27 277	7931,	19346	28 109	8065,	20044	28 961	13024,	15937
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29 201	3287, 25914	30 137	868, 29269	30 893	7625, 23268
29 209	5276, 23933	30 161	6782, 23379	30 937	14395 16542
29 221	9215, 20006	30 169	3517, 26652	30 941	5826, 25115
29 269	9576, 19693	30 181	4745, 25436	30 949	12026, 18923
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36 713	11912,	24801	37 489	4491,	32998	38 333	5945,	32388
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39 229	15652,	23577	40 177	11710,	28467	41 141	19704,	21437
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39 241	12163,	27078	40 193	1169,	39024	41 161	19842,	21319
39 293	1795,	37498	40 213	6373,	33840	41 177	14271,	26906
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39 569	17048,	22521	40 577	637,	39940	41 381	6500,	34881
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41 941	19882, 22059	42 689	462, 42227	43 609	15330, 28279
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41 957	17574, 24383	42 701	14789, 27912	43 633	7799, 35834
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42 169	3497, 38672	42 901	854, 42047	43 793	1895, 41898
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42 209	3588, 38621	42 961	17541, 25420	43 913	17738, 26175
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42 433	14731, 27702	43 201	13677, 29524	44 053	8505, 35548
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44 293	10485	33808	45 281	10183	35098	46 073	11297	34776
44 357	1698	42659	45 289	8056	37233	46 093	7217	38876
44 381	16584	27797	45 293	673	44620	46 133	18956	27177
44 389	7821	36568	45 317	14947	30370	46 141	6745	39396
44 417	16002	28415	45 329	14279	31050	46 153	8234	37919
44 449	18896	25553	45 337	21557	23780	46 181	10164	36017
44 453	15734	28719	45 341	20185	25156	46 229	23007	23222
44 497	4870	39627	45 361	20646	24715	46 237	11019	35218
44 501	12076	32425	45 377	12541	32836	46 261	7746	38515
44 533	4273	40260	45 389	11735	33654	46 273	22896	23377
44 537	11187	33350	45 413	6035	39378	46 301	19252	27049
44 549	10947	33602	45 433	17064	28369	46 309	14775	31534
44 617	10436	34181	45 481	16772	28709	46 337	2601	43736
44 621	4441	40180	45 497	13726	31771	46 349	4838	41511
44 633	9656	34977	45 533	2774	42759	46 381	13349	33032
44 641	19476	25165	45 541	7768	37773	46 441	22832	23609
44 657	15265	29392	45 553	880	44673	46 457	5284	41173
44 701	299	44402	45 557	19961	25596	46 477	7247	39230
44 729	14619	30110	45 569	3654	41915	46 489	12635	33854
44 741	4938	39803	45 589	1245	44344	46 549	2598	43951
44 753	7380	37373	45 613	3637	41976	46 573	12674	33899
44 773	20613	24160	45 641	19328	26313	46 589	6304	40285
44 777	14006	30771	45 673	11849	33824	46 601	18269	28332
44 789	2413	42376	45 677	6711	38966	46 633	17705	28928
44 797	8429	36368	45 697	478	45219	46 649	683	45966
44 809	9712	35097	45 737	6712	39025	46 681	9293	37388
44 893	10938	33955	45 757	6825	38932	46 757	15076	31681
44 909	16400	28509	45 817	8903	36914	46 769	9606	37163
44 917	5938	38979	45 821	9207	36614	46 817	12544	34273
44 953	15055	29898	45 833	677	45156	46 829	1082	45747
45 013	3613	41400	45 841	20449	25392	46 853	15779	31074
45 053	7990	37063	45 853	6243	39610	46 861	13458	33403
45 061	13089	31972	45 869	9621	36248	46 877	12142	34735
45 077	17668	27409	45 893	7312	38581	46 889	12426	34463
45 121	4678	40443	45 949	18161	27788	46 901	19851	27050
45 137	18208	26929	45 953	17844	28109	46 933	6463	40470
45 161	2423	42738	45 989	21249	24740	46 957	10187	36770
45 181	4382	40799	46 021	12258	33763	46 993	18181	28812
45 197	15637	29560	46 049	13526	32523	46 997	19129	27868

p	y, y	p	y, y	p	y, y
47 017	19808, 27209	47 809	21958, 25851	48 677	17250, 31427
47 041	15941, 31100	47 837	18000, 29837	48 733	2517, 46216
47 057	11417, 35640	47 857	7272, 40585	48 757	6554, 42203
47 093	23655, 23438	47 869	22024, 25845	48 761	17953, 30808
47 129	15812, 31317	47 881	8202, 39679	48 781	11764, 37017
47 137	9366, 37771	47 917	7227, 40690	48 809	16105, 32704
47 149	13515, 33634	47 933	12278, 35655	48 817	20851, 27966
47 161	23189, 23972	47 969	6109, 41860	48 821	7985, 40836
47 189	14135, 33054	47 977	12049, 35928	48 857	12159, 36698
47 221	21952, 25269	47 981	15405, 32576	48 869	9516, 39353
47 237	12617, 34620	48 017	4437, 43580	48 889	14338, 34551
47 269	5543, 41726	48 029	4518, 43511	48 953	7234, 41719
47 293	3029, 44264	48 049	15651, 32398	48 973	23778, 25195
47 297	22286, 25011	48 073	19410, 28663	48 989	16434, 32555
47 309	12491, 34818	48 109	21590, 26519	49 009	7046, 41963
47 317	2432, 44885	48 121	23262, 24859	49 033	11736, 37297
47 353	10402, 36951	48 157	10335, 37822	49 037	17529, 31508
47 381	4187, 43194	48 193	21141, 27052	49 057	22030, 27027
47 389	12349, 35040	48 197	18693, 29504	49 069	7525, 41544
47 417	3443, 43974	48 221	3121, 45100	49 081	14605, 34476
47 441	7988, 39453	48 281	2655, 45626	49 109	12477, 36632
47 497	14733, 32764	48 313	15350, 32963	49 117	13008, 36109
47 501	8342, 39159	48 337	17679, 30658	49 121	2527, 46594
47 513	1271, 46242	48 341	3148, 45193	49 157	11290, 37867
47 521	1326, 46195	48 353	16587, 31766	49 169	17395, 31774
47 533	15917, 31616	48 397	4749, 43648	49 177	11448, 37729
47 569	5470, 42099	48 409	16063, 32346	49 193	6386, 42807
47 581	20165, 27416	48 413	492, 47921	49 201	20408, 28793
47 609	7739, 39870	48 437	8762, 39675	49 253	11636, 37617
47 629	488, 47141	48 449	13874, 34575	49 261	11620, 37641
47 653	17284, 30369	48 473	22600, 25873	49 277	6678, 42599
47 657	11067, 36590	48 481	10798, 37683	49 297	7875, 41422
47 681	15207, 32474	48 497	19240, 29257	49 333	21111, 28222
47 701	10068, 37633	48 533	8946, 39587	49 369	17689, 31680
47 713	17521, 30192	48 541	7648, 40893	49 393	23672, 25721
47 717	2249, 45468	48 589	10875, 37714	49 409	8152, 41257
47 737	12284, 35453	48 593	13316, 35277	49 417	10286, 39131
47 741	309, 47432	48 649	10460, 38189	49 429	5414, 44015
47 777	11201, 36576	48 661	21320, 27341	49 433	4430, 45003
47 797	1665, 46132	48 673	13616, 35057	49 477	16060, 33417

p	y	y	p	y	y	p	y	y
49 481	14252,	35229	50 461	21198,	29263	51 413	12980,	38433
49 529	6013,	43516	50 497	23877,	26620	51 421	28113,	23308
49 537	13356,	36181	50 513	1812,	48701	51 437	24353,	27084
49 549	14319,	35230	50 549	24811,	25738	51 449	11061,	40388
49 597	1426,	48171	50 581	23761,	26820	51 461	3248,	48213
49 613	315,	49298	50 593	9776,	40817	51 473	10131,	41342
49 633	4394,	45239	50 741	2937,	47804	51 481	20857,	30624
49 669	14758,	34911	50 753	1593,	49160	51 517	9802,	41715
49 681	12650,	37031	50 773	15139,	35634	51 521	321,	51200
49 697	19090,	30607	50 777	1149,	49628	51 577	25177,	26400
49 741	18231,	31510	50 789	15381,	35408	51 581	4660,	46921
49 757	11689,	38068	50 821	3614,	47207	51 593	19319,	32274
49 789	3406,	46383	50 833	13441,	37392	51 613	508,	51105
49 801	24651,	25150	50 849	23178,	27671	51 637	14129,	37508
49 853	6131,	43722	50 857	1444,	49413	51 673	4325,	47348
49 877	6492,	43385	50 873	2640,	48233	51 713	15967,	35746
49 921	17926,	31995	50 893	20421,	30472	51 721	22699,	29022
49 937	7320,	42617	50 909	19899,	31010	51 749	13531,	38218
49 957	1802,	48155	50 929	19279,	31650	51 769	1944,	49825
49 993	11031,	38962	50 957	5534,	45423	51 797	20024,	31773
50 021	18466,	31555	50 969	814,	50155	51 817	25654,	26163
50 033	24143,	25890	50 989	10441,	40548	51 829	6415,	45414
50 053	13916,	36137	50 993	23818,	27175	51 853	14391,	37462
50 069	12110,	37959	51 001	15996,	35005	51 869	22744,	29125
50 077	22866,	27211	51 061	24170,	26891	51 893	25377,	26516
50 093	19401,	30692	51 109	20898,	30211	51 913	21214,	30699
50 101	18687,	31414	51 133	7140,	43993	51 929	7778,	44151
50 129	17534,	32595	51 137	14839,	36298	51 941	1780,	50161
50 153	1206,	48947	51 157	5659,	45498	51 949	10124,	41825
50 177	224,	49953	51 169	19047,	32122	51 973	1838,	50135
50 221	16471,	33750	51 193	16369,	34824	51 977	17639,	34338
50 261	24073,	26188	51 197	9288,	41909	52 009	20758,	31251
50 273	14465,	35808	51 217	7389,	43828	52 021	14615,	37406
50 321	19660,	30661	51 229	4583,	46646	52 057	15451,	36606
50 329	11590,	38739	51 241	19747,	31494	52 069	17819,	34250
50 333	8543,	41790	51 257	1132,	50125	52 081	4245,	47836
50 341	24212,	26129	51 329	8977,	42352	52 121	23928,	28193
50 377	19824,	30553	51 341	19514,	31827	52 153	8006,	44147
50 417	11148,	39269	51 349	8747,	42602	52 177	10500,	41677
50 441	23965,	26476	51 361	23185,	28176	52 181	8179,	44002

Least Roots (y) *of* $y^2+1\equiv 0$ (*mod* p^κ).

modulus	p	y	y
modulus = p^2	101	515,	9686
	109	5744,	6137
	113	1710,	11059
	137	6613,	12156
	149	1744,	20457
	157	11018,	13631
	173	13241,	16688
	181	3458,	29303
	193	5099,	32150
	197	1393,	37416
	229	16610,	35831
	233	26884,	27405
	241	15006,	43075
	257	2072,	63977
	269	13637,	58724
	277	31361,	45368
	281	4443,	74518
	293	26508,	59341
	313	7850,	90119

modulus	p	y	y
modulus = p^2	317	62969,	37520
	337	31152,	82417
	349	39922,	81879
	353	37107,	87502
	373	6072,	133057
	389	4005,	147316
	397	32491,	125118
	401	4030,	156771
	409	43211,	124070
	421	12238,	165003
	433	55603,	131886
	449	51119,	150482
	457	21131,	187718
	461	55368,	157153

modulus	p	y	y
mod = p^3	29	10133,	14256
	37	9466,	41187
	41	11389,	57532
mod = p^4	13	239,	28322
	17	27493,	56028
mod	5^6	1068,	14557
	5^7	32318,	45807

Least Roots (y) *of* $y^4+1\equiv 0$ (*mod* p^κ).

modulus	p	y	y	y	y
modulus = p^2	113	1990,	6346,	6423,	10779
	137	3141,	5850,	12919,	15628
	193	3851,	6026,	31223,	33398
	233	3825,	7934,	46355,	50464
	241	5573,	8202,	49879,	52508

modulus	p	y	y	y	y
mod = p^2	257	261,	12400,	53649,	65788
	281	1907,	23974,	54987,	77054
	313	9828,	48197,	49772,	88141
mod	41^3	22629,	26062,	42859,	46292
	17^4	20051,	23543,	59978,	63470

Least Roots (y) *of* $y^8+1\equiv 0$ (*mod* p^κ).

modulus	p	y	y	y	y	y	y	y	y
mod = p^2	113	1057,	3455,	3997,	5728,	7041,	8772,	9314,	11712
	193	436,	2831,	11776,	17343,	19906,	25473,	34418,	36813
	241	7104,	11765,	20650,	23989,	34092,	37431,	46316,	50977
	257	779,	5172,	16446,	28912,	37137,	49603,	60877,	65270
	$p^4=17^4$	4260,	15541,	23738,	24723,	58798,	59783,	67980,	79261

p	y	y	y	y	p	y	y	y	y
17	2,	8,	9,	15	1 097	79,	486,	611,	1018
41	3,	14,	27,	38	1 129	31,	437,	692,	1098
73	10,	22,	51,	63	1 153	75,	123,	1030,	1078
89	12,	37,	52,	77	1 193	362,	524,	669,	831
97	33,	47,	50,	64	1 201	7,	343,	858,	1194
113	18,	44,	69,	95	1 217	239,	387,	830,	978
137	10,	41,	96,	127	1 249	338,	388,	861,	911
193	9,	43,	150,	184	1 289	402,	497,	792,	887
233	12,	97,	136,	221	1 297	6,	216,	1081,	1291
241	8,	30,	211,	233	1 321	235,	371,	950,	1086
257	4,	64,	193,	253	1 361	114,	585,	776,	1247
281	60,	89,	192,	221	1 409	72,	137,	1272,	1337
313	5,	125,	188,	308	1 433	342,	507,	926,	1091
337	85,	111,	226,	252	1 481	511,	655,	826,	970
353	70,	116,	237,	283	1 489	15,	397,	1092,	1474
401	45,	98,	303,	356	1 553	483,	672,	881,	1070
409	31,	66,	343,	378	1 601	310,	408,	1193,	1291
433	79,	148,	285,	354	1 609	355,	630,	979,	1254
449	92,	122,	327,	357	1 657	104,	239,	1418,	1553
457	170,	207,	250,	287	1 697	292,	401,	1296,	1405
521	43,	206,	315,	478	1 721	232,	408,	1313,	1489
569	76,	277,	292,	493	1 753	190,	489,	1264,	1563
577	152,	186,	391,	425	1 777	108,	181,	1596,	1669
593	59,	201,	392,	534	1 801	464,	524,	1277,	1337
601	59,	163,	438,	542	1 873	219,	325,	1548,	1654
617	139,	182,	435,	478	1 889	85,	200,	1689,	1804
641	256,	318,	323,	385	1 913	305,	922,	991,	1608
673	64,	326,	347,	609	1 993	546,	960,	1033,	1447
761	62,	135,	626,	699	2 017	438,	548,	1469,	1579
769	40,	173,	596,	729	2 081	868,	947,	1134,	1213
809	44,	239,	570,	765	2 089	84,	572,	1517,	2005
857	188,	351,	506,	669	2 113	663,	835,	1278,	1450
881	177,	219,	662,	704	2 129	380,	846,	1283,	1749
929	18,	258,	671,	911	2 137	265,	629,	1508,	1872
937	14,	67,	870,	923	2 153	246,	1059,	1094,	1907
953	156,	336,	617,	797	2 161	335,	458,	1703,	1826
977	227,	439,	538,	750	2 273	465,	743,	1530,	1808
1 009	192,	247,	762,	817	2 281	686,	1074,	1207,	1595
1 033	231,	398,	635,	802	2 297	890,	973,	1324,	1407
1 049	223,	461,	588,	826	2 377	580,	709,	1668,	1797

p	y	y	y	y	p	y	y	y	y
2 393	58	1114	1279	2335	3 673	1010	1211	2462	2663
2 417	345	1205	1212	2072	3 697	529	615	3082	3168
2 441	285	1122	1319	2156	3 761	490	1159	2602	3271
2 473	574	978	1495	1899	3 769	409	728	3041	3360
2 521	159	1205	1316	2362	3 793	294	916	2877	3499
2 593	625	697	1896	1968	3 833	19	807	3026	3814
2 609	271	1059	1550	2338	3 881	977	1581	2300	2904
2 617	99	608	2009	2518	3 889	427	592	3297	3462
2 633	885	1077	1556	1748	3 929	1643	1937	1992	2286
2 657	379	666	1991	2278	4 001	70	1086	2915	3931
2 689	653	873	1816	2036	4 049	665	755	3294	3384
2 713	60	1040	1673	2653	4 057	315	747	3310	3742
2 729	66	951	1778	2663	4 073	514	1149	2924	3559
2 753	286	1338	1415	2467	4 129	777	1743	2386	3352
2 777	224	905	1872	2553	4 153	104	599	3554	4049
2 801	576	851	1950	2225	4 177	1395	1566	2611	2782
2 833	450	1278	1555	2383	4 201	107	1649	2552	4094
2 857	933	1133	1724	1924	4 217	590	1551	2666	3627
2 897	311	680	2217	2586	4 241	473	1856	2385	3768
2 953	456	939	2014	2497	4 273	325	1157	3116	3948
2 969	544	1097	1872	2425	4 289	271	1551	2738	4018
3 001	711	1338	1663	2290	4 297	1008	1735	2562	3289
3 041	185	263	2778	2856	4 337	777	1161	3176	3560
3 049	137	1046	2003	2912	4 409	693	808	3601	3716
3 089	273	826	2263	2816	4 441	1096	1714	2727	3345
3 121	285	668	2453	2836	4 457	420	711	3746	4037
3 137	1099	1196	1941	2038	4 481	995	1279	3202	3486
3 169	133	1239	1930	3036	4 513	260	2135	2378	4253
3 209	22	1021	2188	3187	4 561	143	606	3955	4418
3 217	633	1423	1794	2584	4 649	1797	2124	2525	2852
3 257	753	904	2353	2504	4 657	188	867	3790	4469
3 313	450	935	2378	2863	4 673	358	1475	3198	4315
3 329	40	749	2580	3289	4 721	1771	1890	2831	2950
3 361	30	112	3249	3331	4 729	58	1223	3506	4671
3 433	1338	1619	1814	2095	4 793	192	1373	3420	4601
3 449	76	953	2496	3373	4 801	1484	1582	3219	3317
3 457	1521	1716	1741	1936	4 817	971	1141	3676	3846
3 529	1312	1396	2133	2217	4 889	1431	1616	3273	3458
3 593	799	1439	2154	2794	4 937	95	1663	3274	4842
3 617	676	1343	2274	2941	4 969	161	679	4290	4808

p	y	y	y	y	p	y	y	y	y
4 993	932,	2459,	2534,	4061	6 529	1095,	2707,	3822,	5434
5 009	1018,	2288,	2721,	3991	6 553	645,	2672,	3881,	5908
5 081	1284,	2402,	2679,	3797	6 569	736,	2508,	4061,	5833
5 113	45,	909,	4204,	5068	6 577	731,	3284,	3293,	5846
5 153	925,	1298,	3855,	4228	6 673	2137,	2929,	3744,	4536
5 209	1642,	1767,	3442,	3567	6 689	1924,	2750,	3939,	4765
5 233	696,	1812,	3421,	4537	6 737	875,	1332,	5405,	5862
5 273	1539,	2532,	2741,	3734	6 761	1784,	2452,	4309,	4977
5 281	940,	1118,	4163,	4341	6 793	78,	958,	5835,	6715
5 297	91,	1397,	3900,	5206	6 833	428,	910,	5923,	6405
5 393	196,	908,	4485,	5197	6 841	1240,	3095,	3746,	5601
5 417	979,	2667,	2750,	4438	6 857	596,	1139,	5718,	6261
5 441	896,	1172,	4269,	4545	6 961	528,	646,	6315,	6433
5 449	78,	489.	4960,	5371	6 977	2169,	2390,	4587,	4808
5 521	2067,	2249,	3272,	3454	7 001	149,	3477,	3524,	6852
5 569	1010,	2586,	2983,	4559	7 057	877,	3098,	3959,	6180
5 641	583,	1761,	3880,	5058	7 121	225,	2975,	4146,	6896
5 657	617,	816,	4841,	5040	7 129	86,	1575,	5554,	7043
5 689	1340,	1660,	4029,	4349	7 177	976,	1581,	5596,	6201
5 737	395,	2716,	3021,	5342	7 193	2324,	3092,	4101,	4869
5 801	1011,	2605,	3196,	4790	7 297	2067,	3269,	4028,	5230
5 849	2014,	2576,	3273,	3835	7 321	11,	1331,	5990,	7310
5 857	102,	1091,	4766,	5755	7 369	1171,	3373,	3996,	6198
5 881	779,	2597,	3284,	5102	7 393	499,	2652,	4741,	6894
5 897	661,	794,	5103,	5236	7 417	739,	2198,	5219,	6678
5 953	968,	1519,	4434,	4985	7 433	146,	2291,	5142,	7287
6 073	716,	2570,	3503,	5357	7 457	297,	1632,	5825,	7160
6 089	1798,	2164,	3925,	4291	7 481	348,	3719,	3762,	7133
6 113	33,	741,	5372,	6080	7 489	773,	1647,	5842,	6716
6 121	1880,	2087,	4034,	4241	7 529	292,	1315,	6214,	7237
6 217	935,	1649,	4568,	5282	7 537	1771,	3677,	3860,	5766
6 257	925,	1062,	5195,	5332	7 561	715,	3109,	4452,	6846
6 329	475,	2918,	3411,	5854	7 577	1271,	2474,	5103,	6306
6 337	338,	3131,	3206,	5999	7 649	1138,	3354,	4295,	6511
6 353	509,	3008,	3345,	5844	7 673	86,	803,	6870,	7587
6 361	2454,	3082,	3279,	3907	7 681	1213,	1925,	5756,	6468
6 449	2207,	3112,	3337,	4242	7 753	1531,	3560,	4193,	6222
6 473	91,	2703,	3770,	6382	7 793	1815,	2778,	5015,	5978
6 481	27,	240,	6241,	6454	7 817	955,	1899,	5918,	6862
6 521	187,	1360,	5161,	6334	7 841	1479,	2725,	5116,	6362

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7 873	2242,	2574,	5299,	5631
7 937	3323,	3449,	4488,	4614
7 993	1654,	3001,	4992,	6339
8 009	2017,	2172,	5837,	5992
8 017	300,	1256,	6761,	7717
8 081	1876,	2675,	5406,	6205
8 089	1455,	3647,	4442,	6634
8 161	664,	3552,	4609,	7497
8 209	1030,	2383,	5826,	7179
8 233	1074,	2752,	5481,	7159
8 273	1593,	2498,	5775,	6680
8 297	3658,	3899,	4398,	4639
8 329	2825,	3594,	4735,	5504
8 353	190,	1187,	7166,	8163
8 369	1692,	2943,	5426,	6677
8 377	290,	3553,	4824,	8087
8 513	156,	382,	8131,	8357
8 521	774,	2433,	6088,	7747
8 537	3053,	3445,	5092,	5484
8 609	510,	3528,	5081,	8099
8 641	2103,	2264,	6377,	6538
8 681	2072,	4219,	4462,	6609
8 689	2201,	2720,	5969,	6488
8 713	1345,	1477,	7236,	7368
8 737	3140,	3876,	4861,	5597
8 753	3109,	4285,	4468,	5644
8 761	1900,	4339,	4422,	6861
8 849	1559,	4223,	4626,	7290
8 929	2953,	3444,	5485,	5976
8 969	591,	3536,	5433,	8378
9 001	2159,	2614,	6387,	6842
9 041	3066,	4031,	5010,	5975
9 049	413,	1468,	7581,	8636
9 137	531,	2409,	6728,	8606
9 161	1227,	4084,	5077,	7934
9 209	629,	3382,	5827,	8580
9 241	1579,	4442,	4799,	7662
9 257	2883,	3243,	6014,	6374
9 281	1674,	2822,	6459,	7607
9 337	1847,	3387,	5950,	7490
9 377	1816,	4136,	5241,	7561
9 433	2117,	4086,	5347,	7316
9 473	3423,	4677,	4796,	6050
9 497	872,	2799,	6698,	8625
9 521	4607,	4745,	4776,	4914
9 601	3435,	4405,	5196,	6166
9 649	1386,	3641,	6008,	8263
9 689	3007,	4830,	4859,	6682
9 697	1792,	2592,	7105,	7905
9 721	3670,	3997,	5724,	6051
9 769	1673,	4140,	5629,	8096
9 817	1165,	1837,	7980,	8652
9 833	475,	2008,	7825,	9358
9 857	1573,	4211,	5646,	8284
9 929	1273,	4945,	4984,	8656

p^κ	y	y	y	y
17^2	110,	134,	155,	179
41^2	776,	834,	847,	905
17^3	399,	1022,	3891,	4514
73^2	1032,	1482,	3847,	4297
89^2	927,	1145,	6776,	6994
97^2	2569,	3234,	6175,	6840

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10 009	792,	3627,	6382,	9217	11 777	4201,	4976,	6801,	7576
10 169	3532,	4120,	6049,	6637	11 801	2867,	5174,	6627,	8934
10 177	777,	1205,	8972,	9400	11 833	1933,	5136,	6697,	9900
10 193	2213,	2934,	7259,	7980	11 897	2399,	3035,	8862,	9498
10 273	1866,	3595,	6678,	8407	11 953	3174,	5148,	6805,	8779
10 289	578,	3400,	6889,	9711	11 969	4049,	5850,	6119,	7920
10 313	41,	3270,	7043,	10272	12 041	411,	1875,	10166,	11630
10 321	234,	4543,	5778,	10087	12 049	3596,	4768,	7281,	8453
10 337	2526,	2795,	7542,	7811	12 073	3324,	5844,	6229,	8749
10 369	2982,	3126,	7243,	7387	12 097	5564,	5855,	6242,	6533
10 433	176,	4683,	5750,	10257	12 113	635,	3281,	8832,	11478
10 457	499,	1425,	9032,	9958	12 161	2457,	2648,	9513,	9704
10 513	178,	4784,	5729,	10335	12 241	1615,	3858,	8383,	10626
10 529	37,	1992,	8537,	10492	12 281	1443,	3566,	8715,	10838
10 601	2357,	2892,	7709,	8244	12 289	4043,	5146,	7143,	8246
10 657	1185,	1331,	9326,	9472	12 329	1480,	4740,	7589,	10849
10 729	1488,	4319,	6410,	9241	12 377	202,	674,	11703,	12175
10 753	67,	321,	10432,	10686	12 401	1773,	3721,	8680,	10628
10 889	3832,	4413,	6476,	7057	12 409	773,	2119,	10290,	11636
10 937	2241,	4285,	6652,	8696	12 433	2317,	4765,	7668,	10116
10 993	395,	3117,	7876,	10598	12 457	575,	3098,	9359,	11882
11 057	2971,	5348,	5709,	8086	12 473	2583,	5785,	6688,	9890
11 113	359,	4860,	6253,	10754	12 497	1846,	3852,	8645,	10651
11 161	1288,	2227,	8934,	9873	12 553	1999,	4873,	7680,	10554
11 177	4336,	5091,	6086,	6841	12 569	999,	4781,	7788,	11570
11 257	1545,	2827,	8430,	9712	12 577	2530,	6030,	6547,	10047
11 273	3161,	3659,	7614,	8112	12 601	2737,	3034,	9567,	9864
11 321	2817,	4272,	7049,	8504	12 641	1130,	2696,	9945,	11511
11 329	185,	1286,	10043,	11144	12 689	5129,	5893,	6796,	7560
11 353	925,	1436,	9917,	10428	12 697	316,	2451,	10246,	12381
11 369	868,	4414,	6955,	10501	12 713	1532,	2448,	10265,	11181
11 393	4292,	5370,	6023,	7101	12 721	662,	2402,	10319,	12059
11 489	25,	4136,	7353,	11464	12 809	2521,	5965,	6844,	10288
11 497	1018,	1615,	9882,	10479	12 841	5129,	5553,	7288,	7712
11 593	2339,	3296,	8297,	9254	12 889	1636,	6153,	6736,	11253
11 617	77,	3470,	8147,	11540	12 953	684,	3314,	9639,	12269
11 633	1796,	4398,	7235,	9837	13 001	1321,	4852,	8149,	11680
11 657	1782,	1969,	9688,	9875	13 009	2645,	3192,	9817,	10364
11 681	4383,	5562,	6119,	7298	13 033	452,	6430,	6603,	12581
11 689	1201,	2112,	9577,	10488	13 049	863,	5897,	7152,	12186

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13 121	2874,	3036,	10085,	10247	14 593	2916,	3368,	11225,	11677
13 177	500,	2978,	10199,	12677	14 633	459,	6918,	7715,	14174
13 217	652,	5899,	7318,	12565	14 657	5330,	7323,	7334,	9327
13 241	4527,	5569,	7672,	8714	14 713	3230,	5662,	9051,	11483
13 249	111,	2984,	10265,	13138	14 737	1487,	6759,	7978,	13250
13 297	322,	2519,	10778,	12975	14 753	3409,	6868,	7885,	11344
13 313	1455,	2626,	10687,	11858	14 897	3727,	6771,	8126,	11170
13 337	4188,	4831,	8506,	9149	14 929	4602,	5836,	9093,	10327
13 417	217,	5441,	7976,	13200	14 969	3847,	7288,	7681,	11122
13 441	617,	3638,	9803,	12824	15 017	4810,	6191,	8826,	10207
13 457	3198,	5828,	7629,	10259	15 073	361,	3048,	12025,	14712
13 513	2063,	5810,	7703,	11450	15 121	96,	7403,	7718,	15025
13 537	1922,	5219,	8318,	11615	15 137	3728,	3902,	11235,	11409
13 553	1758,	3554,	9999,	11795	15 161	2566,	6387,	8774,	12595
13 577	2041,	6712,	6865,	11536	15 193	3269,	6581,	8612,	11924
13 633	3916,	4202,	9431,	9717	15 217	1627,	3844,	11373,	13590
13 649	2888,	4797,	8852,	10761	15 233	3437,	4534,	10699,	11796
13 681	881,	5901,	7780,	12800	15 241	3465,	4878,	10363,	11776
13 697	342,	6448,	7249,	13355	15 289	2673,	3266,	12023,	12616
13 721	75,	3476,	10245,	13646	15 313	4668,	5311,	10002,	10645
13 729	5056,	6688,	7041,	8673	15 329	1159,	3452,	11877,	14170
13 841	770,	1456,	12385,	13071	15 361	1319,	4309,	11052,	14042
13 873	1347,	2513,	11360,	12526	15 377	4541,	5865,	9512,	10836
13 913	3443,	4243,	9670,	10470	15 401	5171,	7288,	8113,	10230
13 921	1898,	5963,	7958,	12023	15 473	3877,	5747,	9726,	11596
14 009	227,	432,	13577,	13782	15 497	360,	5467,	10030,	15137
14 033	4148,	5917,	8116,	9885	15 569	2086,	4814,	10755,	13483
14 057	692,	5830,	8227,	13365	15 601	107,	7436,	8165,	15494
14 081	5314,	5758,	8323,	8767	15 641	3183,	6108,	9533,	12458
14 153	3289,	3847,	10306,	10864	15 649	1159,	6265,	9384,	14490
14 177	881,	1400,	12777,	13296	15 737	182,	1297,	14440,	15555
14 249	4099,	7088,	7161,	10150	15 761	3916,	5687,	10074,	11845
14 281	13,	2197,	12084,	14268	15 809	2139,	2742,	13067,	13670
14 321	2203,	4778,	9543,	12118	15 817	5209,	7409,	8408,	10608
14 369	1760,	4972,	9397,	12609	15 881	5697,	7783,	8098,	10184
14 401	588,	1445,	12956,	13813	15 889	3750,	7453,	8436,	12139
14 449	522,	692,	13757,	13927	15 913	3074,	3546,	12367,	12839
14 489	4161,	5251,	9238,	10328	15 937	2658,	4347,	11590,	13279
14 537	6666,	6985,	7552,	7871	16 001	4873,	6889,	9112,	11128
14 561	3278,	3487,	11074,	11283	16 033	3061,	3766,	12267,	12972

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16 057	848,	3503,	12554,	15209	17 681	1538,	7369,	10312,	16143
16 073	1175,	2558,	13515,	14898	17 713	4823,	5149,	12564,	12890
16 097	196,	3860,	12237,	15901	17 729	1739,	6851,	10878,	15990
16 193	3860,	4602,	11591,	12333	17 737	2757,	3963,	13774,	14980
16 217	390,	2786,	13431,	15827	17 761	4794,	7732,	10029,	12967
16 249	4054,	6401,	9848,	12195	17 881	978,	3163,	14718,	16903
16 273	975,	1886,	14387,	15298	17 921	481,	4769,	13152,	17440
16 361	2275,	6717,	9644,	14086	17 929	1429,	6336,	11593,	16500
16 369	3154,	4427,	11942,	13215	17 977	183,	1670,	16307,	17794
16 417	3256,	3595,	12822,	13161	18 041	5667,	6297,	11744,	12374
16 433	7409,	7723,	8710,	9024	18 049	6623,	6813,	11236,	11426
16 481	3851,	5662,	10819,	12630	18 089	274,	3631,	14458,	17815
16 529	2450,	7293,	9236,	14079	18 097	1023,	1256,	16841,	17074
16 553	329,	5786,	10767,	16224	18 121	4213,	8882,	9239,	13908
16 561	1657,	1839,	14722,	14904	18 169	1783,	5574,	12595,	16386
16 633	1293,	7088,	9545,	15340	18 217	221,	8820,	9397,	17996
16 649	1589,	6800,	9849,	15060	18 233	3788,	5261,	12972,	14445
16 657	4312,	5806,	10851,	12345	18 257	1699,	6297,	11960,	16558
16 673	3938,	8074,	8599,	12735	18 289	1424,	5741,	12548,	16865
16 729	2372,	5621,	11108,	14357	18 313	4638,	5287,	13026,	13675
16 889	1473,	4013,	12876,	15416	18 329	274,	5686,	12643,	18055
16 921	5541,	7552,	9369,	11380	18 353	99,	2410,	15943,	18254
16 937	1153,	6860,	10077,	15784	18 401	4205,	5575,	12826,	14196
16 993	2167,	5301,	11692,	14826	18 433	158,	350,	18083,	18275
17 033	2294,	8368,	8665,	14739	18 457	2550,	6203,	12254,	15907
17 041	3215,	7956,	9085,	13826	18 481	102,	7791,	10690,	18379
17 137	662,	5255,	11882,	16475	18 521	6666,	7599,	10922,	11855
17 209	4131,	6557,	10652,	13078	18 553	935,	7699,	10854,	17618
17 257	1190,	4307,	12950,	16067	18 593	1079,	1413,	17180,	17514
17 321	922,	2198,	15123,	16399	18 617	1819,	2180,	16437,	16798
17 377	2130,	4022,	13355,	15247	18 713	2169,	6091,	12622,	16544
17 393	2453,	7913,	9480,	14940	18 793	5129,	6108,	12685,	13664
17 401	5225,	6664,	10737,	12176	18 913	289,	4581,	14332,	18624
17 417	2306,	5068,	12349,	15111	19 001	5805,	7967,	11034,	13196
17 449	6103,	7568,	9881,	11346	19 009	3465,	7055,	11954,	15544
17 489	1924,	3336,	14153,	15565	19 073	72,	8212,	10861,	19001
17 497	2979,	7565,	9932,	14518	19 081	7317,	8264,	10817,	11764
17 569	147,	3466,	14103,	17422	19 121	83,	1843,	17278,	19038
17 609	5172,	6840,	10769,	12437	19 249	3419,	6604,	12645,	15830
17 657	559,	3822,	13835,	17098	19 273	4171,	6469,	12804,	15102

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19 289	1346,	3511,	15778,	17943	20 849	3651,	8383,	12466,	17198
19 417	62,	5324,	14093,	19355	20 857	3616,	9546,	11311,	17241
19 433	4698,	7276,	12157,	14735	20 873	720,	2986,	17887,	20153
19 441	1411,	3913,	15528,	18030	20 897	6994,	9588,	11309,	13903
19 457	3051,	4598,	14859,	16406	20 921	4904,	8673,	12248,	16017
19 489	6782,	8164,	11325,	12707	20 929	6059,	7278,	13651,	14870
19 553	7826,	8962,	10591,	11727	21 001	222,	473,	20528,	20779
19 577	5926,	9696,	9881,	13651	21 017	1062,	9519,	11498,	19955
19 609	3805,	7622,	11987,	15804	21 089	4102,	4735,	16354,	16987
19 681	5693,	7647,	12034,	13988	21 121	901,	8650,	12471,	20220
19 697	3901,	8720,	10977,	15796	21 169	1601,	10525,	10644,	19568
19 753	3148,	9180,	10573,	16605	21 193	1094,	7342,	13851,	20099
19 777	2768,	3551,	16226,	17009	21 313	6002,	7766,	13547,	15311
19 793	1925,	9696,	10097,	17868	21 377	8037,	10235,	11142,	13340
19 801	8006,	9114,	10687,	11795	21 401	3855,	5768,	15633,	17546
19 841	1360,	5821,	14020,	18481	21 433	1276,	9020,	12413,	20157
19 889	1203,	8812,	11077,	18686	21 481	5582,	7862,	13619,	15899
19 913	7221,	7749,	12164,	12692	21 521	3069,	9628,	11893,	18452
19 937	6391,	7646,	12291,	13546	21 529	3724,	8955,	12574,	17805
19 961	410,	4333,	15628,	19551	21 569	1950,	7975,	13594,	19619
19 993	3928,	5777,	14216,	16065	21 577	2956,	8387,	13190,	18621
20 089	8955,	9330,	10759,	11134	21 601	4984,	5890,	15711,	16617
20 113	136,	1331,	18782,	19977	21 617	5173,	5562,	16055,	16444
20 129	175,	5061,	15068,	19954	21 649	2411,	3897,	17752,	19238
20 161	4806,	6498,	13663,	15355	21 673	4972,	6168,	15505,	16701
20 177	5089,	9694,	10483,	15088	21 713	5584,	10386,	11327,	16129
20 201	921,	3312,	16889,	19280	21 737	520,	8653,	13084,	21217
20 233	5959,	8064,	12169,	14274	21 817	780,	10433,	11384,	21037
20 249	1212,	7301,	12948,	19037	21 841	5672,	6034,	15807,	16169
20 297	7846,	8467,	11830,	12451	21 881	4123,	5790,	16091,	17758
20 353	862,	4982,	15371,	19491	21 929	273,	3695,	18234,	21656
20 369	3280,	9135,	11234,	17089	21 937	1922,	9713,	12224,	20015
20 393	3582,	9946,	10447,	16811	21 961	6377,	8668,	13293,	15584
20 441	3204,	7088,	13353,	17237	21 977	1918,	10851,	11126,	20059
20 521	1454,	1990,	18531,	19067	22 073	6358,	7419,	14654,	15715
20 593	290,	6888,	13705,	20303	22 129	8347,	10069,	12060,	13782
20 641	3864,	6672,	13969,	16777	22 153	6726,	9265,	12888,	15427
20 681	8153,	9180,	11501,	12528	22 193	3303,	7868,	14325,	18890
20 753	7689,	8529,	12224,	13064	22 273	1254,	4991,	17282,	21019
20 809	1455,	1659,	19150,	19354	22 369	7284,	10211,	12158,	15085

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22 409	2330,	5684,	16725,	20079	24 001	4260,	5065,	18936,	19741
22 433	838,	4417,	18016,	21595	24 049	4875,	6418,	17631,	19174
22 441	5595,	6622,	15819,	16846	24 097	907,	3135,	20962,	23190
22 481	394,	7817,	14664,	22087	24 113	783,	7083,	17030,	23330
22 697	3637,	5136,	17561,	19060	24 121	4285,	4554,	19567,	19836
22 721	6600,	7164,	15557,	16121	24 137	1034,	8567,	15570,	23103
22 769	766,	4964,	17805,	22003	24 169	1615,	10790,	13379,	22554
22 777	3588,	10887,	11890,	19189	24 281	1580,	9236,	15045,	22701
22 817	9944,	10314,	12503,	12873	24 329	6469,	8052,	16277,	17860
22 921	3556,	6710,	16211,	19365	24 337	7706,	9623,	14714,	16631
22 937	4107,	10399,	12538,	18830	24 473	1856,	5604,	18869,	22617
22 961	3430,	6085,	16876,	19531	24 481	11489,	11760,	12721,	12992
22 993	2759,	5742,	17251,	20234	24 593	5805,	9350,	15243,	18788
23 017	9214,	10869,	12148,	13803	24 697	260,	8264,	16433,	24437
23 041	4343,	4987,	18054,	18698	24 793	3325,	8150,	16643,	21468
23 057	389,	652,	22405,	22668	24 809	4121,	5978,	18831,	20688
23 081	3999,	8115,	14966,	19082	24 841	2237,	2654,	22187,	22604
23 201	5380,	5783,	17418,	17821	24 889	5012,	10334,	14555,	19877
23 209	645,	6333,	16876,	22564	24 953	10125,	11418,	13535,	14828
23 297	175,	1065,	22232,	23122	24 977	2103,	11283,	13694,	22874
23 321	412,	5151,	18170,	22909	25 033	10871,	11463,	13570,	14162
23 369	6073,	9697,	13672,	17296	25 057	5329,	5854,	19203,	19728
23 417	3812,	7820,	15597,	19605	25 073	1863,	8250,	16823,	23210
23 473	634,	6257,	17216,	22839	25 097	3628,	11919,	13178,	21469
23 497	3641,	9080,	14417,	19856	25 121	7070,	8318,	16803,	18051
23 537	7176,	10099,	13438,	16361	25 153	47,	3211,	21942,	25106
23 561	1830,	11729,	11832,	21731	25 169	2271,	3347,	21822,	22898
23 593	6431,	8071,	15522,	17162	25 321	3563,	6197,	19124,	21758
23 609	3050,	10148,	13461,	20559	25 409	809,	2387,	23022,	24600
23 633	299,	1976,	21657,	23334	25 457	6335,	11967,	13490,	19122
23 689	7779,	11508,	12181,	15910	25 537	2606,	2832,	22705,	22931
23 753	6606,	9302,	14451,	17147	25 561	10896,	11319,	14242,	14665
23 761	8678,	8896,	14865,	15083	25 577	8515,	8792,	16785,	17062
23 801	8579,	10290,	13511,	15222	25 601	5833,	11644,	13957,	19768
23 833	84,	3121,	20712,	23749	25 609	10510,	11401,	14208,	15099
23 857	4030,	6352,	17505,	19827	25 633	5068,	5599,	20034,	20565
23 873	1672,	2413,	21460,	22201	25 657	557,	8798,	16859,	25100
23 929	596,	8873,	15056,	23333	25 673	293,	5783,	19890,	25380
23 977	4238,	7089,	16888,	19739	25 793	791,	2413,	23380,	25002
23 993	868,	5169,	18824,	23125	25 801	1545,	10287,	15514,	24256

p	y	y	y	y	p	y	y	y	y
25 841	7878,	10690,	15151,	17963	27 281	6290,	6727,	20554,	20991
25 849	7843,	12178,	13671,	18006	27 329	5410,	6269,	21060,	21919
25 873	11250,	11644,	14229,	14623	27 337	1195,	4987,	22350,	26142
25 889	4209,	10641,	15248,	21680	27 361	1796,	7084,	20277,	25565
25 913	2967,	10070,	15843,	22946	27 409	11140,	12275,	15134,	16269
25 969	6782,	9814,	16155,	19187	27 449	3884,	10622,	16827,	23565
26 017	5775,	11024,	14993,	20242	27 457	13275,	13562,	13895,	14182
26 041	2018,	9175,	16866,	24023	27 481	6299,	11714,	15767,	21182
26 113	619,	7720,	18393,	25494	27 529	2413,	6617,	20912,	25116
26 153	5653,	7888,	18265,	20500	27 617	4401,	13341,	14276,	23216
26 161	945,	7087,	19074,	25216	27 673	1416,	7485,	20188,	26257
26 177	824,	4797,	21380,	25353	27 689	8340,	11059,	16630,	19349
26 209	841,	10066,	16143,	25368	27 697	5799,	10221,	17476,	21898
26 249	4977,	5754,	20495,	21272	27 737	783,	4428,	23309,	26954
26 297	3744,	4432,	21865,	22553	27 793	2730,	4510,	23283,	25063
26 321	12547,	12828,	13493,	13774	27 809	915,	6352,	21457,	26894
26 393	11383,	12145,	14248,	15010	27 817	1403,	6067,	21750,	26414
26 417	1980,	5697,	20720,	24437	27 953	4678,	6806,	21147,	23275
26 449	5613,	5843,	20606,	20836	27 961	593,	5281,	22680,	27368
26 489	8020,	12323,	14166,	18469	28 001	5539,	7229,	20772,	22462
26 497	7925,	12314,	14183,	18572	28 057	7091,	12155,	15902,	20966
26 513	1768,	7573,	18940,	24745	28 081	4325,	10252,	17829,	23756
26 561	3811,	9388,	17173,	22750	28 097	4489,	6378,	21719,	23608
26 633	2300,	6913,	19720,	24333	28 201	4296,	8501,	19700,	23905
26 641	7868,	12281,	14360,	18773	28 289	8528,	8973,	19316,	19761
26 681	1592,	5782,	20899,	25089	28 297	3496,	9203,	19094,	24801
26 713	8204,	13223,	13490,	18509	28 393	11483,	13226,	15167,	16910
26 729	10471,	12480,	14249,	16258	28 409	7067,	11686,	16723,	21342
26 737	5862,	6983,	19754,	20875	28 433	1399,	2134,	26299,	27034
26 777	1491,	3951,	22826,	25286	28 513	1075,	13978,	14535,	27438
26 801	1537,	11526,	15275,	25264	28 537	5906,	11524,	17013,	22631
26 833	2146,	11741,	15092,	24687	28 649	545,	11775,	16874,	28104
26 849	4990,	7861,	18988,	21859	28 657	5986,	12854,	15803,	22671
26 881	26,	9305,	17576,	26855	28 697	10986,	12520,	16177,	17711
26 921	5872,	11998,	14923,	21049	28 729	5058,	10434,	18295,	23671
26 953	7080,	9681,	17272,	19873	28 753	1783,	6773,	21980,	26970
26 993	11505,	12463,	14530,	15488	28 793	1179,	10428,	18365,	27614
27 017	3263,	11807,	15210,	23754	28 817	2217,	7201,	21616,	26600
27 073	1938,	5965,	21108,	25135	28 921	7040,	9691,	19230,	21881
27 241	4155,	12483,	14758,	23086	28 961	3163,	12370,	16591,	25798

p	y	y	y	y	p	y	y	y	y
29 009	8761,	9056,	19953,	20248	30 817	14236,	14666,	16151,	16581
29 017	1560,	5822,	23195,	27457	30 841	2615,	7324,	23517,	28226
29 033	11362,	13014,	16019,	17671	30 881	1326,	10643,	20238,	29555
29 129	3595,	7973,	21156,	25534	30 937	4809,	11451,	19486,	26128
29 137	2768,	9779,	19358,	26369	30 977	8879,	13854,	17123,	22098
29 153	1621,	6996,	22157,	27532	31 033	4760,	11846,	19187,	26273
29 201	4463,	10979,	18222,	24738	31 081	6339,	12807,	18274,	24742
29 209	389,	7734,	21475,	28820	31 121	4102,	12632,	18489,	27019
29 297	7794,	8747,	20550,	21503	31 153	4684,	15224,	15929,	26469
29 401	3833,	11360,	18041,	25568	31 177	8000,	11251,	19926,	23177
29 473	546,	8043,	21430,	28927	31 193	183,	14659,	16534,	31010
29 537	8567,	14653,	14884,	20970	31 249	2334,	2584,	28665,	28915
29 569	1602,	12828,	16741,	27967	31 321	4352,	11220,	20101,	26969
29 633	2424,	12995,	16638,	27209	31 337	1696,	14061,	17276,	29641
29 641	5071,	12234,	17407,	24570	31 393	660,	1094,	30299,	30733
29 753	1578,	7146,	22607,	28175	31 481	9360,	15199,	16282,	22121
29 761	9439,	12634,	17127,	20322	31 489	1561,	7726,	23763,	29928
29 833	5780,	14261,	15572,	24053	31 513	9035,	12375,	19138,	22478
29 873	5594,	10910,	18963,	24279	31 601	2985,	8374,	23227,	28616
29 881	6267,	12721,	17160,	23614	31 649	983,	12299,	19350,	30666
29 921	6220,	8654,	21267,	23701	31 657	12147,	13797,	17860,	19510
30 089	11437,	14004,	16085,	18652	31 721	10276,	14635,	17086,	21445
30 097	8726,	9185,	20912,	21371	31 729	7613,	10361,	21368,	24116
30 113	240,	2133,	27980,	29873	31 769	5020,	7398,	24371,	26749
30 137	4004,	9717,	20420,	26133	31 793	6922,	11873,	19920,	24871
30 161	1016,	13804,	16357,	29145	31 817	13135,	14638,	17179,	18682
30 169	13034,	13867,	16302,	17135	31 849	6325,	13223,	18626,	25524
30 241	6874,	7888,	22353,	23367	31 873	2575,	3243,	28630,	29298
30 313	2633,	9049,	21264,	27680	32 009	4532,	12579,	19430,	27477
30 449	3428,	6120,	24329,	27021	32 057	6809,	7853,	24204,	25248
30 497	792,	3042,	27455,	29705	32 089	4547,	5307,	26782,	27542
30 529	4513,	15207,	15322,	26016	32 233	7807,	11482,	20751,	24426
30 553	4117,	9180,	21373,	26436	32 257	9519,	9773,	22484,	22738
30 577	510,	7974,	22603,	30067	32 297	11474,	12934,	19363,	20823
30 593	269,	7961,	22632,	30324	32 321	1677,	4587,	27734,	30644
30 649	4865,	12247,	18402,	25784	32 353	218,	7272,	25081,	32135
30 689	297,	10333,	20356,	30392	32 369	6134,	8459,	23910,	26235
30 697	10980,	14630,	16067,	19717	32 377	13295,	14580,	17797,	19082
30 713	13503,	14325,	16388,	17210	32 401	4830,	5427,	26974,	27571
30 809	4215,	5694,	25115,	26594	32 441	571,	9488,	22953,	31870

☞ *For solutions of* $y^4+1\equiv 0$ (*mod* $p^\kappa > 10^4$), *see page* 22.

p	y	y	y	y	y	y	y	y
17	3,	5,	6,	7,	10,	11,	12,	14
97	8,	12,	18,	27,	70,	79,	85,	89
113	35,	40,	42,	48,	65,	71,	73,	78
193	3,	27,	50,	64,	129,	143,	166,	190
241	44,	76,	111,	115,	126,	130,	165,	197
257	2,	8,	32,	128,	129,	225,	249,	255
337	30,	40,	59,	146,	191,	278,	297,	307
353	36,	49,	60,	100,	253,	293,	304,	317
401	30,	133,	147,	199,	202,	254,	268,	371
433	151,	168,	183,	195,	238,	250,	265,	282
449	35,	77,	100,	220,	229,	349,	372,	414
577	27,	65,	71,	171,	406,	506,	512,	550
593	82,	94,	122,	209,	384,	471,	499,	511
641	16,	40,	100,	250,	391,	541,	601,	625
673	8,	84,	161,	209,	464,	512,	589,	665
769	27,	57,	136,	311,	458,	633,	712,	742
881	68,	85,	114,	298,	583,	767,	796,	813
929	40,	46,	101,	209,	720,	828,	883,	889
977	52,	80,	357,	403,	574,	620,	897,	925
1 009	62,	179,	183,	204,	805,	826,	830,	947
1 153	67,	156,	170,	413,	740,	983,	997,	1086
1 201	104,	292,	358,	473,	728,	843,	909,	1097
1 217	287,	322,	441,	480,	737,	776,	895,	930
1 249	98,	124,	554,	599,	650,	695,	1125,	1151
1 297	157,	190,	355,	464,	833,	942,	1107,	1140
1 361	63,	108,	377,	574,	787,	984,	1253,	1298
1 409	100,	112,	155,	390,	1019,	1254,	1297,	1309
1 489	143,	189,	583,	656,	833,	906,	1300,	1346
1 553	99,	251,	326,	605,	948,	1227,	1302,	1454
1 601	257,	380,	674,	791,	810,	927,	1221,	1344
1 697	36,	330,	369,	837,	860,	1328,	1367,	1661
1 777	189,	446,	761,	865,	912,	1016,	1331,	1588
1 873	151,	377,	645,	780,	1093,	1228,	1496,	1722
1 889	458,	478,	739,	928,	961,	1150,	1411,	1431
2 017	108,	528,	691,	913,	1104,	1326,	1489,	1909
2 081	155,	725,	838,	966,	1115,	1243,	1356,	1926
2 113	348,	407,	623,	1014,	1099,	1490,	1706,	1765
2 129	105,	551,	588,	738,	1391,	1541,	1578,	2024
2 161	227,	238,	410,	954,	1207,	1751,	1923,	1934
2 273	598,	672,	764,	1079,	1194,	1509,	1601,	1675

p	y	y	y	y	y	y	y	y
2 417	67,	974,	992,	1055,	1362,	1425,	1443,	2350
2 593	25,	67,	387,	726,	1867,	2206,	2526,	2568
2 609	460,	572,	743,	1081,	1528,	1866,	2037,	2149
2 657	169,	283,	960,	977,	1680,	1697,	2374,	2488
2 689	250,	441,	466,	779,	1910,	2223,	2248,	2439
2 753	342,	598,	999,	1296,	1457,	1754,	2155,	2411
2 801	24,	181,	619,	817,	1984,	2182,	2620,	2777
2 833	423,	509,	539,	1088,	1745,	2294,	2324,	2410
2 897	286,	381,	861,	1247,	1650,	2036,	2516,	2611
3 041	221,	344,	758,	1352,	1689,	2283,	2697,	2820
3 089	627,	709,	1050,	1276,	1813,	2039,	2380,	2462
3 121	113,	436,	580,	995,	2126,	2541,	2685,	3008
3 137	107,	282,	645,	1524,	1613,	2492,	2855,	3030
3 169	206,	416,	1123,	1455,	1714,	2046,	2753,	2963
3 217	279,	328,	1326,	1481,	1736,	1891,	2889,	2938
3 313	506,	536,	649,	897,	2416,	2664,	2777,	2807
3 329	630,	687,	848,	1432,	1897,	2481,	2642,	2699
3 361	57,	338,	885,	1651,	1710,	2476,	3023,	3304
3 457	39,	44,	550,	1241,	2216,	2907,	3413,	3418
3 617	26,	469,	509,	1252,	2365,	3108,	3148,	3591
3 697	23,	134,	643,	1076,	2621,	3054,	3563,	3674
3 761	693,	1080,	1101,	1667,	2094,	2660,	2681,	3068
3 793	141,	194,	269,	567,	3226,	3524,	3599,	3652
3 889	123,	1075,	1396,	1925,	1964,	2493,	2814,	3766
4 001	1115,	1413,	1866,	1970,	2031,	2135,	2588,	2886
4 049	276,	1044,	1335,	1881,	2168,	2714,	3005,	3773
4 129	230,	377,	600,	1163,	2966,	3529,	3752,	3899
4 177	236,	750,	763,	2000,	2177,	3414,	3427,	3941
4 241	783,	1061,	1392,	1415,	2826,	2849,	3180,	3458
4 273	582,	1138,	1760,	1901,	2372,	2513,	3135,	3691
4 289	181,	1210,	1872,	1946,	2343,	2417,	3079,	4108
4 337	1432,	1481,	1945,	1989,	2348,	2392,	2856,	2905
4 481	74,	545,	1934,	1981,	2500,	2547,	3936,	4407
4 513	300,	346,	1279,	1422,	3091,	3234,	4167,	4213
4 561	1044,	1221,	1285,	1315,	3246,	3276,	3340,	3517
4 657	236,	296,	497,	2202,	2455,	4160,	4361,	4421
4 673	112,	1088,	1645,	1961,	2712,	3028,	3585,	4561
4 721	958,	1702,	1779,	2244,	2477,	2942,	3019,	3763
4 801	1282,	1292,	1729,	2102,	2699,	3072,	3509,	3519
4 817	550,	637,	1340,	1951,	2866,	3477,	4180,	4267

p	y	y	y	y	y	y	y	y
4 993	269,	1058,	2395,	2435,	2558,	2598,	3935,	4724
5 009	463,	488,	893,	2445,	2564,	4116,	4521,	4546
5 153	119,	128,	1248,	1862,	3291,	3905,	5025,	5034
5 233	77,	792,	1262,	1767,	3466,	3971,	4441,	5156
5 281	677,	1703,	2487,	2621,	2660,	2794,	3578,	4604
5 297	385,	609,	2044,	2449,	2848,	3253,	4688,	4912
5 393	14,	1476,	1926,	2649,	2744,	3467,	3917,	5379
5 441	968,	1260,	2209,	2673,	2768,	3232,	4181,	4473
5 521	1518,	1778,	1860,	2004,	3517,	3661,	3743,	4003
5 569	770,	1960,	2482,	2605,	2964,	3087,	3609,	4799
5 857	788,	1271,	1448,	1622,	4235,	4409,	4586,	5069
5 953	216,	689,	733,	1137,	4816,	5220,	5264,	5737
6 113	888,	1178,	1261,	2196,	3917,	4852,	4935,	5225
6 257	627,	1695,	1926,	2632,	3625,	4331,	4562,	5630
6 337	738,	1713,	2301,	2327,	4010,	4036,	4624,	5599
6 353	161,	640,	1460,	1757,	4596,	4893,	5713,	6192
6 449	2026,	2210,	2225,	2886,	3563,	4224,	4239,	4423
6 481	79,	483,	738,	2133,	4348,	5743,	5998,	6402
6 529	153,	1017,	2219,	2844,	3685,	4310,	5512,	6376
6 577	541,	851,	854,	2734,	3843,	5723,	5726,	6036
6 673	413,	1142,	1745,	1864,	4809,	4928,	5531,	6260
6 689	474,	855,	2272,	3278,	3411,	4417,	5834,	6215
6 737	1480,	1496,	2022,	2581,	4156,	4715,	5241,	5257
6 833	330,	352,	831,	2253,	4580,	6002,	6481,	6503
6 961	1311,	1481,	2336,	3069,	3892,	4625,	5480,	5650
6 977	2054,	2363,	2748,	3177,	3800,	4229,	4614,	4923
7 057	273,	517,	1086,	1761,	5296,	5971,	6540,	6784
7 121	15,	1899,	2572,	3375,	3746,	4549,	5222,	7106
7 297	1240,	1668,	1833,	3572,	3725,	5464,	5629,	6057
7 393	1398,	2660,	3400,	3603,	3790,	3993,	4733,	5995
7 457	1068,	1962,	2931,	3455,	4002,	4526,	5495,	6389
7 489	971,	1683,	2127,	3409,	4080,	5362,	5806,	6518
7 537	937,	940,	1287,	3103,	4434,	6250,	6597,	6600
7 649	695,	1915,	2250,	3063,	4586,	5399,	5734,	6954
7 681	527,	583,	849,	1728,	5953,	6832,	7098,	7154
7 793	2501,	3578,	3609,	3789,	4004,	4184,	4215,	5292
7 841	131,	2276,	2415,	3711,	4130,	5426,	5565,	7710
7 873	654,	1426,	1706,	1890,	5983,	6167,	6447,	7219
7 937	2595,	2781,	3603,	3773,	4164,	4334,	5156,	5342
8 017	1160,	2134,	2626,	3269,	4748,	5391,	5883,	6857

p	y	y	y	y	y	y	y	y
8 081	327,	704,	1977,	3501,	4580,	6104,	7377,	7754
8 161	314,	1860,	2729,	3690,	4471,	5432,	6301,	7847
8 209	1201,	2529,	2617,	2958,	5251,	5592,	5680,	7008
8 273	138,	528,	2742,	3537,	4736,	5531,	7745,	8135
8 353	388,	1141,	1181,	1457,	6896,	7172,	7212,	7965
8 369	253,	262,	1118,	1257,	7112,	7251,	8107,	8116
8 513	161,	423,	1911,	2116,	6397,	6602,	8090,	8352
8 609	579,	663,	2379,	2584,	6025,	6230,	7946,	8030
8 641	987,	1600,	1821,	3451,	5190,	6820,	7041,	7654
8 689	1758,	2753,	2810,	3120,	5569,	5879,	5936,	6931
8 737	1599,	2915,	3191,	3276,	5461,	5546,	5822,	7138
8 753	573,	1533,	4155,	4288,	4465,	4598,	7220,	8180
8 849	472,	1381,	2231,	2672,	6177,	6618,	7468,	8377
8 929	494,	3355,	3875,	4103,	4826,	5054,	5574,	8435
9 041	375,	1543,	1778,	2395,	6646,	7263,	7498,	8666
9 137	720,	1434,	1550,	3083,	6054,	7587,	7703,	8417
9 281	2448,	3192,	4027,	4250,	5031,	5254,	6089,	6833
9 377	3844,	4216,	4608,	4624,	4753,	4769,	5161,	5533
9 473	269,	1796,	1906,	2659,	6814,	7567,	7677,	9204
9 521	408,	2812,	3197,	4019,	5502,	6324,	6709,	9113
9 601	187,	922,	1260,	1951,	7650,	8341,	8679,	9414
9 649	1743,	2779,	3462,	3548,	6101,	6187,	6870,	7906
9 697	291,	2094,	2166,	2672,	7025,	7531,	7603,	9406
9 857	698,	1892,	2756,	3827,	6030,	7101,	7965,	9159

p^κ	y	y	y	y	y	y	y	y
17^2	40,	65,	75,	131,	158,	214,	224,	249
17^3	158,	653,	802,	827,	4086,	4111,	4260,	4755
97^2	279,	978,	1428,	1667,	7742,	7981,	8431,	9130

p	y	y	y	y	y	y	y	y
10 177	495,	1750,	2111,	3968,	6209,	8066,	8427,	9682
10 193	620,	3604,	3995,	4726,	5467,	6198,	6589,	9573
10 273	252,	1916,	2326,	5110,	5163,	7947,	8357,	10021
10 289	83,	117,	3471,	4397,	5892,	6818,	10172,	10206
10 321	1519,	2575,	3932,	4532,	5789,	6389,	7746,	8802
10 337	485,	1178,	1428,	4993,	5344,	8909,	9159,	9852
10 369	1792,	2532,	3485,	3709,	6660,	6884,	7837,	8577
10 433	103,	1970,	2431,	2738,	7695,	8002,	8463,	10330
10 513	1702,	1921,	4991,	5207,	5306,	5522,	8592,	8811
10 529	543,	967,	2831,	4192,	6337,	7698,	9562,	9986
10 657	1875,	1887,	3455,	5219,	5438,	7202,	8770,	8782
10 753	1656,	3422,	3461,	4679,	6074,	7292,	7331,	9097
10 993	217,	1015,	2229,	5177,	5816,	8764,	9978,	10776
11 057	1289,	1308,	3897,	5061,	5996,	7160,	9749,	9768
11 329	1308,	4071,	5396,	5421,	5908,	5933,	7258,	10021
11 393	1902,	1955,	5397,	5612,	5781,	5996,	9438,	9491
11 489	5,	125,	2298,	3125,	8364,	9191,	11364,	11484
11 617	715,	970,	3030,	4988,	6629,	8587,	10647,	10902
11 633	2406,	4109,	4442,	5333,	6300,	7191,	7524,	9227
11 681	2611,	2899,	3367,	4458,	7223,	8314,	8782,	9070
11 777	337,	2497,	3410,	4578,	7199,	8367,	9280,	11440
11 953	1854,	2284,	3720,	5898,	6055,	8233,	9669,	10099
11 969	2423,	3254,	3853,	5190,	6779,	8116,	8715,	9546
12 049	3339,	3623,	3802,	5809,	6240,	8247,	8426,	8710
12 097	257,	2502,	2519,	4707,	7390,	9578,	9595,	11840
12 113	2338,	2627,	3449,	5269,	6844,	8664,	9486,	9775
12 161	1204,	2010,	3105,	4038,	8123,	9056,	10151,	10957
12 241	2914,	4196,	4974,	5566,	6675,	7267,	8045,	9327
12 289	722,	1305,	4134,	5736,	6553,	8155,	10984,	11567
12 401	61,	1394,	3456,	3763,	8638,	8945,	11007,	12340
12 433	698,	976,	1414,	6074,	6359,	11019,	11457,	11735
12 497	1650,	3368,	5173,	6178,	6319,	7324,	9129,	10847
12 577	2423,	2842,	3784,	5191,	7386,	8793,	9735,	10154
12 641	386,	1831,	4094,	6255,	6386,	8547,	10810,	12255
12 689	1330,	3974,	4112,	5112,	7577,	8577,	8715,	11359
12 721	476,	1538,	2913,	5186,	7535,	9808,	11183,	12245
13 009	3406,	3572,	5940,	6367,	6642,	7069,	9437,	9603
13 121	3503,	3815,	4846,	6023,	7098,	8275,	9306,	9618
13 217	5504,	6073,	6416,	6557,	6660,	6801,	7144,	7713
13 249	735,	2091,	6094,	6381,	6868,	7155,	11158,	12514

p	y	y	y	y	y	y	y	y
13 297	2004,	3814,	4784,	6265,	7032,	8513,	9483,	11293
13 313	3024,	5275,	6476,	6630,	6683,	6837,	8038,	10289
13 441	1497,	2481,	3780,	6474,	6967,	9661,	10960,	11944
13 457	154,	4107,	4407,	5417,	8040,	9050,	9350,	13303
13 537	4779,	6089,	6385,	6447,	7090,	7152,	7448,	8758
13 553	3521,	3803,	4045,	4215,	9338,	9508,	9750,	10032
13 633	2856,	3872,	5036,	5975,	7658,	8597,	9761,	10777
13 649	2719,	2795,	4297,	5401,	8248,	9352,	10854,	10930
13 681	4680,	4851,	5099,	5259,	8422,	8582,	8830,	9001
13 697	2580,	5184,	5752,	6015,	7682,	7945,	8513,	11117
13 729	1561,	1759,	2888,	5928,	7801,	10841,	11970,	12168
13 841	355,	587,	3470,	4763,	9078,	10371,	13254,	13486
13 873	1098,	1453,	2790,	5405,	8468,	11083,	12420,	12775
13 921	1509,	2425,	3644,	5201,	8720,	10277,	11496,	12412
14 033	228,	1908,	5533,	6929,	7104,	8500,	12125,	13805
14 081	1412,	1805,	2609,	5559,	8522,	11472,	12276,	12669
14 177	953,	1562,	3150,	3542,	10635,	11027,	12615,	13224
14 321	813,	914,	3523,	5719,	8602,	10798,	13407,	13508
14 369	127,	718,	792,	6384,	7985,	13577,	13651,	14242
14 401	243,	358,	1126,	5511,	8890,	13275,	14043,	14158
14 449	1417,	1968,	2775,	3650,	10799,	11674,	12481,	13032
14 561	180,	1071,	1537,	6961,	7600,	13024,	13490,	14381
14 593	54,	3059,	3721,	6756,	7837,	10872,	11534,	14539
14 657	461,	4376,	4793,	5246,	9411,	9864,	10281,	14196
14 737	1045,	1573,	4132,	6530,	8207,	10605,	13164,	13692
14 753	2763,	3926,	4716,	6653,	8100,	10037,	10827,	11990
14 897	3656,	4038,	4843,	5303,	9594,	10054,	10859,	11241
14 929	4475,	5212,	5279,	6859,	8070,	9650,	9717,	10454
15 073	19,	2380,	4127,	6859,	8214,	10946,	12693,	15054
15 121	852,	1899,	4233,	6187,	8934,	10888,	13222,	14269
15 137	1009,	1498,	2314,	7561,	7576,	12823,	13639,	14128
15 217	62,	5144,	5645,	6653,	8564,	9572,	10073,	15155
15 233	3334,	3742,	4602,	5220,	10013,	10631,	11491,	11899
15 313	1497,	1654,	3120,	5268,	10045,	12193,	13659,	13816
15 329	393,	4383,	5998,	7645,	7684,	9331,	10946,	14936
15 361	179,	3261,	3666,	5686,	9675,	11695,	12100,	15182
15 377	257,	359,	1114,	1615,	13762,	14263,	15018,	15120
15 473	2105,	2451,	5467,	6814,	8659,	10006,	13022,	13368
15 569	2198,	4759,	5748,	7742,	7827,	9821,	10810,	13371
15 601	424,	1435,	1462,	2465,	13136,	14139,	14166,	15177

p	y	y	y	y	y	y	y	y
15 649	1858,	2486,	4035,	6140,	9509,	11614,	13163,	13791
15 761	2050,	4790,	5451,	5722,	10039,	10310,	10971,	13711
15 809	2406,	4592,	4899,	7300,	8509,	10910,	11217,	13403
15 889	2871,	3052,	4920,	6492,	9397,	10969,	12837,	13018
15 937	4302,	6491,	6693,	7887,	8050,	9244,	9446,	11635
16 001	83,	4249,	4434,	5532,	10469,	11567,	11752,	15918
16 033	2007,	2788,	4512,	6819,	9214,	11521,	13245,	14026
16 097	14,	2744,	5749,	6623,	9474,	10348,	13353,	16083
16 193	463,	5326,	5950,	6750,	9443,	10243,	10867,	15730
16 273	2827,	3983,	5822,	6188,	10085,	10451,	12290,	13446
16 369	172,	2311,	4689,	7899,	8470,	11680,	14058,	16197
16 417	1586,	4971,	7339,	7368,	9049,	9078,	11446,	14831
16 433	836,	1317,	1741,	3549,	12884,	14692,	15116,	15597
16 481	1846,	3098,	5092,	5635,	10846,	11389,	13383,	14635
16 529	3875,	3945,	4215,	6104,	10425,	12314,	12584,	12654
16 561	4014,	4245,	4460,	6324,	10237,	12101,	12316,	12547
16 657	2174,	3603,	3762,	4845,	11812,	12895,	13054,	14483
16 673	310,	1990,	3651,	5512,	11161,	13022,	14683,	16363
16 993	1726,	1782,	4183,	7292,	9701,	12810,	15211,	15267
17 041	637,	3035,	6795,	6968,	10073,	10246,	14006,	16404
17 137	668,	2745,	3346,	4379,	12758,	13791,	14392,	16469
17 377	1661,	5805,	6978,	7774,	9603,	10399,	11572,	15716
17 393	2614,	4305,	5875,	7422,	9971,	11518,	13088,	14779
17 489	1026,	2233,	5108,	6002,	11487,	12381,	15256,	16463
17 569	459,	2803,	7885,	7954,	9615,	9684,	14766,	17110
17 681	4155,	5297,	6055,	7549,	10132,	11626,	12384,	13526
17 713	4167,	5440,	6307,	6814,	10899,	11406,	12273,	13546
17 729	514,	6657,	7396,	8119,	9610,	10333,	11072,	17215
17 761	2195,	3047,	7776,	8318,	9443,	9985,	14714,	15566
17 921	814,	2728,	3935,	6891,	11030,	13986,	15193,	17107
18 049	106,	218,	1873,	5216,	12833,	16176,	17831,	17943
18 097	2844,	4205,	5371,	6955,	11142,	12726,	13892,	15253
18 257	2541,	6151,	7545,	8507,	9750,	10712,	12106,	15716
18 289	1814,	4387,	7733,	7750,	10539,	10556,	13902,	16475
18 353	887,	3952,	5835,	8722,	9631,	12518,	14401,	17466
18 401	3163,	3508,	5567,	6462,	11939,	12834,	14893,	15238
18 433	651,	6342,	6654,	7740,	10693,	11779,	12091,	17782
18 481	1839,	2768,	4874,	5121,	13360,	13607,	15713,	16642
18 593	819,	4481,	8560,	8763,	9830,	10033,	14112,	17774
18 913	17,	1382,	2225,	4913,	14000,	16688,	17531,	18896

p	y	y	y	y	y	y	y	y
19 009	1189,	3696,	5068,	5426,	13583,	13941,	15313,	17820
19 073	7036,	7048,	7515,	8379,	10694,	11558,	12025,	12037
19 121	1813,	2489,	3744,	4816,	14305,	15377,	16632,	17308
19 249	1522,	3310,	6488,	7624,	11625,	12761,	15939,	17727
19 441	187,	7027,	7077,	8317,	11124,	12364,	12414,	19254
19 457	4019,	4059,	4788,	9357,	10100,	14669,	15398,	15438
19 489	1105,	2187,	2744,	9155,	10334,	16745,	17302,	18384
19 553	3187,	5039,	7912,	8166,	11387,	11641,	14514,	16366
19 681	1849,	2978,	8345,	8413,	11268,	11336,	16703,	17832
19 697	2527,	4268,	5503,	9327,	10370,	14194,	15429,	17170
19 777	5456,	7204,	7420,	9743,	10034,	12357,	12573,	14321
19 793	3714,	4177,	4767,	7477,	12316,	15026,	15616,	16079
19 841	1549,	3494,	8915,	9841,	10000,	10926,	16347,	18292
19 889	1111,	2550,	3970,	4744,	15145,	15919,	17339,	18778
19 937	1667,	6139,	7096,	7439,	12498,	12841,	13798,	18270
20 113	875,	1678,	1929,	6965,	13148,	18184,	18435,	19238
20 129	1251,	2494,	6388,	9324,	10805,	13741,	17635,	18878
20 161	824,	4361,	8474,	8588,	11573,	11687,	15800,	19337
20 177	265,	3274,	4816,	6431,	13746,	15361,	16903,	19912
20 353	5068,	5774,	7279,	9297,	11056,	13074,	14579,	15285
20 369	568,	5415,	9461,	10093,	10276,	10908,	14954,	19801
20 593	6014,	6345,	7273,	8684,	11909,	13320,	14248,	14579
20 641	1009,	2373,	3082,	4668,	15973,	17559,	18268,	19632
20 753	2125,	6514,	6756,	9157,	11596,	13997,	14239,	18628
20 849	3705,	4046,	5995,	9995,	10854,	14854,	16803,	17144
20 897	984,	2813,	6983,	10045,	10852,	13914,	18084,	19913
20 929	4565,	8803,	9747,	10285,	10644,	11182,	12126,	16364
21 089	2244,	3516,	9039,	10084,	11005,	12050,	17573,	18845
21 121	681,	1072,	2109,	5694,	15427,	19012,	20049,	20440
21 169	3255,	3681,	7433,	8299,	12870,	13736,	17488,	17914
21 313	1070,	2450,	5809,	6927,	14386,	15504,	18863,	20243
21 377	4725,	5601,	6879,	9273,	12104,	14498,	15776,	16652
21 521	3318,	3509,	8540,	8621,	12900,	12981,	18012,	18203
21 569	861,	3432,	6010,	7533,	14036,	15559,	18137,	20708
21 601	514,	3320,	5895,	8743,	12858,	15706,	18281,	21087
21 617	1461,	1910,	8197,	9473,	12144,	13420,	19707,	20156
21 649	3297,	3884,	9693,	10552,	11097,	11956,	17765,	18352
21 713	127,	1115,	5471,	7361,	14352,	16242,	20598,	21586
21 841	1534,	4428,	7009,	8130,	13711,	14832,	17413,	20307
21 937	214,	5433,	5495,	9693,	12244,	16442,	16504,	21723

p	y	y	y	y	y	y	y	y
22 129	2836,	4474,	5938,	9274,	12855,	16191,	17655,	19293
22 193	2568,	4378,	9302,	9394,	12799,	12891,	17815,	19625
22 273	3598,	5580,	8530,	9527,	12746,	13743,	16693,	18675
22 369	796,	1165,	4493,	8009,	14360,	17876,	21204,	21573
22 433	1653,	5632,	8686,	10576,	11857,	13747,	16801,	20780
22 481	983,	4040,	4391,	5125,	17356,	18090,	18441,	21498
22 721	2432,	4159,	7845,	10174,	12547,	14876,	18562,	20289
22 769	3818,	7502,	8744,	10156,	12613,	14025,	15267,	18951
22 817	181,	2679,	4160,	10280,	12537,	18657,	20138,	22636
22 961	1404,	1848,	5810,	6090,	16871,	17151,	21113,	21557
22 993	3495,	4599,	8638,	11494,	11499,	14355,	18394,	19498
23 041	3010,	8183,	9547,	11179,	11862,	13494,	14858,	20031
23 057	504,	5810,	6772,	11457,	11600,	16285,	17247,	22553
23 201	5375,	5715,	9054,	11580,	11621,	14147,	17486,	17826
23 297	3660,	5633,	7301,	11481,	11816,	15996,	17664,	19637
23 473	3787,	6712,	6795,	11002,	12471,	16678,	16761,	19686
23 537	389,	2168,	5222,	9439,	14098,	18315,	21369,	23148
23 633	3843,	7575,	8511,	8960,	14673,	15122,	16058,	19790
23 761	7301,	10883,	10907,	11252,	12509,	12854,	12878,	16460
23 857	1057,	7048,	10247,	10693,	13164,	13610,	16809,	22800
23 873	149,	1442,	5912,	10398,	13475,	17961,	22431,	23724
24 001	2596,	3808,	5501,	9284,	14717,	18500,	20193,	21405
24 049	5750,	9696,	9884,	11715,	12334,	14165,	14353,	18299
24 097	309,	712,	4835,	8901,	15196,	19262,	23385,	23788
24 113	5282,	8424,	10970,	11630,	12483,	13143,	15689,	18831
24 337	1808,	2571,	9996,	11684,	12653,	14341,	21766,	22529
24 481	4747,	4778,	5385,	8040,	16441,	19096,	19703,	19734
24 593	3645,	5148,	5299,	9245,	15348,	19294,	19445,	20948
24 977	2521,	4360,	6539,	10810,	14167,	18438,	20617,	22456
25 057	73,	1373,	5755,	11895,	13162,	19302,	23684,	24984
25 073	9967,	10572,	11690,	11742,	13331,	13383,	14501,	15106
25 121	928,	4379,	6957,	10458,	14663,	18164,	20742,	24193
25 153	707,	2277,	6407,	8076,	17077,	18746,	22876,	24446
25 169	2930,	9200,	9414,	10713,	14456,	15755,	15969,	22239
25 409	1486,	3882,	7951,	10178,	15231,	17458,	21527,	23923
25 457	1447,	2225,	5489,	7803,	17654,	19968,	23232,	24010
25 537	4765,	6608,	8510,	10944,	14593,	17027,	18929,	20772
25 601	213,	3125,	8479,	12020,	13581,	17122,	22476,	25388
25 633	5834,	8124,	11863,	12299,	13334,	13770,	17509,	19799
25 793	4246,	5496,	5777,	11681,	14112,	20016,	20297,	21547

p	y	y	y	y	y	y	y	y
25 841	1038,	3475,	10431,	11608,	14233,	15410,	22366,	24803
25 873	3677,	4677,	4827,	9432,	16441,	21046,	21196,	22196
25 889	2346,	3809,	6790,	10605,	15284,	19099,	22080,	23543
25 969	1003,	1191,	1532,	2424,	23545,	24437,	24778,	24966
26 017	3767,	4180,	4213,	4276,	21741,	21804,	21837,	22250
26 113	1910,	5428,	7205,	8645,	17468,	18908,	20685,	24203
26 161	1729,	8156,	10075,	11923,	14238,	16086,	18005,	24432
26 177	4918,	5003,	6169,	12683,	13494,	20008,	21174,	21259
26 209	29,	1820,	3615,	10498,	15711,	22594,	24389,	26180
26 321	1552,	3299,	4596,	10380,	15941,	21725,	23022,	24769
26 417	4278,	4742,	9417,	11125,	15292,	17000,	21675,	22139
26 449	3566,	5674,	5935,	12585,	13864,	20514,	20775,	22883
26 497	6024,	7394,	12064,	12583,	13914,	14433,	19103,	20473
26 513	603,	5584,	6283,	9676,	16837,	20230,	20929,	25910
26 561	4483,	5990,	11991,	12781,	13780,	14570,	20571,	22078
26 641	1435,	2145,	5204,	13107,	13534,	21437,	24496,	25206
26 737	4313,	7729,	10396,	11817,	14920,	16341,	19008,	22424
26 801	1300,	2041,	6712,	11975,	14826,	20089,	24760,	25501
26 833	6707,	7968,	10734,	12450,	14383,	16099,	18865,	20126
26 849	7707,	9381,	10162,	13383,	13466,	16687,	17468,	19142
26 881	898,	3533,	4101,	11215,	15666,	22780,	23348,	25983
26 993	3325,	3730,	5044,	5220,	21773,	21949,	23263,	23668
27 073	674,	5064,	6708,	13467,	13606,	20365,	22009,	26399
27 281	2443,	7267,	10899,	13526,	13755,	16382,	20014,	24838
27 329	3155,	7501,	9440,	12075,	15254,	17889,	19828,	24174
27 361	4429,	7567,	8031,	8085,	19276,	19330,	19794,	22932
27 409	4127,	7093,	9887,	11818,	15591,	17522,	20316,	23282
27 457	2293,	2700,	10238,	11115,	16342,	17219,	24757,	25164
27 617	3535,	9164,	9401,	9944,	17673,	18216,	18453,	24082
27 697	1805,	2271,	2703,	13454,	14243,	24994,	25426,	25892
27 793	226,	5534,	9081,	11572,	16221,	18712,	22259,	27567
27 809	2875,	8513,	11230,	13880,	13929,	16579,	19296,	24934
27 953	5523,	7247,	8022,	13963,	13990,	19931,	20706,	22430
28 001	4224,	12100,	12494,	13795,	14206,	15507,	15901,	23777
28 081	2147,	4460,	8052,	9036,	19045,	20029,	23621,	25934
28 097	67,	5871,	8063,	8304,	19793,	20034,	22226,	28030
28 289	3089,	5623,	5933,	12397,	15892,	22356,	22666,	25200
28 433	558,	3414,	6628,	12951,	15482,	21805,	25019,	27875
28 513	2800,	9221,	9949,	12378,	16135,	18564,	19292,	25713
28 657	1155,	2044,	4893,	7493,	21164,	23764,	26613,	27502

p	y	y	y	y	y	y	y	y
28 753	502,	3723,	3834,	7192,	21561,	24919,	25030,	28251
28 817	4292,	5754,	9313,	13949,	14868,	19504,	23063,	24525
28 961	3330,	9014,	9558,	13658,	15303,	19403,	19947,	25631
29 009	8366,	9012,	10355,	11217,	17792,	18654,	19997,	20643
29 137	1793,	6727,	8013,	9734,	19403,	21124,	22410,	27344
29 153	2322,	3225,	6491,	9338,	19815,	22662,	25928,	26831
29 201	2039,	10655,	10986,	14037,	15164,	18215,	18546,	27162
29 297	885,	6687,	12895,	14377,	14920,	16402,	22610,	28412
29 473	4058,	5193,	5970,	11883,	17590,	23503,	24280,	25415
29 537	122,	11379,	11793,	14091,	15446,	17744,	18158,	29415
29 569	6859,	7094,	10092,	11550,	18019,	19477,	22475,	22710
29 633	2084,	2982,	8874,	14006,	15627,	20759,	26651,	27549
29 761	885,	4924,	9046,	9326,	20435,	20715,	24837,	28876
29 873	1448,	2922,	4529,	5137,	24736,	25344,	26951,	28425
29 921	4458,	8007,	11363,	14925,	14996,	18558,	21914,	25463
30 097	2345,	3490,	4424,	10627,	19470,	25673,	26607,	27752
30 113	4711,	5357,	9179,	13654,	16459,	20934,	24756,	25402
30 161	3497,	6046,	10108,	14988,	15173,	20053,	24115,	26664
30 241	3974,	5968,	9653,	13005,	17236,	20588,	24273,	26267
30 449	6437,	6566,	8760,	9489,	20960,	21689,	23883,	24012
30 497	2959,	3741,	4663,	4741,	25756,	25834,	26756,	27538
30 529	310,	2216,	5304,	12704,	17825,	25225,	28313,	30219
30 577	6253,	9022,	9665,	14670,	15907,	20912,	21555,	24324
30 593	6000,	7429,	9856,	10327,	20266,	20737,	23164,	24593
30 689	320,	2973,	7000,	7852,	22837,	23689,	27716,	30369
30 817	2882,	10725,	13512,	13682,	17135,	17305,	20092,	27935
30 881	1263,	7164,	8874,	11884,	18997,	22007,	23717,	29618
30 977	4526,	5756,	8826,	9185,	21792,	22151,	25221,	26451
31 121	2272,	6342,	6690,	14565,	16556,	24431,	24779,	28849
31 153	2623,	5594,	9246,	11850,	19303,	21907,	25559,	28530
31 249	3853,	6810,	11201,	12279,	18970,	20048,	24439,	27396
31 393	8104,	9057,	11830,	12950,	18443,	19563,	22336,	23289
31 489	2211,	11165,	12419,	15148,	16341,	19070,	20324,	29278
31 601	1092,	4717,	11719,	13801,	17800,	19882,	26884,	30509
31 649	5752,	8105,	8333,	10955,	20694,	23316,	23544,	25897
31 729	2868,	4532,	12693,	14725,	17004,	19036,	27197,	28861
31 793	1587,	10798,	15164,	15278,	16515,	16629,	20995,	30206
31 873	441,	4122,	11853,	12859,	19014,	20020,	27751,	31432
32 257	4627,	4643,	9560,	13608,	18649,	22697,	27614,	27630
32 321	1415,	5916,	12948,	13522,	18799,	19373,	26405,	30906

☞ *For solutions of* $y^8+1\equiv 0$ *(mod* $p^\kappa > 10^4$*), see page* 22.

y	y	p	y'	y'	y	y	p	y'	y'
2,	4	7	3,	5	171,	267	439	172,	268
3,	9	13	4,	10	133,	323	457	134,	324
7,	11	19	8,	12	21,	441	463	22,	442
5,	25	31	6,	26	232,	254	487	233,	255
10,	26	37	11,	27	139,	359	499	140,	360
6,	36	43	7,	37	60,	462	523	61,	463
13,	47	61	14,	48	129,	411	541	130,	412
29,	37	67	30,	38	40,	506	547	41,	507
8,	64	73	9,	65	109,	461	571	110,	462
23,	55	79	24,	56	213,	363	577	214,	364
35,	61	97	36,	62	24,	576	601	25,	577
46,	56	103	47,	57	210,	396	607	211,	397
45,	63	109	46,	64	65,	547	613	66,	548
19,	107	127	20,	108	252,	366	619	253,	367
42,	96	139	43,	97	43,	587	631	44,	588
32,	118	151	33,	119	177,	465	643	178,	466
12,	144	157	13,	145	296,	364	661	297,	365
58,	104	163	59,	105	255,	417	673	256,	418
48,	132	181	49,	133	253,	437	691	254,	438
84,	108	193	85,	109	227,	481	709	228,	482
92,	106	199	93,	107	281,	445	727	282,	446
14,	196	211	15,	197	307,	425	733	308,	426
39,	183	223	40,	184	320,	418	739	321,	419
94,	134	229	95,	135	72,	678	751	73,	679
15,	225	241	16,	226	27,	729	757	28,	730
28,	242	271	29,	243	360,	408	769	361,	409
116,	160	277	117,	161	379,	407	787	380,	408
44,	238	283	45,	239	130,	680	811	131,	681
17,	289	307	18,	290	174,	648	823	175,	649
98,	214	313	99,	215	125,	703	829	126,	704
31,	299	331	32,	300	220,	632	853	221,	633
128,	208	337	129,	209	260,	598	859	261,	599
122,	226	349	123,	227	282,	594	877	283,	595
83,	283	367	84,	284	337,	545	883	338,	546
88,	284	373	89,	285	384,	522	907	385,	523
51,	327	379	52,	328	52,	866	919	53,	867
34,	362	397	35,	363	322,	614	937	323,	615
53,	355	409	54,	356	142,	824	967	143,	825
20,	400	421	21,	401	113,	877	991	114,	878
198,	234	433	199,	235	304,	692	997	305,	693

y	y	p	y'	y'	y	y	p	y'	y'
374,	634	1 009	375,	635	222,	1374	1 597	223,	1375
368,	652	1 021	369,	653	250,	1358	1 609	251,	1359
195,	837	1 033	196,	838	184,	1436	1 621	185,	1437
140,	898	1 039	141,	899	264,	1362	1 627	265,	1363
180,	870	1 051	181,	871	70,	1586	1 657	71,	1587
343,	719	1 063	344,	720	318,	1344	1 663	319,	1345
86,	982	1 069	87,	983	248,	1420	1 669	249,	1421
257,	829	1 087	258,	830	433,	1259	1 693	434,	1260
151,	941	1 093	152,	942	397,	1301	1 699	398,	1302
120,	996	1 117	121,	997	41,	1681	1 723	42,	1682
33,	1089	1 123	34,	1090	356,	1384	1 741	357,	1385
387,	741	1 129	388,	742	371,	1375	1 747	372,	1376
502,	650	1 153	503,	651	182,	1570	1 753	183,	1571
420,	750	1 171	421,	751	508,	1250	1 759	509,	1251
570,	630	1 201	571,	631	629,	1147	1 777	630,	1148
217,	995	1 213	218,	996	193,	1589	1 783	194,	1590
126,	1104	1 231	127,	1105	152,	1636	1 789	153,	1637
300,	936	1 237	301,	937	73,	1727	1 801	74,	1728
93,	1155	1 249	94,	1156	672,	1158	1 831	673,	1159
504,	774	1 279	505,	775	454,	1406	1 861	455,	1407
346,	944	1 291	347,	945	834,	1032	1 867	835,	1033
365,	931	1 297	366,	932	114,	1758	1 873	115,	1759
95,	1207	1 303	96,	1208	488,	1390	1 879	489,	1391
297,	1023	1 321	298,	1024	591,	1341	1 933	592,	1342
347,	979	1 327	348,	980	76,	1874	1 951	77,	1875
354,	1026	1 381	355,	1027	647,	1339	1 987	648,	1340
390,	1008	1 399	391,	1009	312,	1680	1 993	313,	1681
643,	779	1 423	644,	780	808,	1190	1 999	809,	1191
664,	764	1 429	665,	765	205,	1805	2 011	206,	1806
704,	742	1 447	705,	743	294,	1722	2 017	295,	1723
693,	759	1 453	694,	760	975,	1053	2 029	976,	1054
339,	1119	1 459	340,	1120	197,	1855	2 053	198,	1856
251,	1219	1 471	252,	1220	449,	1633	2 083	450,	1634
38,	1444	1 483	39,	1445	826,	1262	2 089	827,	1263
483,	1005	1 489	484,	1006	438,	1674	2 113	439,	1675
646,	884	1 531	647,	885	468,	1662	2 131	469,	1663
681,	861	1 543	682,	862	201,	1935	2 137	202,	1936
275,	1273	1 549	276,	1274	349,	1793	2 143	350,	1794
535,	1031	1 567	536,	1032	593,	1567	2 161	594,	1568
639,	939	1 579	640,	940	123,	2055	2 179	124,	2056

y y	p	y' y'	y y	p	y' y'
285, 1917	2 203	286, 1918	1300, 1532	2 833	1301, 1533
543, 1677	2 221	544, 1678	1014, 1836	2 851	1015, 1837
295, 1943	2 239	296, 1944	350, 2506	2 857	351, 2507
708, 1542	2 251	709, 1543	698, 2188	2 887	699, 2189
82, 2186	2 269	83, 2187	247, 2669	2 917	248, 2670
663, 1617	2 281	664, 1618	800, 2152	2 953	801, 2153
804, 1482	2 287	805, 1483	54, 2916	2 971	55, 2917
989, 1303	2 293	990, 1304	934, 2066	3 001	935, 2067
882, 1428	2 311	883, 1429	239, 2779	3 019	240, 2780
1106, 1234	2 341	1107, 1235	745, 2291	3 037	746, 2292
1062, 1284	2 347	1063, 1285	532, 2516	3 049	533, 2517
464, 1906	2 371	465, 1907	561, 2499	3 061	562, 2500
721, 1655	2 377	722, 1656	973, 2093	3 067	974, 2094
1103, 1279	2 383	1104, 1280	546, 2532	3 079	547, 2533
689, 1699	2 389	690, 1700	1085, 2023	3 109	1086, 2024
85, 2351	2 437	86, 2352	1121, 1999	3 121	1122, 2000
216, 2250	2 467	217, 2251	536, 2626	3 163	537, 2627
1015, 1457	2 473	1016, 1458	97, 3071	3 169	98, 3072
1226, 1276	2 503	1227, 1277	440, 2740	3 181	441, 2741
675, 1845	2 521	676, 1846	1315, 1871	3 187	1316, 1872
306, 2232	2 539	307, 2233	204, 3012	3 217	205, 3013
50, 2500	2 551	51, 2501	914, 2314	3 229	915, 2315
835, 1721	2 557	836, 1722	1439, 1813	3 253	1440, 1814
1137, 1455	2 593	1138, 1456	852, 2406	3 259	853, 2407
1064, 1552	2 617	1065, 1553	842, 2428	3 271	843, 2429
185, 2461	2 647	186, 2462	1574, 1726	3 301	1575, 1727
903, 1755	2 659	904, 1756	57, 3249	3 307	58, 3250
544, 2126	2 671	545, 2127	1123, 2189	3 313	1124, 2190
1033, 1643	2 677	1034, 1644	1527, 1791	3 319	1528, 1792
636, 2046	2 683	637, 2047	1463, 1867	3 331	1464, 1868
391, 2297	2 689	392, 2298	1424, 1918	3 343	1425, 1919
1327, 1379	2 707	1328, 1380	892, 2468	3 361	893, 2469
1211, 1501	2 713	1212, 1502	654, 2718	3 373	655, 2719
1265, 1453	2 719	1266, 1454	555, 2835	3 391	556, 2836
446, 2284	2 731	447, 2285	268, 3164	3 433	269, 3165
595, 2153	2 749	596, 2154	722, 2734	3 457	723, 2735
328, 2438	2 767	329, 2439	367, 3095	3 463	368, 3096
91, 2699	2 791	92, 2700	1683, 1785	3 469	1684, 1786
1100, 1696	2 797	1101, 1697	156, 3342	3 499	157, 3343
413, 2389	2 803	414, 2390	756, 2754	3 511	757, 2755

y y	p	y' y'	y y	p	y' y'
258, 3258	3 517	259, 3259	170, 3982	4 153	171, 3983
448, 3080	3 529	449, 3081	1604, 2554	4 159	1605, 2555
59, 3481	3 541	60, 3482	1102, 3074	4 177	1103, 3075
1162, 2384	3 547	1163, 2385	1124, 3076	4 201	1125, 3077
1435, 2123	3 559	1436, 2124	112, 4106	4 219	113, 4107
103, 3467	3 571	104, 3468	620, 3610	4 231	621, 3611
1038, 2544	3 583	1039, 2545	298, 3944	4 243	299, 3945
1399, 2207	3 607	1400, 2208	1647, 2613	4 261	1648, 2614
1675, 1937	3 613	1676, 1938	1610, 2662	4 273	1611, 2663
335, 3295	3 631	336, 3296	1410, 2886	4 297	1411, 2887
695, 2941	3 637	696, 2942	627, 3699	4 327	628, 3700
422, 3220	3 643	423, 3221	237, 4101	4 339	238, 4102
1151, 2521	3 673	1152, 2522	1318, 3038	4 357	1319, 3039
474, 3216	3 691	475, 3217	412, 3950	4 363	413, 3951
519, 3177	3 697	520, 3178	66, 4356	4 423	67, 4357
498, 3210	3 709	499, 3211	901, 3539	4 441	902, 3540
1188, 2538	3 727	1189, 2539	115, 4331	4 447	116, 4332
948, 2784	3 733	949, 2785	505, 3977	4 483	506, 3978
694, 3044	3 739	695, 3045	791, 3715	4 507	792, 3716
463, 3305	3 769	464, 3306	814, 3698	4 513	815, 3699
1068, 2724	3 793	1069, 2725	1056, 3462	4 519	1057, 3463
1184, 2638	3 823	1185, 2639	1744, 2804	4 549	1745, 2805
1892, 1954	3 847	1893, 1955	243, 4317	4 561	244, 4318
1139, 2713	3 853	1140, 2714	1112, 3454	4 567	1113, 3455
224, 3652	3 877	225, 3653	310, 4280	4 591	311, 4281
1890, 1998	3 889	1891, 1999	377, 4219	4 597	378, 4220
62, 3844	3 907	63, 3845	179, 4423	4 603	180, 4424
1169, 2749	3 919	1170, 2750	1763, 2857	4 621	1764, 2858
617, 3313	3 931	618, 3314	1360, 3278	4 639	1361, 3279
1135, 2807	3 943	1136, 2808	786, 3864	4 651	787, 3865
888, 3078	3 967	889, 3079	967, 3689	4 657	968, 3690
822, 3180	4 003	823, 3181	2092, 2570	4 663	2093, 2571
1812, 2208	4 021	1813, 2209	717, 4005	4 723	718, 4006
1820, 2206	4 027	1821, 2207	2036, 2692	4 729	2037, 2693
797, 3253	4 051	798, 3254	1525, 3233	4 759	1526, 3234
1408, 2648	4 057	1409, 2649	1745, 3037	4 783	1746, 3038
902, 3190	4 093	903, 3191	1679, 3109	4 789	1680, 3110
2017, 2081	4 099	2018, 2082	2340, 2460	4 801	2341, 2461
1055, 3055	4 111	1056, 3056	1888, 2924	4 813	1889, 2925
1979, 2149	4 129	1980, 2150	69, 4761	4 831	70, 4762

y, y	p	y', y'	y, y	p	y', y'
319, 4541	4 861	320, 4542	2242, 3326	5 569	2243, 3327
2416, 2486	4 903	2417, 2487	2458, 3122	5 581	2459, 3123
573, 4335	4 909	574, 4336	2013, 3609	5 623	2014, 3610
2131, 2801	4 933	2132, 2802	2044, 3596	5 641	2045, 3597
2261, 2689	4 951	2262, 2690	853, 4793	5 647	854, 4794
2282, 2674	4 957	2283, 2675	740, 4912	5 653	741, 4913
186, 4782	4 969	187, 4783	1464, 4194	5 659	1465, 4195
1136, 3850	4 987	1137, 3851	1110, 4572	5 683	1111, 4573
2342, 2650	4 993	2343, 2651	2419, 3269	5 689	2420, 3270
2337, 2661	4 999	2338, 2662	75. 5625	5 701	76, 5626
2103, 2907	5 011	2104, 2908	2469, 3267	5 737	2470, 3268
953, 4069	5 023	954, 4070	200, 5542	5 743	201, 5543
1912, 3146	5 059	1913, 3147	330, 5418	5 749	331, 5419
1629, 3447	5 077	1630, 3448	2851, 2927	5 779	2852, 2928
1614, 3486	5 101	1615, 3487	1572, 4218	5 791	1573, 4219
311, 4795	5 107	312, 4796	2148, 3672	5 821	2149, 3673
71, 5041	5 113	72, 5042	1350, 4476	5 827	1351, 4477
1682, 3436	5 119	1683, 3437	1854, 3984	5 839	1855, 3985
124, 5042	5 167	125, 5043	577, 5273	5 851	578, 5274
1497, 3681	5 179	1498, 3682	1264, 4592	5 857	1265, 4593
1878, 3318	5 197	1879, 3319	777, 5091	5 869	778, 5092
1192, 4016	5 209	1193, 4017	276, 5604	5 881	277, 5605
451, 4775	5 227	452, 4776	428, 5494	5 923	429, 5495
331, 4901	5 233	332, 4902	869, 5083	5 953	870, 5084
1403, 3877	5 281	1404, 3878	77, 5929	6 007	78, 5930
1282, 4040	5 323	1283, 4041	509, 5527	6 037	510, 5528
479, 4867	5 347	480, 4868	1715, 4327	6 043	1716, 4328
1042, 4364	5 407	1043, 4365	665, 5401	6 067	666, 5402
1224, 4188	5 413	1225, 4189	1842, 4230	6 073	1843, 4231
127, 5291	5 419	128, 5292	1553, 4525	6 079	1554, 4526
1533, 3897	5 431	1534, 3898	744, 5346	6 091	745, 5347
2271, 3165	5 437	2272, 3166	1152, 4968	6 121	1153, 4969
2588, 2854	5 443	2589, 2855	949, 5183	6 133	950, 5184
1474, 3974	5 449	1475, 3975	207, 5943	6 151	208, 5944
2702, 2776	5 479	2703, 2777	78, 6084	6 163	79, 6085
929, 4573	5 503	930, 4574	2645, 3553	6 199	2646, 3554
340, 5180	5 521	341, 5181	136, 6074	6 211	137, 6075
876, 4650	5 527	877, 4651	2459, 3757	6 217	2460, 3758
1827, 3729	5 557	1828, 3730	930, 5298	6 229	931, 5299
711, 4851	5 563	712, 4852	2316, 3930	6 247	2317, 3931

y y	p	y' y'
2020, 4250	6 271	2021, 4251
2308, 3968	6 277	2309, 3969
2977, 3323	6 301	2978, 3324
1512, 4824	6 337	1513, 4825
557, 5785	6 343	558, 5786
1858, 4502	6 361	1859, 4503
769, 5597	6 367	770, 5598
623, 5749	6 373	624, 5750
3006, 3372	6 379	3007, 3373
2294, 4102	6 397	2295, 4103
3104, 3316	6 421	3105, 3317
1084, 5342	6 427	1085, 5343
212, 6238	6 451	213, 6239
1476, 4992	6 469	1477, 4993
80, 6400	6 481	81, 6401
491, 6037	6 529	492, 6038
2332, 4214	6 547	2333, 4215
1944, 4608	6 553	1945, 4609
2979, 3591	6 571	2980, 3592
353, 6223	6 577	354, 6224
1518, 5088	6 607	1519, 5089
569, 6049	6 619	570, 6050
1370, 5266	6 637	1371, 5267
1348, 5312	6 661	1349, 5313
1393, 5279	6 673	1394, 5280
942, 5736	6 679	943, 5737
2918, 3772	6 691	2919, 3773
1480, 5222	6 703	1481, 5223
1239, 5469	6 709	1240, 5470
619, 6113	6 733	620, 6114
2155, 4607	6 763	2156, 4608
2926, 3854	6 781	2927, 3855
1168, 5624	6 793	1169, 5625
2685, 4137	6 823	2686, 4138
734, 6094	6 829	735, 6095
2808, 4032	6 841	2809, 4033
1466, 5404	6 871	1467, 5405
219, 6663	6 883	220, 6664
1856, 5050	6 907	1857, 5051
1942, 5006	6 949	1943, 5007
507, 6453	6 961	508, 6454
382, 6584	6 967	383, 6585
1381, 5609	6 991	1382, 5610
2908, 4088	6 997	2909, 4089
523, 6503	7 027	524, 6504
302, 6736	7 039	303, 6737
145, 6911	7 057	146, 6912
2040, 5028	7 069	2041, 5029
1249, 5879	7 129	1250, 5880
2880, 4278	7 159	2881, 4279
2038, 5138	7 177	2039, 5139
1838, 5368	7 207	1839, 5369
2602, 4610	7 213	2603, 4611
2725, 4493	7 219	2726, 4494
1830, 5406	7 237	1831, 5407
3426, 3816	7 243	3427, 3817
3535, 3761	7 297	3536, 3762
3416, 3892	7 309	3417, 3893
308, 7012	7 321	309, 7013
3062, 4270	7 333	3063, 4271
148, 7202	7 351	149, 7203
2559, 4809	7 369	2560, 4810
1717, 5675	7 393	1718, 5676
394, 7016	7 411	395, 7017
2312, 5104	7 417	2313, 5105
228, 7230	7 459	229, 7231
3468, 4008	7 477	3469, 4009
2467, 5021	7 489	2468, 5022
606, 6900	7 507	607, 6901
1962, 5574	7 537	1963, 5575
528, 7020	7 549	529, 7021
1298, 6262	7 561	1299, 6263
2057, 5515	7 573	2058, 5516
2453, 5137	7 591	2454, 5138
2094, 5508	7 603	2095, 5509
3124, 4496	7 621	3125, 4497
2975, 4663	7 639	2976, 4664
2070, 5598	7 669	2071, 5599
684, 6996	7 681	685, 6997
2274, 5412	7 687	2275, 5413

y	y	p	y'	y'	y	y	p	y'	y'
2269,	5429	7 699	2270,	5430	1035,	7407	8 443	1036,	7408
3439,	4277	7 717	3440,	4278	1776,	6684	8 461	1777,	6685
917,	6805	7 723	918,	6806	4187,	4279	8 467	4188,	4280
2452,	5288	7 741	2453,	5289	2024,	6496	8 521	2025,	6497
403,	7349	7 753	404,	7350	2976,	5550	8 527	2977,	5551
1759,	5999	7 759	1760,	6000	2552,	5986	8 539	2553,	5987
233,	7555	7 789	234,	7556	2823,	5739	8 563	2824,	5740
1465,	6401	7 867	1466,	6402	424,	8156	8 581	425,	8157
1394,	6478	7 873	1395,	6479	1205,	7393	8 599	1206,	7394
1366,	6512	7 879	1367,	6513	1545,	7077	8 623	1546,	7078
3759,	4167	7 927	3760,	4168	3306,	5322	8 629	3307,	5323
2005,	5927	7 933	2006,	5928	3572,	5068	8 641	3573,	5069
321,	7629	7 951	322,	7630	794,	7852	8 647	795,	7853
1623,	6339	7 963	1624,	6340	1378,	7298	8 677	1379,	7299
3244,	4748	7 993	3245,	4749	1903,	6785	8 689	1904,	6786
89,	7921	8 011	90,	7922	2413,	6293	8 707	2414,	6294
2642,	5374	8 017	2643,	5375	2537,	6175	8 713	2538,	6176
3497,	4555	8 053	3498,	4556	2281,	6437	8 719	2282,	6438
2765,	5293	8 059	2766,	5294	3658,	5072	8 731	3659,	5073
1413,	6675	8 089	1414,	6676	2268,	6468	8 737	2269,	6469
2217,	5883	8 101	2218,	5884	1733,	7027	8 761	1734,	7028
2903,	5257	8 161	2904,	5258	1267,	7511	8 779	1268,	7512
3092,	5074	8 167	3093,	5075	988,	7814	8 803	989,	7815
1096,	7082	8 179	1097,	7083	2436,	6384	8 821	2437,	6385
90,	8100	8 191	91,	8101	4372,	4466	8 839	4373,	4467
3265,	4943	8 209	3266,	4944	1461,	7401	8 863	1462,	7402
415,	7805	8 221	416,	7806	4227,	4659	8 887	4228,	4660
2612,	5620	8 233	2613,	5621	249,	8643	8 893	250,	8644
240,	8022	8 263	241,	8023	3847,	5075	8 923	3848,	5076
157,	8111	8 269	158,	8112	4339,	4589	8 929	4340,	4590
568,	7718	8 287	569,	7719	1640,	7300	8 941	1641,	7301
2050,	6242	8 293	2051,	6243	341,	8629	8 971	342,	8630
1266,	7044	8 311	1267,	7045	3798,	5202	9 001	3799,	5203
1286,	7030	8 317	1287,	7031	1094,	7912	9 007	1095,	7913
1052,	7276	8 329	1053,	7277	3325,	5687	9 013	3326,	5688
1736,	6616	8 353	1737,	6617	3152,	5890	9 043	3153,	5891
813,	7563	8 377	814,	7564	845,	8203	9 049	846,	8204
691,	7697	8 389	692,	7698	1081,	7985	9 067	1082,	7986
1961,	6457	8 419	1962,	6458	3388,	5702	9 091	3389,	5703
2147,	6283	8 431	2148,	6284	4379,	4723	9 103	4380,	4724

y y	p	y' y'
3120, 5988	9 109	3121, 5989
3010, 6116	9 127	3011, 6117
3797, 5335	9 133	3798, 5336
1175, 7975	9 151	1176, 7976
1294, 7862	9 157	1295, 7863
1009, 8171	9 181	1010, 8172
4121, 5065	9 187	4122, 5066
3767, 5431	9 199	3768, 5432
166, 9074	9 241	167. 9075
601, 8675	9 277	602, 8676
2843, 6439	9 283	2844, 6440
3138, 6180	9 319	3139, 6181
4399, 4937	9 337	4400, 4938
3230, 6112	9 343	3231, 6113
4020, 5328	9 349	4021, 5329
983, 8407	9 391	984, 8408
2624, 6772	9 397	2625, 6773
3410, 5992	9 403	3411, 5993
2379, 7041	9 421	2380, 7042
926, 8506	9 433	927, 8507
733, 8705	9 439	734, 8706
607, 8855	9 463	608, 8856
3490, 6020	9 511	3491, 6021
3268, 6278	9 547	3269, 6279
4206, 5394	9 601	4207, 5395
3086, 6526	9 613	3087, 6527
4426, 5192	9 619	4427, 5193
1621, 8009	9 631	1622, 8010
4596, 5046	9 643	4597, 5047
3415, 6233	9 649	3416, 6234
1653, 8007	9 661	1654, 8008
2913, 6765	9 679	2914, 6766
1991, 7705	9 697	1992, 7706
1924, 7796	9 721	1925, 7797
1550, 8182	9 733	1551, 8183
2768, 6970	9 739	2769, 6971
261, 9507	9 769	262, 9508
4307, 5473	9 781	4308, 5474
3031, 6755	9 787	3032, 6756
602, 9208	9 811	603, 9209

y y	p	y' y'
955, 8861	9 817	956, 8862
748, 9080	9 829	749, 9081
4750, 5108	9 859	4751, 5109
651, 9219	9 871	652, 9220
2536, 7346	9 883	2537, 7347
99, 9801	9 901	100, 9802
3335, 6571	9 907	3336, 6572
4231, 5699	9 931	4232, 5700
3633, 6315	9 949	3634, 6316
457, 9509	9 967	458, 9510
1569, 8403	9 973	1570, 8404

y	y	p^κ	y'	y'
18,	30	7^2	19,	31
22,	146	13^2	23,	147
18,	324	7^3	19,	325
68,	292	19^2	69,	293
439,	521	31^2	440,	522
581,	787	37^2	582,	788
423,	1425	43^2	424,	1426
1036,	1160	13^3	1037,	1161
1047,	1353	7^4	1048,	1354
1660,	2060	61^2	1661,	2061
699,	3789	67^2	700,	3790
2198,	3130	73^2	2199,	3131
1714,	4526	79^2	1715,	4527
2819,	4039	19^3	2820,	4040
4620,	4788	97^2	4621,	4789

☞ *The roots (y) are placed to left of the Argument (p) throughout this Table.*

y	y	p	y	y	p	y	y	p
1044,	8964	10 009	4123,	6713	10 837	1671,	9921	11 593
3731,	6307	10 039	4198,	6662	10 861	2593,	9023	11 617
4705,	5363	10 069	2879,	7987	10 867	2158,	9518	11 677
4959,	5133	10 093	1099,	9791	10 891	1900,	9788	11 689
3947,	6151	10 099	478,	10424	10 903	4057,	7643	11 701
4281,	5829	10 111	1641,	9267	10 909	187,	11531	11 719
2185,	7955	10 141	4396,	6542	10 939	3378,	8352	11 731
4593,	5565	10 159	3145,	7811	10 957	3157,	8585	11 743
4773,	5403	10 177	4358,	6628	10 987	3962,	7816	11 779
563,	9679	10 243	1544,	9448	10 993	1048,	10772	11 821
175,	10091	10 267	1691,	9355	11 047	3740,	8086	11 827
2019,	8253	10 273	4306,	6752	11 059	2617,	9215	11 833
101,	10201	10 303	1738,	9332	11 071	679,	11159	11 839
2084,	8236	10 321	4377,	6705	11 083	3225,	8637	11 863
3566,	6766	10 333	3857,	7255	11 113	4533,	7353	11 887
3806,	6550	10 357	2355,	8763	11 119	4447,	7475	11 923
833,	9535	10 369	105,	11025	11 131	3191,	8749	11 941
444,	9954	10 399	4438,	6710	11 149	3113,	8839	11 953
2414,	8014	10 429	3813,	7347	11 161	765,	11193	11 959
270,	10182	10 453	4037,	7135	11 173	5225,	6745	11 971
1329,	9129	10 459	4402,	6794	11 197	1810,	10196	12 007
716,	9760	10 477	5566,	5672	11 239	1293,	10743	12 037
4730,	5770	10 501	1076,	10174	11 251	3163,	8879	12 043
3538,	6974	10 513	4321,	6935	11 257	685,	11363	12 049
5080,	5450	10 531	4197,	7089	11 287	2270,	9802	12 073
1185,	9381	10 567	1499,	9799	11 299	190,	11906	12 097
4807,	5789	10 597	1120,	10190	11 311	3432,	8676	12 109
4286,	6340	10 627	487,	10829	11 317	5588,	6568	12 157
1893,	8745	10 639	1227,	10101	11 329	5695,	6467	12 163
984,	9666	10 651	2307,	9045	11 353	110,	12100	12 211
3983,	6673	10 657	805,	10577	11 383	2933,	9307	12 241
806,	9856	10 663	2417,	9019	11 437	482,	11770	12 253
3653,	7033	10 687	2931,	8511	11 443	399,	11877	12 277
4500,	6210	10 711	3401,	8065	11 467	6048,	6240	12 289
1255,	9467	10 723	386,	11104	11 491	2226,	10074	12 301
1742,	8986	10 729	1450,	10046	11 497	3422,	8920	12 343
5150,	5602	10 753	467,	11035	11 503	1496,	10876	12 373
3873,	6897	10 771	5517,	6009	11 527	5769,	6609	12 379
3471,	7317	10 789	3979,	7571	11 551	4228,	8162	12 391
3367,	7463	10 831	753,	10833	11 587	2811,	9597	12 409

☞ *The roots (y) are placed to left of the Argument (p) throughout this Table.*

y	y	p	y	y	p	y	y	p
5862,	6558	12 421	4616,	8482	13 099	656,	13246	13 903
111,	12321	12 433	1894,	11252	13 147	2968,	10952	13 921
2512,	9938	12 451	303,	12855	13 159	2134,	11798	13 933
486,	11970	12 457	5866,	7304	13 171	541,	13421	13 963
5033,	7453	12 487	3017,	10159	13 177	4211,	9787	13 999
3394,	9116	12 511	1737,	11445	13 183	1141,	12869	14 011
6110,	6406	12 517	5724,	7494	13 219	5207,	8821	14 029
2383,	10157	12 541	3208,	10040	13 249	1013,	13057	14 071
6217,	6329	12 547	199,	13067	13 267	2775,	11307	14 083
5157,	7395	12 553	3246,	10044	13 291	5148,	8958	14 107
4814,	7762	12 577	2399,	10897	13 297	5171,	8971	14 143
1697,	10885	12 583	5103,	8205	13 309	1652,	12496	14 149
5337,	7251	12 589	3142,	10184	13 327	3513,	10659	14 173
847,	11753	12 601	3911,	9427	13 339	5123,	9073	14 197
5475,	7137	12 613	1402,	11978	13 381	4845,	9375	14 221
3774,	8844	12 619	2921,	10477	13 399	5849,	8401	14 251
6073,	6563	12 637	1975,	11435	13 411	119,	14161	14 281
1299,	11397	12 697	2508,	10908	13 417	1840,	12452	14 293
5354,	7348	12 703	645,	12795	13 441	4630,	9692	14 323
4930,	7790	12 721	2330,	11146	13 477	6360,	7980	14 341
5585,	7153	12 739	5595,	7917	13 513	4470,	9876	14 347
2366,	10390	12 757	419,	13117	13 537	432,	13956	14 389
3033,	9729	12 763	4758,	8808	13 567	317,	14083	14 401
3223,	9557	12 781	2741,	10849	13 591	3315,	11091	14 407
2418,	10380	12 799	4429,	9167	13 597	6630,	7788	14 419
1391,	11431	12 823	3224,	10402	13 627	550,	13880	14 431
1668,	11160	12 829	3042,	10590	13 633	4484,	9952	14 437
5752,	7088	12 841	202,	13466	13 669	1934,	12514	14 449
6072,	6780	12 853	1925,	11755	13 681	6501,	7959	14 461
4258,	8630	12 889	2675,	11011	13 687	524,	13954	14 479
2938,	9968	12 907	3008,	10684	13 693	4173,	10329	14 503
5520,	7398	12 919	3148,	10562	13 711	6691,	7841	14 533
5717,	7249	12 967	2150,	11572	13 723	3835,	10715	14 551
5886,	7086	12 973	6709,	7019	13 729	6221,	8335	14 557
3841,	9137	12 979	713,	13045	13 759	4295,	10267	14 563
1687,	11315	13 003	2426,	11362	13 789	6683,	7909	14 593
4011,	8997	13 009	117,	13689	13 807	7154,	7474	14 629
2222,	10810	13 033	2234,	11596	13 831	3693,	10959	14 653
1347,	11715	13 063	6834,	7038	13 873	3298,	11384	14 683
1289,	11803	13 093	2878,	11000	13 879	757,	13955	14 713

☞ *The roots (y) are placed to left of the Argument (p) throughout this Table.*

y	y	p	y	y	p	y	y	p
1907,	12823	14 731	1557,	13893	15 451	7516,	8732	16 249
4341,	10395	14 737	6825,	8667	15 493	796,	15470	16 267
5029,	9737	14 767	5060,	10450	15 511	5145,	11127	16 273
6688,	8090	14 779	4551,	10989	15 541	1549,	14783	16 333
851,	13945	14 797	3634,	11924	15 559	7113,	9225	16 339
2011,	12809	14 821	5930,	9652	15 583	5899,	10463	16 363
7134,	7692	14 827	3358,	12242	15 601	338,	16030	16 369
2834,	12016	14 851	3187,	12419	15 607	3393,	12987	16 381
2867,	12001	14 869	4345,	11273	15 619	2756,	13654	16 411
7382,	7504	14 887	4922,	10720	15 643	558,	15858	16 417
4233,	10689	14 923	5448,	10200	15 649	4251,	12195	16 447
4202,	10726	14 929	6823,	8837	15 661	1667,	14785	16 453
763,	14183	14 947	6555,	9111	15 667	5900,	10576	16 477
2656,	12326	14 983	2960,	12718	15 679	5463,	11055	16 519
4963,	10049	15 013	5584,	10142	15 727	3455,	13105	16 561
5359,	9671	15 031	1321,	14411	15 733	5016,	11550	16 567
1183,	13877	15 061	5013,	10725	15 739	7351,	9221	16 573
6913,	8159	15 073	4528,	11258	15 787	7558,	9044	16 603
3691,	11399	15 091	2973,	12843	15 817	4010,	12622	16 633
7292,	7828	15 121	6716,	9106	15 823	223,	16427	16 651
5650,	9488	15 139	2515,	13343	15 859	6707,	9949	16 657
962,	14224	15 187	7337,	8539	15 877	5153,	11539	16 693
4590,	10602	15 193	333,	15555	15 889	4376,	12322	16 699
5214,	9984	15 199	2083,	13817	15 901	976,	15752	16 729
863,	14353	15 217	3696,	12210	15 907	6179,	10561	16 741
6387,	8853	15 241	2099,	13813	15 913	2289,	14457	16 747
5127,	10131	15 259	7420,	8498	15 919	5700,	11058	16 759
3029,	12241	15 271	7801,	8135	15 937	1884,	14946	16 831
810,	14466	15 277	1034,	14938	15 973	6422,	10420	16 843
2166,	13122	15 289	4052,	11938	15 991	4163,	12715	16 879
4547,	10759	15 307	6936,	9096	16 033	8386,	8516	16 903
6819,	8493	15 313	4688,	11368	16 057	4347,	12573	16 921
539,	14779	15 319	3555,	12507	16 063	2799,	14127	16 927
3370,	11960	15 331	1951,	14117	16 069	1846,	15116	16 963
6547,	8801	15 349	5619,	10467	16 087	7466,	9514	16 981
7293,	8067	15 361	5666,	10444	16 111	8001,	8985	16 987
2507,	12865	15 373	4003,	12137	16 141	912,	16080	16 993
2964,	12426	15 391	5209,	10973	16 183	6641,	10369	17 011
3236,	12190	15 427	5917,	10271	16 189	5519,	11509	17 029
7495,	7943	15 439	6044,	10186	16 231	2697,	14343	17 041

☞ *The roots (y) are placed to left of the Argument (p) throughout this Table.*

y y	p	y y	p	y y	p
6097, 10949	17 047	2836, 14924	17 761	8374, 10076	18 451
345, 16707	17 053	4505, 13285	17 791	1734, 16722	18 457
1161, 15915	17 077	1883, 15943	17 827	4292, 14188	18 481
6797, 10309	17 107	5290, 12548	17 839	3651, 14841	18 493
4779, 12357	17 137	3361, 14489	17 851	5071, 13445	18 517
5965, 11201	17 167	8172, 9690	17 863	3064, 15458	18 523
1821, 15369	17 191	5579, 12301	17 881	2733, 15807	18 541
4387, 12815	17 203	5048, 12862	17 911	7689, 10863	18 553
2167, 15041	17 209	613, 17309	17 923	6030, 12552	18 583
2063, 15175	17 239	2805, 15123	17 929	1385, 17251	18 637
1371, 15885	17 257	8912, 9046	17 959	2595, 16065	18 661
131, 17161	17 293	2737, 15233	17 971	3906, 14772	18 679
5722, 11576	17 299	4817, 13159	17 977	896, 17794	18 691
6418, 10898	17 317	6289, 11699	17 989	4083, 14673	18 757
8383, 8957	17 341	2328, 15684	18 013	8204, 10582	18 787
1967, 15391	17 359	6209, 11833	18 043	2154, 16638	18 793
1257, 16119	17 377	3260, 14788	18 049	2565, 16293	18 859
3172, 14210	17 383	2796, 15264	18 061	6258, 12654	18 913
3642, 13746	17 389	5987, 12109	18 097	2035, 16883	18 919
604, 16796	17 401	749, 17371	18 121	2744, 16228	18 973
6689, 10729	17 419	7749, 10377	18 127	600, 18378	18 979
2984, 14446	17 431	3529, 14603	18 133	1625, 17383	19 009
6122, 11320	17 443	6574, 11594	18 169	2995, 16055	19 051
803, 16645	17 449	6623, 11557	18 181	2853, 16215	19 069
6891, 10575	17 467	1018, 17180	18 199	4928, 14152	19 081
1688, 15802	17 491	2077, 16133	18 211	3890, 15196	19 087
4719, 12777	17 497	8397, 9819	18 217	9138, 10002	19 141
4127, 13381	17 509	2172, 16050	18 223	138, 19044	19 183
477, 17061	17 539	3763, 14465	18 229	7415, 11791	19 207
5733, 11817	17 551	9009, 9243	18 253	8990, 10222	19 213
5052, 12516	17 569	3248, 15040	18 289	8256, 10962	19 219
5191, 12389	17 581	1290, 17010	18 301	9365, 9865	19 231
6786, 10812	17 599	5376, 12930	18 307	909, 18327	19 237
3350, 14272	17 623	2992, 15320	18 313	6857, 12391	19 249
7887, 9771	17 659	6167, 12199	18 367	7149, 12117	19 267
4070, 13612	17 683	7324, 11054	18 379	1739, 17533	19 273
4634, 13072	17 707	2647, 15749	18 397	8922, 10386	19 309
6367, 11345	17 713	1565, 16861	18 427	4798, 14534	19 333
2200, 15536	17 737	3784, 14648	18 433	7557, 11823	19 381
6977, 10771	17 749	8089, 10349	18 439	3147, 16239	19 387

☞ *The roots* (y) *are placed to left of the Argument* (p) *throughout this Table.*

y	y	p	y	y	p	y	y	p
4452,	14964	19 417	6965,	13141	20 107	1631,	19327	20 959
9039,	10383	19 423	5472,	14640	20 113	1287,	19695	20 983
1607,	17821	19 429	375,	19767	20 143	7174,	13826	21 001
241,	19199	19 441	7760,	12388	20 149	8328,	12684	21 013
7730,	11716	19 447	6174,	13986	20 161	4713,	16305	21 019
8828,	10642	19 471	9963,	10209	20 173	1851,	19179	21 031
3620,	15856	19 477	3475,	16757	20 233	3375,	17685	21 061
4477,	15005	19 483	5471,	14797	20 269	3680,	17386	21 067
4019,	15469	19 489	7508,	12778	20 287	7168,	13952	21 121
503,	18997	19 501	9904,	10418	20 323	8617,	12521	21 139
3827,	15679	19 507	2356,	17984	20 341	3420,	17736	21 157
4124,	15406	19 531	998,	19348	20 347	4337,	16825	21 163
7219,	12323	19 543	3883,	16469	20 353	10458,	10710	21 169
5477,	14119	19 597	6660,	13698	20 359	5069,	16117	21 187
9731,	9871	19 603	2376,	18012	20 389	2144,	19048	21 193
874,	18734	19 609	5676,	14730	20 407	8073,	13137	21 211
6356,	13324	19 681	8791,	11639	20 431	9501,	11745	21 247
4982,	14704	19 687	3985,	16457	20 443	8319,	12957	21 277
7417,	12281	19 699	7412,	13066	20 479	10378,	10904	21 283
6448,	13268	19 717	6463,	14045	20 509	5495,	15817	21 313
5755,	13997	19 753	516,	20004	20 521	10586,	10732	21 319
7875,	11883	19 759	1627,	18905	20 533	4480,	16898	21 379
5630,	14146	19 777	9839,	10711	20 551	2124,	19266	21 391
2184,	17616	19 801	3065,	17497	20 563	913,	20483	21 397
7510,	12302	19 813	143,	20449	20 593	6087,	15345	21 433
8962,	10856	19 819	6818,	13780	20 599	4325,	17155	21 481
7724,	12118	19 843	4093,	16517	20 611	8700,	12786	21 487
6291,	13569	19 861	9021,	11619	20 641	2543,	18949	21 493
4595,	15271	19 867	5496,	15210	20 707	6163,	15335	21 499
1626,	18264	19 891	3176,	17542	20 719	2190,	19326	21 517
244,	19682	19 927	3871,	16859	20 731	9549,	11973	21 523
6345,	13617	19 963	10041,	10701	20 743	3972,	17556	21 529
5960,	14032	19 993	3119,	17629	20 749	5980,	15578	21 559
9471,	10539	20 011	1419,	19353	20 773	6894,	14682	21 577
141,	19881	20 023	1232,	19576	20 809	6631,	14957	21 589
6723,	13305	20 029	2047,	18809	20 857	4185,	17415	21 601
510,	19536	20 047	7010,	13876	20 887	2310,	19302	21 613
4785,	15285	20 071	8388,	12510	20 899	7039,	14609	21 649
8165,	11923	20 089	6015,	14913	20 929	6280,	15380	21 661
6313,	13787	20 101	1881,	19065	20 947	389,	21283	21 673

☞ *The roots (y) are placed to left of the Argument (p) throughout this Table.*

y	y	p	y	y	p	y	y	p
7456,	14270	21 727	1844,	20686	22 531	2770,	20702	23 473
6777,	14961	21 739	7069,	15473	22 543	265,	23231	23 497
8041,	13709	21 751	10094,	12454	22 549	3320,	20188	23 509
147,	21609	21 757	9283,	13283	22 567	5452,	18086	23 539
4435,	17351	21 787	688,	21884	22 573	958,	22598	23 557
4978,	16820	21 799	9532,	13088	22 621	153,	23409	23 563
2440,	19376	21 817	542,	22096	22 639	4685,	18895	23 581
8529,	13311	21 841	150,	22500	22 651	6836,	16756	23 593
7609,	14249	21 859	9037,	13631	22 669	6764,	16834	23 599
10163,	11707	21 871	10281,	12417	22 699	5232,	18390	23 623
8723,	13213	21 937	9377,	13339	22 717	3070,	20558	23 629
9367,	12575	21 943	10350,	12390	22 741	8927,	14743	23 671
7962,	13998	21 961	1740,	21036	22 777	4483,	19193	23 677
8058,	13932	21 991	1235,	21547	22 783	5478,	18210	23 689
10391,	11605	21 997	7995,	14811	22 807	11782,	11936	23 719
2573,	19429	22 003	3743,	19117	22 861	3931,	19811	23 743
4629,	17397	22 027	8452,	14468	22 921	8775,	14985	23 761
1319,	20719	22 039	6466,	16496	22 963	6008,	17758	23 767
1564,	20486	22 051	1446,	21546	22 993	8580,	15192	23 773
8644,	13418	22 063	7100,	15910	23 011	1472,	22354	23 827
6861,	15231	22 093	5617,	17399	23 017	8772,	15060	23 833
7858,	14252	22 111	401,	22627	23 029	10979,	12877	23 857
8770,	13352	22 123	4081,	18959	23 041	9020,	14848	23 869
2502,	19626	22 129	3100,	19952	23 053	3289,	20597	23 887
8604,	13542	22 147	10936,	12122	23 059	5681,	18211	23 893
10588,	11564	22 153	2237,	20833	23 071	5872,	18026	23 899
3519,	18639	22 159	3880,	19250	23 131	3495,	20415	23 911
2852,	19318	22 171	7531,	15611	23 143	1756,	22160	23 917
10965,	11223	22 189	697,	22469	23 167	2489,	21439	23 929
9638,	12634	22 273	7076,	16096	23 173	709,	23261	23 971
10797,	11481	22 279	11174,	12022	23 197	9276,	14700	23 977
5671,	16619	22 291	5901,	17301	23 203	6596,	17404	24 001
831,	21471	22 303	2254,	20954	23 209	11240,	12766	24 007
8977,	13391	22 369	549,	22677	23 227	5262,	18756	24 019
10235,	12145	22 381	4971,	18279	23 251	10342,	13700	24 043
2026,	20414	22 441	8638,	14630	23 269	6657,	17391	24 049
259,	22187	22 447	10004,	13288	23 293	6586,	17474	24 061
4542,	17910	22 453	9457,	13853	23 311	11150,	12940	24 091
10518,	11964	22 483	10559,	12811	23 371	6398,	17698	24 097
1132,	21368	22 501	8167,	15263	23 431	5655,	18447	24 103

☞ *The roots (y) are placed to left of the Argument (p) throughout this Table.*

y	y	p	y	y	p	y	y	p
9816,	14292	24 109	2904,	22128	25 033	10722,	15096	25 819
6333,	17787	24 121	9201,	15855	25 057	4527,	21321	25 849
10042,	14090	24 133	3083,	22003	25 087	5258,	20608	25 867
9854,	14296	24 151	8979,	16131	25 111	2827,	23045	25 873
6985,	17183	24 169	274,	24842	25 117	8157,	17745	25 903
155,	24025	24 181	11405,	13741	25 147	6523,	19409	25 933
6527,	17695	24 223	2248,	22904	25 153	7855,	18083	25 939
5122,	19106	24 229	11820,	13350	25 171	6557,	19393	25 951
11578,	12668	24 247	2780,	22402	25 183	9758,	16210	25 969
11828,	12508	24 337	11837,	13351	25 189	1587,	24393	25 981
7897,	16475	24 373	9118,	16100	25 219	11856,	14142	25 999
9608,	14770	24 379	8117,	17119	25 237	1555,	24461	26 017
8948,	15442	24 391	5686,	19556	25 243	2182,	23846	26 029
5562,	18858	24 421	8560,	16700	25 261	739,	25301	26 041
8924,	15514	24 439	7147,	18155	25 303	12522,	13530	26 053
951,	23517	24 469	3072,	22236	25 309	161,	25921	26 083
6299,	18181	24 481	1302,	24018	25 321	3018,	23088	26 107
8218,	16280	24 499	8952,	16386	25 339	11434,	14678	26 113
5385,	19131	24 517	9006,	16350	25 357	7439,	18679	26 119
4826,	19720	24 547	730,	24680	25 411	11809,	14351	26 161
271,	24299	24 571	6529,	18893	25 423	4085,	22117	26 203
1513,	23117	24 631	8535,	16911	25 447	2028,	24180	26 209
2680,	22010	24 691	1204,	24248	25 453	280,	25946	26 227
3067,	21629	24 697	6851,	18619	25 471	6989,	19261	26 251
10183,	14525	24 709	9349,	16187	25 537	10770,	15492	26 263
7461,	17271	24 733	972,	24588	25 561	6972,	19320	26 293
5221,	19541	24 763	4752,	20826	25 579	9710,	16606	26 317
6845,	17935	24 781	12721,	12881	25 603	11190,	15156	26 347
2319,	22473	24 793	11961,	13647	25 609	1995,	24375	26 371
5877,	18921	24 799	4977,	20643	25 621	162,	26244	26 407
4667,	20173	24 841	1310,	24322	25 633	7815,	18615	26 431
6143,	18703	24 847	11115,	14523	25 639	10037,	16399	26 437
3141,	21717	24 859	12064,	13592	25 657	1445,	25003	26 449
8521,	16355	24 877	7665,	18027	25 693	2295,	24183	26 479
10814,	14074	24 889	9977,	15739	25 717	9620,	16876	26 497
7800,	17106	24 907	1530,	24210	25 741	6742,	19796	26 539
7699,	17219	24 919	1850,	23896	25 747	9464,	17092	26 557
1233,	23709	24 943	10860,	14898	25 759	588,	26052	26 641
12404,	12562	24 967	12279,	13491	25 771	5842,	20804	26 647
6491,	18487	24 979	2231,	23569	25 801	4919,	21763	26 683

☞ *The roots (y) are placed to left of the Argument (p) throughout this Table.*

y y	p	y y	p	y y	p
11718, 14982	26 701	4569, 23013	27 583	8098, 20330	28 429
12901, 13811	26 713	2746, 24884	27 631	1988, 26458	28 447
11928, 14802	26 731	2738, 24934	27 673	13215, 15261	28 477
5100, 21636	26 737	5723, 21967	27 691	12370, 16142	28 513
2412, 24408	26 821	5914, 21782	27 697	9906, 18630	28 537
8234, 18598	26 833	7523, 20209	27 733	7759, 20789	28 549
1613, 25225	26 839	9446, 18292	27 739	7185, 21387	28 573
1563, 25299	26 863	1606, 26144	27 751	8252, 20326	28 579
2844, 24036	26 881	2454, 25308	27 763	6744, 21846	28 591
751, 26141	26 893	11630, 16162	27 793	6052, 22544	28 597
7556, 19390	26 947	10801, 16997	27 799	10587, 18015	28 603
1990, 24962	26 953	5360, 22456	27 817	5244, 23376	28 621
13183, 13775	26 959	11653, 16169	27 823	9296, 19330	28 627
9582, 17448	27 031	9429, 18417	27 847	6263, 22393	28 657
9646, 17396	27 043	2848, 25034	27 883	12077, 16585	28 663
164, 26896	27 061	6933, 20967	27 901	11959, 16709	28 669
8967, 18099	27 067	1261, 26657	27 919	10875, 17811	28 687
12212, 14860	27 073	5646, 22296	27 943	11099, 17611	28 711
10123, 16967	27 091	11418, 16542	27 961	2727, 25995	28 723
5974, 21128	27 103	10143, 17823	27 967	6924, 21804	28 729
10994, 16114	27 109	11651, 16345	27 997	1786, 26966	28 753
3505, 23621	27 127	10310, 17716	28 027	12978, 15780	28 759
3311, 23899	27 211	2374, 25676	28 051	5175, 23595	28 771
2220, 25020	27 241	167, 27889	28 057	7517, 21271	28 789
6344, 20908	27 253	12216, 15852	28 069	10746, 18060	28 807
1246, 26012	27 259	1765, 26315	28 081	14259, 14553	28 813
8755, 18515	27 271	13659, 14427	28 087	8680, 20156	28 837
9617, 17659	27 277	443, 27655	28 099	10117, 18725	28 843
6711, 20571	27 283	5435, 22675	28 111	5683, 23183	28 867
4276, 23060	27 337	13094, 15028	28 123	7567, 21311	28 879
286, 27074	27 361	10852, 17330	28 183	10917, 17991	28 909
10889, 16477	27 367	5057, 23143	28 201	8782, 20138	28 921
9187, 18209	27 397	3355, 24863	28 219	4849, 24077	28 927
10930, 16478	27 409	6886, 21392	28 279	1328, 27604	28 933
3894, 23532	27 427	11197, 17099	28 297	1964, 27052	29 017
3588, 23868	27 457	10519, 17789	28 309	13004, 16018	29 023
4981, 22499	27 481	8250, 20100	28 351	7377, 21681	29 059
4262, 23224	27 487	2711, 25675	28 387	9635, 19441	29 077
13220, 14308	27 529	168, 28224	28 393	11135, 17965	29 101
760, 26780	27 541	5648, 22762	28 411	6492, 22638	29 131

☞ *The roots (y) are placed to left of the Argument (p) throughout this Table.*

y	y	p	y	y	p	y	y	p
5324,	23812	29 137	10409,	19681	30 091	13471,	17345	30 817
2748,	26418	29 167	7325,	22771	30 097	5946,	24882	30 829
9972,	19200	29 173	173,	29929	30 103	4780,	26060	30 841
11997,	17181	29 179	9978,	20130	30 109	7214,	23638	30 853
4762,	24428	29 191	966,	29166	30 133	2902,	27956	30 859
4420,	24788	29 209	12941,	17197	30 139	633,	30237	30 871
1648,	27572	29 221	3469,	26599	30 169	11001,	19929	30 931
452,	28798	29 251	6929,	23251	30 181	11277,	19659	30 937
11222,	18046	29 269	8080,	22106	30 187	7165,	23783	30 949
3435,	25851	29 287	6201,	24009	30 211	9356,	21676	31 033
13394,	15916	29 311	7911,	22311	30 223	14831,	16207	31 039
2830,	26516	29 347	14577,	15663	30 241	3087,	27963	31 051
11662,	17720	29 383	8361,	21891	30 253	11170,	19892	31 063
6480,	22908	29 389	13106,	17152	30 259	13224,	17844	31 069
11208,	18192	29 401	2006,	28264	30 271	2137,	28943	31 081
7659,	21777	29 437	12062,	18244	30 307	2210,	28912	31 123
3427,	26015	29 443	4373,	25939	30 313	9052,	22094	31 147
6530,	22942	29 473	12494,	17824	30 319	176,	30976	31 153
7194,	22332	29 527	3853,	26513	30 367	10327,	20831	31 159
13058,	16510	29 569	8013,	22377	30 391	4114,	27062	31 177
1983,	27597	29 581	4583,	25819	30 403	9715,	21467	31 183
14707,	14879	29 587	9988,	20438	30 427	13341,	17847	31 189
1046,	28552	29 599	12222,	18246	30 469	12150,	19068	31 219
4906,	24704	29 611	8556,	21936	30 493	13828,	17402	31 231
8724,	20904	29 629	3263,	27253	30 517	9340,	21896	31 237
9728,	19912	29 641	1984,	28544	30 529	3386,	27862	31 249
3517,	26153	29 671	7838,	22714	30 553	13259,	18007	31 267
3115,	26567	29 683	10340,	20218	30 559	8340,	22980	31 321
8188,	21572	29 761	2365,	28211	30 577	771,	30555	31 327
752,	29050	29 803	11544,	19086	30 631	9983,	21349	31 333
7795,	22037	29 833	7615,	23021	30 637	10608,	20748	31 357
2449,	27401	29 851	11586,	19056	30 643	12896,	18490	31 387
13036,	16826	29 863	14750,	15898	30 649	986,	30406	31 393
13154,	16726	29 881	8916,	21744	30 661	11905,	19571	31 477
4030,	25886	29 917	10557,	20139	30 697	10980,	20508	31 489
4091,	25855	29 947	13831,	16871	30 703	10563,	20949	31 513
7773,	22185	29 959	5226,	25500	30 727	4460,	27070	31 531
793,	29189	29 983	15146,	15610	30 757	4531,	27011	31 543
6859,	23129	29 989	4285,	26477	30 763	14049,	17517	31 567
2042,	27970	30 013	7320,	23460	30 781	9131,	22441	31 573

☞ *The roots (y) are placed to left of the Argument (p) throughout this Table.*

y y	p	y y	p	y y	p
6419, 25207	31 627	1719, 30771	32 491	9500, 23842	33 343
12534, 19122	31 657	2343, 30153	32 497	1197, 32151	33 349
14973, 16689	31 663	15620, 16882	32 503	316, 33074	33 391
15932, 15754	31 687	8797, 23735	32 533	9498, 23904	33 403
1167, 30531	31 699	2127, 30435	32 563	9155, 24253	33 409
1876, 29846	31 723	13309, 19259	32 569	13792, 19634	33 427
1083, 30645	31 729	10922, 21664	32 587	659, 32797	33 457
4823, 26917	31 741	1741, 30869	32 611	5497, 27971	33 469
11175, 20595	31 771	15909, 16737	32 647	13707, 19779	33 487
1114, 30734	31 849	9422, 23230	32 653	5332, 28160	33 493
5146, 26726	31 873	12593, 20113	32 707	13615, 19913	33 529
2012, 29878	31 891	13858, 18854	32 713	9380, 24166	33 547
14131, 17825	31 957	1781, 30937	32 719	15914, 17662	33 577
7872, 24090	31 963	4354, 28394	32 749	2949, 30639	33 589
12636, 19344	31 981	8314, 24464	32 779	8121, 25479	33 601
473, 31555	32 029	15444, 17352	32 797	12687, 20925	33 613
12659, 19399	32 059	5022, 27780	32 803	13967, 19651	33 619
15305, 16771	32 077	5812, 27020	32 833	8017, 25619	33 637
7008, 25074	32 083	6821, 26017	32 839	11287, 22391	33 679
4033, 28055	32 089	13710, 19158	32 869	10249, 23453	33 703
14414, 17704	32 119	9234, 23652	32 887	6912, 26808	33 721
6366, 25776	32 143	15680, 17230	32 911	1503, 32235	33 739
3627, 28545	32 173	3937, 28979	32 917	16232, 17518	33 751
3707, 28483	32 191	7432, 25508	32 941	2388, 31368	33 757
8449, 23753	32 203	11358, 21612	32 971	9149, 24619	33 769
6032, 26200	32 233	480, 32502	32 983	11537, 22273	33 811
10534, 21716	32 251	15584, 17428	33 013	4627, 29201	33 829
4582, 27674	32 257	16012, 17024	33 037	3139, 30731	33 871
10001, 22297	32 299	9246, 23802	33 049	13943, 19945	33 889
7830, 24492	32 323	12561, 20511	33 073	3411, 30519	33 931
14544, 17796	32 341	12448, 20642	33 091	5857, 28079	33 937
4714, 27638	32 353	5555, 27595	33 151	844, 33116	33 961
7544, 24814	32 359	16193, 16987	33 181	7903, 26063	33 967
15299, 17071	32 371	9941, 23257	33 199	12989, 21007	33 997
6256, 26120	32 377	12603, 20607	33 211	6870, 27162	34 033
8262, 24138	32 401	10809, 22413	33 223	14542, 19496	34 039
3948, 28464	32 413	15280, 17966	33 247	15509, 18547	34 057
10187, 22255	32 443	9762, 23526	33 289	1028, 33094	34 123
5757, 26709	32 467	3228, 30072	33 301	5414, 28714	34 129
8786, 23692	32 479	9364, 23966	33 331	8186, 25954	34 141

☞ *The roots* (y) *are placed to left of the Argument* (p) *throughout this Table.*

y	y	p	y	y	p	y	y	p
7444,	26702	34 147	6055,	28967	35 023	12350,	23548	35 899
11867,	22291	34 159	5544,	29508	35 053	189,	35721	35 911
666,	33504	34 171	10603,	24455	35 059	6452,	29470	35 923
5456,	28726	34 183	15647,	19435	35 083	14456,	21520	35 977
10112,	24100	34 213	6854,	28234	35 089	5093,	30889	35 983
11573,	22657	34 231	6444,	28662	35 107	13354,	22652	36 007
4059,	30201	34 261	5775,	29373	35 149	2795,	33217	36 013
16570,	17696	34 267	10484,	24736	35 221	2757,	33279	36 037
12875,	21397	34 273	8545,	26681	35 227	7574,	28486	36 061
12366,	21930	34 297	1417,	33833	35 251	6902,	29164	36 067
7103,	27199	34 303	8983,	26273	35 257	12467,	23605	36 073
5832,	28494	34 327	3103,	32177	35 281	1186,	34910	36 097
8507,	25843	34 351	15036,	20274	35 311	15840,	20268	36 109
7422,	26946	34 369	325,	34991	35 317	13619,	22531	36 151
7714,	26666	34 381	6999,	28323	35 323	1834,	34352	36 187
13734,	20694	34 429	2782,	32570	35 353	11904,	24312	36 217
3809,	30661	34 471	10317,	25083	35 401	16513,	19715	36 229
12044,	22438	34 483	9300,	26106	35 407	3442,	32798	36 241
9392,	25108	34 501	10644,	24774	35 419	8282,	27994	36 277
13903,	20609	34 513	1910,	33526	35 437	6322,	29984	36 307
7725,	26793	34 519	13885,	21563	35 449	12767,	23545	36 313
10612,	23924	34 537	17142,	18318	35 461	3498,	32820	36 319
3712,	30830	34 543	2548,	32942	35 491	2008,	34334	36 343
1792,	32756	34 549	7835,	27673	35 509	3352,	33020	36 373
3958,	30632	34 591	10031,	25489	35 521	17800,	18632	36 433
14067,	20535	34 603	4911,	30615	35 527	9378,	27072	36 451
492,	34158	34 651	188,	35344	35 533	2693,	33763	36 457
13740,	20946	34 687	10189,	25379	35 569	15416,	21052	36 469
3231,	31461	34 693	7930,	27662	35 593	9075,	27417	36 493
1304,	33424	34 729	2143,	33473	35 617	4669,	31853	36 523
14775,	19971	34 747	865,	34805	35 671	13674,	22854	36 529
10993,	23765	34 759	13788,	21888	35 677	1253,	35287	36 541
10601,	24205	34 807	7864,	27866	35 731	4945,	31613	36 559
4947,	29871	34 819	13315,	22481	35 797	11188,	25382	36 571
15506,	19336	34 843	13200,	22602	35 803	9709,	26873	36 583
14576,	20272	34 849	12576,	23232	35 809	7032,	29574	36 607
13598,	21298	34 897	6411,	29427	35 839	14588,	22048	36 637
2753,	32185	34 939	15160,	20690	35 851	15751,	20891	36 643
9674,	25288	34 963	3424,	32438	35 863	7376,	29314	36 691
8698,	26282	34 981	16033,	19835	35 869	3826,	32870	36 697

☞ *The roots (y) are placed to left of the Argument (p) throughout this Table.*

y	y	p	y	y	p	y	y	p
5271,	31437	36 709	11354,	26134	37 489	10731,	27567	38 299
5815,	30905	36 721	5090,	32410	37 501	4677,	33639	38 317
8885,	27853	36 739	3868,	33638	37 507	5704,	32624	38 329
1829,	34951	36 781	12119,	25417	37 537	16808,	21562	38 371
17644,	19142	36 787	13132,	24416	37 549	14206,	24170	38 377
8978,	27814	36 793	9857,	27703	37 561	15201,	23229	38 431
2582,	34264	36 847	11993,	25573	37 567	2888,	35560	38 449
17851,	19019	36 871	5540,	32032	37 573	6624,	31836	38 461
3738,	33138	36 877	9825,	27753	37 579	7607,	30949	38 557
10278,	26622	36 901	1079,	36511	37 591	3942,	34626	38 569
16910,	20002	36 913	18648,	18984	37 633	8097,	30495	38 593
14591,	22327	36 919	11263,	26393	37 657	13452,	25158	38 611
13652,	23278	36 931	6309,	31353	37 663	19054,	19574	38 629
13600,	23342	36 943	4991,	32701	37 693	7662,	30990	38 653
2007,	34965	36 973	3051,	34647	37 699	5843,	32827	38 671
13825,	23153	36 979	8287,	29429	37 717	17549,	21127	38 677
14937,	22059	36 997	18193,	19553	37 747	2911,	35795	38 707
14105,	22897	37 003	8082,	29700	37 783	4122,	34590	38 713
8781,	28239	37 021	16487,	21325	37 813	4095,	34641	38 737
11606,	25432	37 039	194,	37636	37 831	18826,	19922	38 749
192,	36864	37 057	10389,	27471	37 861	5750,	33016	38 767
7315,	29771	37 087	14741,	23137	37 879	13216,	25574	38 791
4145,	32971	37 117	6967,	30929	37 897	1538,	37264	38 803
3934,	33188	37 123	11090,	26860	37 951	9754,	29066	38 821
17240,	19918	37 159	5854,	32102	37 957	7624,	31208	38 833
7257,	29913	37 171	515,	37447	37 963	17779,	21059	38 839
1578,	35610	37 189	10095,	27891	37 987	5365,	33485	38 851
14639,	22561	37 201	8989,	29003	37 993	3259,	35657	38 917
13496,	23746	37 243	17569,	20441	38 011	1489,	37433	38 923
13166,	24106	37 273	2249,	35797	38 047	4461,	34491	38 953
10707,	26601	37 309	13139,	24913	38 053	15764,	23194	38 959
16664,	20656	37 321	10287,	27795	38 083	2003,	36967	38 971
2846,	34492	37 339	15348,	22764	38 113	18680,	20296	38 977
15581,	21775	37 357	8413,	29705	38 119	15047,	23971	39 019
10266,	27096	37 363	3988,	34160	38 149	16704,	22338	39 043
2178,	35190	37 369	11638,	26528	38 167	7217,	31861	39 079
886,	36536	37 423	15675,	22521	38 197	7126,	31970	39 097
16392,	21048	37 441	15256,	22982	38 239	7364,	31738	39 103
843,	36603	37 447	17035,	21245	38 281	9015,	30117	39 133
3678,	33804	37 483	705,	37581	38 287	13413,	25725	39 139

☞ *The roots* (y) *are placed to left of the Argument* (p) *throughout this Table.*

y	y	p	y	y	p	y	y	p
13567,	25589	39 157	15435,	24627	40 063	881,	40015	40 897
11087,	28075	39 163	8752,	31334	40 087	15496,	25406	40 903
12410,	26770	39 181	16488,	23604	40 093	2810,	38116	40 927
10015,	29183	39 199	19688,	20410	40 099	19501,	21431	40 933
907,	38309	39 217	14759,	25351	40 111	17649,	23289	40 939
6625,	32603	39 229	12936,	27186	40 123	9011,	31981	40 993
11453,	27787	39 241	19799,	20329	40 129	11432,	29578	41 011
12925,	26375	39 301	3476,	36676	40 153	8694,	32322	41 017
7195,	32117	39 313	12870,	27306	40 177	7042,	33980	41 023
5669,	33673	39 343	7169,	33019	40 189	15951,	25095	41 047
17300,	22066	39 367	17813,	22399	40 213	11380,	29696	41 077
9112,	30260	39 373	2786,	37444	40 231	17069,	24043	41 113
18298,	21098	39 397	5719,	34517	40 237	12831,	28299	41 131
13202,	26206	39 409	5438,	34912	40 351	5521,	35621	41 143
5662,	33776	39 439	13629,	26727	40 357	1531,	39617	41 149
13325,	26125	39 451	14285,	26101	40 387	731,	40429	41 161
3123,	36375	39 499	4829,	35593	40 423	18203,	22975	41 179
16598,	22912	39 511	16395,	24033	40 429	16813,	24389	41 203
12259,	27281	39 541	6508,	33950	40 459	10254,	30966	41 221
19347,	20259	39 607	2119,	38351	40 471	13365,	27861	41 227
13971,	25647	39 619	15071,	25411	40 483	12789,	28443	41 233
13753,	25877	39 631	7797,	32709	40 507	5182,	36074	41 257
14115,	25551	39 667	17648,	22870	40 519	4408,	36854	41 263
9833,	29845	39 679	1920,	38610	40 531	885,	40383	41 269
2448,	37254	39 703	17667,	22875	40 543	4646,	36634	41 281
3292,	36416	39 709	5140,	35450	40 591	2001,	39297	41 299
10258,	29468	39 727	15894,	24702	40 597	14845,	26495	41 341
1901,	37831	39 733	13603,	27005	40 609	1270,	40118	41 389
18395,	21373	39 769	18255,	22371	40 627	203,	41209	41 413
13834,	25964	39 799	17445,	23193	40 639	20354,	21088	41 443
10306,	29522	39 829	18478,	22214	40 693	7265,	34201	41 467
5513,	34327	39 841	17164,	23534	40 699	17967,	23511	41 479
18940,	20906	39 847	6297,	34461	40 759	12182,	29308	41 491
2549,	37327	39 877	2731,	38039	40 771	4739,	36781	41 521
13584,	26298	39 883	19375,	21425	40 801	19187,	22351	41 539
17945,	21955	39 901	534,	40278	40 813	11294,	30298	41 593
4517,	35419	39 937	7191,	33627	40 819	14654,	26956	41 611
9862,	30116	39 979	12447,	28401	40 849	9166,	32450	41 617
7006,	33002	40 009	15631,	25235	40 867	1743,	39897	41 641
8363,	31675	40 039	12243,	28635	40 879	15014,	26632	41 647

☞ *The roots (y) are placed to left of the Argument (p) throughout this Table.*

y	y	p	y	y	p	y	y	p
9395,	32263	41 659	1760,	40696	42 457	1985,	41335	43 321
20391,	21327	41 719	7111,	35351	42 463	19021,	24377	43 399
8455,	33281	41 737	10532,	31954	42 487	9287,	34123	43 411
3376,	38384	41 761	15892,	26606	42 499	14177,	29263	43 441
5444,	36364	41 809	5897,	36673	42 571	3279,	40263	43 543
12698,	29152	41 851	6790,	35786	42 577	18622,	24950	43 573
12257,	29605	41 863	3953,	38635	42 589	6920,	36658	43 579
12130,	29756	41 887	206,	42436	42 643	7958,	35632	43 591
13313,	28579	41 893	14147,	28501	42 649	2371,	41225	43 597
3791,	38119	41 911	14404,	28262	42 667	14720,	28888	43 609
1599,	40341	41 941	10078,	32618	42 697	1992,	41634	43 627
12922,	29024	41 947	9512,	33190	42 703	12854,	30778	43 633
14230,	27722	41 953	8054,	34654	42 709	13765,	29885	43 651
1546,	40412	41 959	10456,	32270	42 727	14168,	29500	43 669
7333,	34649	41 983	5275,	37475	42 751	14082,	29628	43 711
15662,	26350	42 013	19281,	23505	42 787	5260,	38456	43 717
8582,	33436	42 019	14716,	28076	42 793	11845,	31907	43 753
2892,	39150	42 043	15360,	27468	42 829	5204,	38554	43 759
1247,	40813	42 061	358,	42482	42 841	7067,	36709	43 777
5233,	36839	42 073	12192,	30660	42 853	5618.	38164	43 783
12090,	30048	42 139	7575,	35283	42 859	3666,	40122	43 789
15297,	26859	42 157	21176,	21724	42 901	18356,	25444	43 801
16191,	25977	42 169	9835,	33101	42 937	21296,	22570	43 867
8207,	33973	42 181	11569,	31373	42 943	209,	43681	43 891
3902,	38284	42 187	4411,	38549	42 961	9824,	34108	43 933
2763,	39429	42 193	4874,	38092	42 967	19510,	24440	43 951
5593,	36629	42 223	548,	42430	42 979	4598,	39364	43 963
6779,	35503	42 283	20031,	22971	43 003	1309,	42659	43 969
5591,	36715	42 307	11888,	31162	43 051	2726,	41260	43 987
19345,	22985	42 331	14423,	28639	43 063	16913,	27103	44 017
17506,	24830	42 337	6126,	36966	43 093	10234,	33794	44 029
4865,	37483	42 349	9558,	33558	43 117	5321,	38719	44 041
18343,	24029	42 373	15776,	27382	43 159	7628,	36424	44 053
13450,	28928	42 379	15074,	28102	43 177	5888,	38170	44 059
8289,	34101	42 391	11189,	31999	43 189	16690,	27380	44 071
5928,	36468	42 397	21420,	21780	43 201	18433,	25655	44 089
15572,	26830	42 403	4014,	39192	43 207	6827,	37273	44 101
7416,	34992	42 409	4153,	39083	43 237	17319,	26799	44 119
10921,	31511	42 433	12670,	30590	43 261	16867,	27263	44 131
13181,	29269	42 451	7180,	36110	43 291	9675,	34503	44 179

☞ *The roots (y) are placed to left of the Argument (p) throughout this Table.*

y y	p	y y	p	y y	p
4344, 39858	44 203	12776, 32404	45 181	5213, 40933	46 147
4221, 39999	44 221	20547, 24699	45 247	4652, 41500	46 153
17854, 26402	44 257	16071, 29187	45 259	18956, 27214	46 171
18366, 25896	44 263	15561, 29727	45 289	4998, 41184	46 183
1588, 42680	44 269	14425, 30881	45 307	8607, 37611	46 219
2072, 42208	44 281	12429, 32889	45 319	11505, 34731	46 237
8931, 35361	44 293	19599, 25737	45 337	7252, 39008	46 261
4953, 39417	44 371	13248, 32094	45 343	17325, 28947	46 273
1872, 42510	44 383	19625, 25735	45 361	20211, 26067	46 279
18322, 26066	44 389	5721, 39681	45 403	7874, 38434	46 309
1382, 43066	44 449	7161, 38265	45 427	13820, 32506	46 327
15322, 29168	44 491	19361, 26071	45 433	17783, 28567	46 351
19959, 24537	44 497	2225, 43213	45 439	776, 45604	46 381
11080, 33452	44 533	21162, 24318	45 481	1412, 44986	46 399
17713, 26849	44 563	564, 44958	45 523	13772, 32638	46 411
11682, 32904	44 587	20975, 24565	45 541	215, 46225	46 441
11663, 32953	44 617	7931, 37621	45 553	15410, 31036	46 447
15704, 28918	44 623	15267, 30321	45 589	17046, 29424	46 471
4241, 40399	44 641	11029, 34583	45 613	19949, 26527	46 477
16812, 27834	44 647	8520, 37110	45 631	20807, 25681	46 489
6674, 38008	44 683	770, 44896	45 667	18677, 27829	46 507
15938, 28762	44 701	4171, 41501	45 673	8052, 38496	46 549
2018, 42754	44 773	8396, 37294	45 691	3565, 43001	46 567
2847, 41949	44 797	5318, 40378	45 697	16459, 30113	46 573
2566, 42242	44 809	12214, 33536	45 751	2432, 44158	46 591
11440, 33398	44 839	370, 45386	45 757	22286, 24346	46 633
970, 43880	44 851	9078, 36684	45 763	1919, 44719	46 639
923, 43963	44 887	6273, 39543	45 817	6868, 39794	46 663
12294, 32598	44 893	7601, 38221	45 823	11475, 35205	46 681
7289, 37627	44 917	20677, 25163	45 841	16066, 30620	46 687
13102, 31850	44 953	9664, 36188	45 853	1203, 45519	46 723
14731, 30227	44 959	16423, 29519	45 943	21319, 25427	46 747
7530, 37440	44 971	1500, 44448	45 949	8898, 37872	46 771
13041, 31941	44 983	17982, 27996	45 979	22745, 24061	46 807
6181, 38825	45 007	16786, 29234	46 021	21532, 25286	46 819
8301, 36711	45 013	15740, 30286	46 027	10470, 36360	46 831
3494, 41566	45 061	1833, 44217	46 051	2496, 44364	46 861
22279, 22841	45 121	7587, 38505	46 093	17113, 29753	46 867
10933, 34193	45 127	7465, 38633	46 099	10405, 36527	46 933
2712, 42426	45 139	2071, 44069	46 141	5745, 41211	46 957

☞ *The roots* (y) *are placed to left of the Argument* (p) *throughout this Table.*

y y	p	y y	p	y y	p
13223, 33769	46 993	578, 47230	47 809	20851, 27929	48 781
16560, 30456	47 017	10475, 37381	47 857	3649, 45137	48 787
3150, 43890	47 041	11063, 36805	47 869	4401, 44397	48 799
13028, 34030	47 059	13875, 34005	47 881	1012, 47804	48 817
5614, 41504	47 119	3179, 44731	47 911	3833, 44989	48 823
17703, 29433	47 137	7035, 40881	47 917	15124, 33722	48 847
22319, 24823	47 143	789, 47157	47 947	12837, 36021	48 859
4850, 42298	47 149	9479, 38497	47 977	14430, 34440	48 871
16741, 30419	47 161	14813, 33235	48 049	12528, 36354	48 883
783, 46437	47 221	22881, 25191	48 073	6083, 42805	48 889
376, 46874	47 251	2850, 45228	48 079	1889, 47017	48 907
22597, 24671	47 269	18962, 29128	48 091	585, 48387	48 973
14264, 33022	47 287	10907, 37201	48 109	3723, 45267	48 991
21856, 25436	47 293	6518, 41602	48 121	7176, 41826	49 003
9578, 37738	47 317	21976, 26180	48 157	20564, 28444	49 009
7446, 39906	47 353	17323, 30839	48 163	9207, 39825	49 033
7097, 40291	47 389	16135, 32051	48 187	24235, 24821	49 057
3777, 43629	47 407	13882, 34310	48 193	16282, 32786	49 069
3504, 43914	47 419	18688, 29558	48 247	9860, 39220	49 081
18667, 28763	47 431	16829, 31429	48 259	2829, 46287	49 117
16839, 30651	47 491	8796, 39474	48 271	5433, 43689	49 123
10916, 36580	47 497	12412, 35900	48 313	11421, 37749	49 171
21789, 25731	47 521	5652, 42684	48 337	14791, 34385	49 177
23654, 23872	47 527	14221, 34175	48 397	10570, 38630	49 201
14916, 32616	47 533	4703, 43705	48 409	9303, 39903	49 207
11746, 35816	47 563	17189, 31273	48 463	15330, 33930	49 261
3300, 44268	47 569	5451, 43029	48 481	5784, 43494	49 279
1214, 46366	47 581	16989, 31497	48 487	5388, 43908	49 297
14301, 33297	47 599	2169, 46353	48 523	15590, 33742	49 333
9945, 37677	47 623	7717, 40823	48 541	20281, 29057	49 339
23625, 24003	47 629	20286, 28284	48 571	1387, 47975	49 363
4727, 42925	47 653	11761, 36827	48 589	10067, 39301	49 369
12772, 34886	47 659	11810, 36808	48 619	2341, 47051	49 393
4648, 43052	47 701	13630, 35018	48 649	23927, 25483	49 411
13306, 34406	47 713	5567, 43093	48 661	19606, 29810	49 417
4831, 42905	47 737	11915, 36757	48 673	8834, 40594	49 429
218, 47524	47 743	16491, 32187	48 679	17193, 32265	49 459
20445, 27333	47 779	14561, 34171	48 733	16688, 32788	49 477
21853, 25937	47 791	10503, 38247	48 751	21802, 27728	49 531
5500, 42296	47 797	18993, 29763	48 757	385, 49151	49 537

☞ *The roots* (y) *are placed to left of the Argument* (p) *throughout this Table.*

y	y	p	y	y	p	y	y	p
7133,	42415	49 549	2164,	48212	50 377	6726,	44514	51 241
18055,	31541	49 597	23732,	26650	50 383	17366,	33916	51 283
17390,	32212	49 603	11238,	39222	50 461	25134,	26172	51 307
10427,	39199	49 627	23663,	26833	50 497	599,	50743	51 343
19072,	30560	49 633	13719,	36783	50 503	6316,	45032	51 349
2530,	47108	49 639	15529,	34997	50 527	15849,	35511	51 361
16286,	33376	49 663	6881,	43657	50 539	20971,	30449	51 421
23771,	25897	49 669	4879,	45671	50 551	9735,	41691	51 427
7568,	42112	49 681	21351,	29229	50 581	21053,	30385	51 439
11971,	37739	49 711	6553,	44033	50 587	19519,	31961	51 481
12379,	37361	49 741	16339,	34253	50 593	12773,	38713	51 487
24002,	25744	49 747	980,	49618	50 599	23880,	27630	51 511
22475,	27307	49 783	6487,	44159	50 647	10423,	41093	51 517
14110,	35678	49 789	8012,	42658	50 671	6475,	45101	51 577
17261,	32539	49 801	17535,	33147	50 683	19277,	32329	51 607
5200,	44606	49 807	4520,	46186	50 707	9742,	41870	51 613
22953,	26877	49 831	17340,	33426	50 767	6276,	45354	51 631
8533,	41309	49 843	2352,	48420	50 773	21496,	30140	51 637
3717,	46173	49 891	11532,	39288	50 821	3499,	48173	51 673
21826,	28094	49 921	20452,	30380	50 833	20316,	31362	51 679
12578,	37348	49 927	5582,	45256	50 839	6337,	45353	51 691
18775,	31163	49 939	17027,	33829	50 857	9683,	42037	51 721
13412,	36544	49 957	3545,	47347	50 893	25138,	26630	51 769
14646,	35346	49 993	14929,	35993	50 923	21683,	30103	51 787
2334,	47664	49 999	16557,	34371	50 929	23004,	28812	51 817
2520,	47502	50 023	8359,	42611	50 971	18151,	33677	51 829
23301,	26745	50 047	4510,	46478	50 989	19547,	32305	51 853
7423,	42629	50 053	597,	50403	51 001	17085,	34773	51 859
4948,	45128	50 077	7046,	43984	51 031	23632,	28238	51 871
2750,	47350	50 101	16520,	34522	51 043	9473,	42433	51 907
16123,	33995	50 119	17478,	33582	51 061	17030,	34882	51 913
4910,	45220	50 131	16022,	35086	51 109	12246,	39702	51 949
19808,	30412	50 221	7836,	43296	51 133	24444,	27528	51 973
4853,	45373	50 227	5092,	46058	51 151	9337,	42653	51 991
3178,	47084	50 263	815,	50341	51 157	20368,	31640	52 009
13287,	36999	50 287	24140,	27028	51 169	21326,	30694	52 021
388,	49922	50 311	8223,	42969	51 193	5488,	46538	52 027
22302,	28026	50 329	9511,	41687	51 199	1045,	51005	52 051
18270,	32070	50 341	15513,	35703	51 217	19941,	32115	52 057
23885,	26473	50 359	3349,	47879	51 229	24632,	27436	52 069

Least Roots (y) *of* $(y^3-1)\div(y-1)\equiv 0 \pmod{p^\kappa}$.

☞ *The roots* (y) *are placed to left of the Argument* (p^κ) *in this Table.*

y	y	p (modulus $= p^2$)
3445,	7163	103
499,	11381	109
4972,	11156	127
6158,	13162	139
7733,	15067	151
6895,	17753	157
710,	25858	163
313,	32447	181
17454,	19794	193
13439,	26161	199
16894,	27626	211
24569,	25159	223
14103,	38337	229
16854,	41226	241
15475,	57965	271
5656,	71072	277
5421,	74667	283
35015,	59233	307
15864,	82104	313

y	y	p (modulus $= p^2$)
20884,	88676	331
23798,	89770	337
48284,	73516	349
29076,	105612	367
44848,	94280	373
43257,	100383	379
27427,	130181	397
9353,	157927	409
64854,	112386	421
89829,	97659	433
22560,	170160	439
84411,	124437	457
64798,	149570	463
78152,	159016	487
67005,	181995	499

y	y		p
6287,	23503	mod $= p^3$	31
14271,	36381	mod $= p^3$	37
34707,	44799	mod $= p^3$	43
7627,	20933	mod $= p^\kappa$	13^4
1353,	15453	mod $= p^\kappa$	7^5

Least Roots (y) *of* $(y^6+1)\div(y^2+1)\equiv 0$ (mod p^κ).

	p	y	y	y	y
modulus $= p^2$	109	2935,	3202,	8679,	8946
	157	892,	11910,	12739,	23757
	181	1241,	4699,	28062,	31520
	193	10085,	15184,	22065,	27164
	229	3117,	13493,	38948,	49324
	241	16625,	26450,	31631,	41456

	p	y	y	y	y
modulus	277^2	11729,	19632,	57097,	65000
	313^2	13801,	21651,	76318,	84168
	37^3	2265,	7201,	43452,	48388
	13^4	4812,	5051,	23510,	23749

Least Roots (y) *of* $(y^{12}+1)\div(y^4+1)\equiv 0$ (*mod* p^κ).

	p	y	y	y	y	y	y	y	y
mod $= p^2$	193	7346,	13372,	15240,	18158,	19091,	22009,	23877,	29903
	241	3728,	10365,	11930,	15938,	42143,	46151,	47716,	54353
	313	2640,	42713,	45428,	47132,	50837,	52541,	55256,	95329

p	y	y	y	y	p	y	y	y	y
13	2,	6,	7,	11	1 093	241,	322,	771,	852
37	8,	14,	23,	29	1 117	11,	203,	914,	1106
61	21,	29,	32,	40	1 129	298,	466,	663,	831
73	3,	24,	49,	70	1 153	53,	87,	1066,	1100
97	6,	16,	81,	91	1 201	307,	356,	845,	894
109	8,	41,	68,	101	1 213	47,	542,	671,	1166
157	22,	50,	107,	135	1 237	175,	516,	721,	1062
181	7,	26,	155,	174	1 249	34,	551,	698,	1215
193	49,	63,	130,	144	1 297	170,	206,	1091,	1127
229	18,	89,	140,	211	1 321	32,	289,	1032,	1289
241	4,	60,	181,	237	1 381	116,	250,	1131,	1265
277	35,	95,	182,	242	1 429	128,	681,	748,	1301
313	29,	54,	259,	284	1 453	60,	557,	896,	1393
337	72,	117,	220,	265	1 489	22,	203,	1286,	1467
349	24,	160,	189,	325	1 549	496,	584,	965,	1053
373	69,	173,	200,	304	1 597	285,	325,	1272,	1312
397	157,	177,	220,	240	1 609	421,	665,	944,	1188
409	49,	192,	217,	360	1 621	89,	255,	1366,	1532
421	159,	188,	233,	262	1 657	129,	745,	912,	1528
433	64,	115,	318,	369	1 669	297,	517,	1152,	1372
457	18,	127,	330,	439	1 693	704,	796,	897,	989
541	216,	268,	273,	325	1 741	112,	171,	1570,	1629
577	57,	81,	496,	520	1 753	44,	757,	996,	1709
601	5,	120,	481,	596	1 777	425,	577,	1200,	1352
613	142,	177,	436,	471	1 789	146,	870,	919,	1643
661	246,	309,	352,	415	1 801	258,	719,	1082,	1543
673	16,	42,	631,	657	1 861	160,	221,	1640,	1701
709	91,	187,	522,	618	1 873	267,	470,	1403,	1606
733	113,	240,	493,	620	1 933	277,	321,	1612,	1656
757	78,	165,	592,	679	1 993	41,	875,	1118,	1952
769	19,	81,	688,	750	2 017	765,	994,	1023,	1252
829	77,	323,	506,	752	2 029	359,	633,	1396,	1670
853	98,	235,	618,	755	2 053	849,	960,	1093,	1204
877	240,	391,	486,	637	2 089	54,	735,	1354,	2035
937	333,	408,	529,	604	2 113	1001,	1047,	1066,	1112
997	91,	252,	745,	906	2 137	44,	340,	1797,	2093
1 009	160,	309,	700,	849	2 161	731,	878,	1283,	1430
1 021	171,	203,	818,	850	2 221	317,	1107,	1114,	1904
1 033	14,	369,	664,	1019	2 269	178,	1109,	1160,	2091
1 069	34,	283,	786,	1035	2 281	727,	844,	1437,	1554

p	y	y	y	y	p	y	y	y	y
2 293	113,	487,	1806,	2180	3 613	1468,	1553,	2060,	2145
2 341	666,	819,	1522,	1675	3 637	913,	1697,	1940,	2724
2 377	74,	1060,	1317,	2303	3 673	888,	1791,	1882,	2785
2 389	467,	752,	1637,	1922	3 697	297,	834,	2863,	3400
2 437	110,	288,	2149,	2327	3 709	138,	1747,	1962,	3571
2 473	137,	704,	1769,	2336	3 733	420,	1271,	2462,	3313
2 521	26,	97,	2424,	2495	3 769	402,	1847,	1922,	3367
2 557	604,	1215,	1342,	1953	3 793	386,	1189,	2604,	3407
2 593	295,	1213,	1380,	2298	3 853	442,	863,	2990,	3411
2 617	481,	1148,	1469,	2136	3 877	15,	517,	3360,	3862
2 677	626,	1176,	1501,	2051	3 889	955,	1409,	2480,	2934
2 689	148,	1290,	1399,	2541	4 021	47,	770,	3251,	3974
2 713	191,	696,	2017,	2522	4 057	252,	1948,	2109,	3805
2 749	671,	1311,	1438,	2078	4 093	1485,	1549,	2544,	2608
2 797	411,	1014,	1783,	2386	4 129	136,	759,	3370,	3993
2 833	498,	859,	1974,	2335	4 153	1059,	1451,	2702,	3094
2 857	226,	670,	2187,	2631	4 177	1346,	1803,	2374,	2831
2 917	1193,	1247,	1670,	1724	4 201	141,	1013,	3188,	4060
2 953	404,	1323,	1630,	2549	4 261	611,	1332,	2929,	3650
3 001	281,	1367,	1634,	2720	4 273	604,	1804,	2469,	3669
3 037	73,	208,	2829,	2964	4 297	361,	1964,	2333,	3936
3 049	108,	367,	2682,	2941	4 357	86,	152,	4205,	4271
3 061	50,	551,	2510,	3011	4 441	584,	1711,	2730,	3857
3 109	164,	891,	2218,	2945	4 513	609,	704,	3809,	3904
3 121	1171,	1250,	1871,	1950	4 549	273,	1533,	3016,	4276
3 169	79,	1404,	1765,	3090	4 561	178,	2178,	2383,	4383
3 181	21,	303,	2878,	3160	4 597	287,	1842,	2755,	4310
3 217	197,	1584,	1633,	3020	4 621	42,	110,	4511,	4579
3 229	817,	1573,	1656,	2412	4 657	75,	1987,	2670,	4582
3 253	349,	1249,	2004,	2904	4 729	147,	1512,	3217,	4582
3 301	290,	922,	2379,	3011	4 789	1108,	2200,	2589,	3681
3 313	133,	274	3039,	3180	4 801	539,	864,	3937,	4262
3 361	421,	479,	2882,	2940	4 813	723,	1145,	3668,	4090
3 373	848,	1420,	1953,	2525	4 861	1715,	2208,	2653,	3146
3 433	389,	1262,	2171,	3044	4 909	1357,	1939,	2970,	3552
3 457	248,	460,	2997,	3209	4 933	180,	1014,	3919,	4753
3 469	351,	1354,	2115,	3118	4 957	1333,	1692,	3265,	3624
3 517	384,	980,	2537,	3133	4 969	1376,	2452,	2517,	3593
3 529	695,	1503,	2026,	2834	4 993	554,	712,	4281,	4439
3 541	694,	1546,	1995,	2847	5 077	1507,	2365,	2712,	3570

p	y	y	y	y	p	y	y	y	y
5 101	117,	218,	4883,	4984	6 553	999,	2368,	4185,	5554
5 113	611,	2477,	2636,	4502	6 577	1073,	2697,	3880,	5504
5 197	513,	2482,	2715,	4684	6 637	1180,	1648,	4989,	5457
5 209	496,	2594,	2615,	4713	6 661	1071,	1729,	4932,	5590
5 233	323,	2576,	2657,	4910	6 673	621,	1816,	4857,	6052
5 281	1153,	2455,	2826,	4128	6 709	377,	2527,	4182,	6332
5 413	35,	464,	4949,	5378	6 733	1008,	1209,	5524,	5725
5 437	799,	1429,	4008,	4638	6 781	2321,	3316,	3465,	4460
5 449	603,	1238,	4211,	4846	6 793	75,	634,	6159,	6718
5 521	613,	1378,	4143,	4908	6 829	1528,	3124,	3705,	5301
5 557	829,	1649,	3908,	4728	6 841	53,	1678,	5163,	6788
5 569	609,	1582,	3987,	4960	6 949	2813,	3204,	3745,	4136
5 581	627,	810,	4771,	4954	6 961	383,	727,	6234,	6578
5 641	267,	1162,	4479,	5374	6 997	2195,	3006,	3991,	4802
5 653	2063,	2373,	3280,	3590	7 057	1850,	1934,	5123,	5207
5 689	789,	2776,	2913,	4900	7 069	1794,	1982,	5087,	5275
5 701	370,	755,	4946,	5331	7 129	1313,	1580,	5549,	5816
5 737	1225,	2351,	3386,	4512	7 177	96,	1869,	5308,	7081
5 749	1526,	2332,	3417,	4223	7 213	825,	2824,	4389,	6388
5 821	1798,	2781,	3040,	4023	7 237	141,	2361,	4876,	7096
5 857	381,	1691,	4166,	5476	7 297	1718,	2026,	5271,	5579
5 869	288,	754,	5115,	5581	7 309	1158,	1559,	5750,	6151
5 881	1666,	2764,	3117,	4215	7 321	663,	784,	6537,	6658
5 953	1107,	1296,	4657,	4846	7 333	672,	2237,	5096,	6661
6 037	232,	2420,	3617,	5805	7 369	939,	1546,	5823,	6430
6 073	186;	2710,	3363,	5887	7 393	2473,	2559,	4834,	4920
6 121	810,	2728,	3393,	5311	7 417	1243,	3437,	3980,	6174
6 133	72,	937,	5196,	6061	7 477	1754,	3406,	4071,	5723
6 217	1202,	2643,	3574,	5015	7 489	761,	2352,	5137,	6728
6 229	812,	2263,	3966,	5417	7 537	537,	1586,	5951,	7000
6 277	63,	1096,	5181,	6214	7 549	23,	2954,	4595,	7526
6 301	864,	1320,	4981,	5437	7 561	1355,	1568,	5993,	6206
6 337	2982,	3160,	3177,	3355	7 573	1353,	2390,	5183,	6220
6 361	1723,	2887,	3474,	4638	7 621	2406,	3177,	4444,	5215
6 373	136,	2015,	4358,	6237	7 669	379,	2671,	4998,	7290
6 397	611,	691,	5706,	5786	7 681	1991,	2307,	5374,	5690
6 421	354,	1179,	5242,	6067	7 717	205,	2748,	4969,	7512
6 469	1601,	1891,	4578,	4868	7 741	2227,	2315,	5426,	5514
6 481	9,	720,	5761,	6472	7 753	1071,	1484,	6269,	6682
6 529	966,	1345,	5184,	5563	7 789	385,	3763,	4026,	7404

p	y	y	y	y	p	y	y	y	y
7 873	822,	2768,	5105,	7051	9 277	3490,	4378,	4899,	5787
7 933	318,	3268,	4665,	7615	9 337	1052,	2352,	6985,	8285
7 993	2832,	3051,	4942,	5161	9 349	73,	3714,	5635,	9276
8 017	2407,	3797,	4220,	5610	9 397	1399,	3251,	6146,	7998
8 053	2273,	2643,	5410,	5780	9 421	478,	2779,	6642,	8943
8 089	1387,	3680,	4409,	6702	9 433	3322,	4336,	5097,	6111
8 101	2905,	2995,	5106,	5196	9 601	153,	251,	9350,	9448
8 161	984,	1186,	6975,	7177	9 613	1475,	4712,	4901,	8138
8 209	1696,	3635,	4574,	6513	9 649	624,	1469,	8180,	9025
8 221	406,	4070,	4151,	7815	9 661	1958,	2097,	7564,	7703
8 233	2648,	3504,	4729,	5585	9 697	1496,	3053,	6644,	8201
8 269	1723,	2366,	5903,	6546	9 721	432,	4838,	4883,	9289
8 293	2167,	2698,	5595,	6126	9 733	2997,	4027,	5706,	6736
8 317	98,	1273,	7044,	8219	9 769	356,	4418,	5351,	9413
8 329	2158,	3601,	4728,	6171	9 781	2059,	3254,	6527,	7722
8 353	271,	2959,	5394,	8082	9 817	1548,	4027,	5790,	8269
8 377	226,	556,	7821,	8151	9 829	2321,	3625,	6204,	7508
8 389	783,	4157,	4232,	7606	9 901	10,	990,	8911,	9891
8 461	847,	939,	7522,	7614	9 949	1359,	3902,	6047,	8590
8 521	45,	2651,	5870,	8476	9 973	1942,	4740,	5233,	8031
8 581	4058,	4189,	4392,	4523					
8 629	941,	3182,	5447,	7688					
8 641	3262,	3796,	4845,	5379					
8 677	1533,	3181,	5496,	7144					
8 689	1066,	3562,	5127,	7623					
8 713	1187,	2092,	6621,	7526					
8 737	1107,	3157,	5580,	7630					
8 761	3261,	3729,	5032,	5500					
8 821	170,	467,	8354,	8651					
8 893	1310,	1541,	7352,	7583					
8 929	1120,	2304,	6625,	7809					
8 941	465,	2615,	6326,	8476					
9 001	396,	841,	8160,	8605					
9 013	1448,	3106,	5907,	7565					
9 049	1667,	3029,	6020,	7382					
9 109	2200,	4186,	4923,	6909					
9 133	280,	4012,	5121,	8853					
9 157	2855,	4099,	5058,	6302					
9 181	2754,	3057,	6124,	6427					
9 241	490,	1339,	7902,	8751					

p^κ	y	y	y	y
13^2	19,	80,	89,	150
37^2	356,	473,	896,	1013
13^3	418,	657,	1540,	1779
61^2	936,	1618,	2103,	2785
73^2	368,	1144,	4185,	4961
97^2	382,	3670,	5739,	9027

p	y	y	y	y	p	y	y	y	y
10 009	1470,	4773,	5236,	8539	11 701	3787,	4233,	7468,	7914
10 069	536,	4903,	5166,	9533	11 821	973,	3098,	8723,	10848
10 093	3003,	4702,	5391,	7090	11 833	96,	2835,	8998,	11737
10 141	315,	998,	9143,	9826	11 941	2568,	2962,	8979,	9373
10 177	1321,	4607,	5570,	8856	11 953	1502,	3605,	8348,	10451
10 273	1561,	2152,	8121,	8712	12 037	602,	4019,	8018,	11435
10 321	2528,	4642,	5679,	7793	12 049	365,	3004,	9045,	11684
10 333	3513,	3762,	6571,	6820	12 073	940,	3121,	8952,	11133
10 357	918,	3599,	6758,	9439	12 097	5057,	5167,	6930,	7040
10 369	916,	3362,	7007,	9453	12 109	3286,	4750,	7359,	8823
10 429	2879,	3358,	7071,	7550	12 157	5191,	5897,	6260,	6966
10 453	531,	2441,	8012,	9922	12 241	2446,	3338,	8903,	9795
10 477	3939,	4527,	5950,	6538	12 253	564,	2846,	9407,	11689
10 501	3742,	5026,	5475,	6759	12 277	20,	4297,	7980,	12257
10 513	1982,	2127,	8386,	8531	12 289	79,	1400,	10889,	12210
10 597	2273,	3077,	7520,	8324	12 301	1249,	1497,	10804,	11052
10 657	121,	2378,	8279,	10536	12 373	1312,	4555,	7818,	11061
10 729	495,	4465,	6264,	10234	12 409	1506,	3403,	9006,	10903
10 753	600,	3889,	6864,	10153	12 421	479,	752,	11669,	11942
10 789	626,	4257,	6532,	10163	12 433	1022,	1545,	10888,	11411
10 837	3586,	3841,	6996,	7251	12 457	907,	4807,	7650,	11550
10 861	161,	2496,	8365,	10700	12 517	3740,	4642,	7875,	8777
10 909	730,	1330,	9579,	10179	12 541	1141,	1253,	11288,	11400
10 957	137,	3679,	7278,	10820	12 553	86,	4233,	8320,	12467
10 993	1998,	4121,	6872,	8995	12 577	5113,	5906,	6671,	7464
11 113	1586,	5052,	6061,	9527	12 589	782,	5232,	7357,	11807
11 149	3128,	4687,	6462,	8021	12 601	810,	5616,	6985,	11791
11 161	917,	3128,	8033,	10244	12 613	74,	1534,	11079,	12539
11 173	251,	3205,	7968,	10922	12 637	1289,	5794,	6843,	11348
11 197	2120,	5139,	6058,	9077	12 697	392,	1328,	11369,	12305
11 257	3254,	3795,	7462,	8003	12 721	1289,	4441,	8280,	11432
11 317	706,	4312,	7005,	10611	12 757	2074,	4361,	8396,	10683
11 329	2290,	2528,	8801,	9039	12 781	637,	4053,	8728,	12144
11 353	3471,	3732,	7621,	7882	12 829	1573,	4608,	8221,	11256
11 437	703,	4962,	6475,	10734	12 841	2450,	5823,	7018,	10391
11 497	403,	1997,	9500,	11094	12 853	384,	5255,	7598,	12469
11 593	4413,	5388,	6205,	7180	12 889	1723,	2693,	10196,	11166
11 617	1082,	4606,	7011,	10535	12 973	2439,	2718,	10255,	10534
11 677	1445,	1996,	9681,	10232	13 009	3017,	5834,	7175,	9992
11 689	1319,	5716,	5973,	10370	13 033	1968,	6192,	6841,	11065

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13 093	1364,	3734,	9359,	11729	14 737	4945,	5564,	9173,	9792
13 177	1480,	1843,	11334,	11697	14 797	5008,	5280,	9517,	9789
13 249	3073,	4001,	9248,	10176	14 821	4168,	6838,	7983,	10653
13 297	1542,	4234,	9063,	11755	14 869	419,	2697,	12172,	14450
13 309	954,	2832,	10477,	12355	14 929	1807,	4040,	10889,	13122
13 381	2829,	5482,	7899,	10552	15 013	3192,	6373,	8640,	11821
13 417	2778,	3801,	9616,	10639	15 061	3336,	3842,	11219,	11725
13 441	4217,	4883,	8558,	9224	15 073	4137,	5600,	9473,	10936
13 477	714,	6663,	6814,	12763	15 121	557,	5348,	9773,	14564
13 513	2603,	3229,	10284,	10910	15 193	319,	6001,	9192,	14874
13 537	616,	901,	12636,	12921	15 217	4356,	4985,	10232,	10861
13 597	4409,	6578,	7019,	9188	15 241	5100,	6458,	8783,	10141
13 633	2479,	4548,	9085,	11154	15 277	2943,	3561,	11716,	12334
13 669	309,	5618,	8051,	13360	15 289	2134,	7100,	8189,	13155
13 681	4259,	5766,	7915,	9422	15 313	914,	1089,	14224,	14399
13 693	2041,	4864,	8829,	11652	15 349	3253,	7021,	8328,	12096
13 729	446,	708,	13021,	13283	15 361	1500,	2468,	12893,	13861
13 789	561,	3564,	10225,	13228	15 373	6219,	6343,	9030,	9154
13 873	2421,	5375,	8498,	11452	15 493	741,	6607,	8886,	14752
13 921	5334,	5452,	8469,	8587	15 541	6963,	7477,	8064,	8578
13 933	4305,	5023,	8910,	9628	15 601	726,	4878,	10723,	14875
14 029	4299,	5391,	8638,	9730	15 649	101,	2634,	13015,	15548
14 149	210,	6805,	7344,	13939	15 661	3807,	6438,	9223,	11854
14 173	5154,	7081,	7092,	9019	15 733	5265,	6344,	9389,	10468
14 197	139,	2247,	11950,	14058	15 817	1269,	6257,	9560,	14548
14 221	4001,	5623,	8598,	10220	15 877	3596,	3722,	12155,	12281
14 281	5830,	5999,	8282,	8451	15 889	6420,	7155,	8734,	9469
14 293	3522,	5020,	9273,	10771	15 901	4752,	7907,	7994,	11149
14 341	2556,	3978,	10363,	11785	15 913	6379,	6688,	9225,	9534
14 389	4009,	5172,	9217,	10380	15 937	4528,	6536,	9401,	11409
14 401	5043,	5163,	9238,	9358	15 973	1982,	6826,	9147,	13991
14 437	3604,	5500,	8937,	10833	16 033	1806,	4643,	11390,	14227
14 449	128,	1919,	12530,	14321	16 057	4998,	7598,	8459,	11059
14 461	2264,	3034,	11427,	12197	16 069	1366,	2388,	13681,	14703
14 533	3815,	6217,	8316,	10718	16 141	1038,	7915,	8226,	15103
14 557	258,	4006,	10551,	14299	16 189	2667,	3636,	12553,	13522
14 593	3156,	6774,	7819,	11437	16 249	2829,	4348,	11901,	13420
14 629	2899,	4476,	10153,	11730	16 273	1564,	7918,	8355,	14709
14 653	4743,	5564,	9089,	9910	16 333	3251,	5235,	11098,	13082
14 713	2308,	3691,	11022,	12405	16 369	180,	4456,	11913,	16189

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16 381	8036,	8164,	8217,	8345	18 013	1805,	6816,	11197,	16208
16 417	713,	4559,	11858,	15704	18 049	613,	4446,	13603,	17436
16 453	800,	1707,	14746,	15653	18 061	1979,	8597,	9464,	16082
16 477	886,	5077,	11400,	15591	18 097	4589,	7686,	10411,	13508
16 561	6310,	6774,	9787,	10251	18 121	4149,	5062,	13059,	13972
16 573	2810,	6352,	10221,	13763	18 133	1352,	3581,	14552,	16781
16 633	877,	7207,	9426,	15756	18 169	2294,	2780,	15389,	15875
16 657	208,	3924,	12733,	16449	18 181	2641,	3862,	14319,	15540
16 693	3254,	8090,	8603,	13439	18 217	1444,	7254,	10963,	16773
16 729	4942,	6347,	10382,	11787	18 229	1136,	7911,	10318,	17093
16 741	3983,	5670,	11071,	12758	18 253	2625,	4638,	13615,	15628
16 921	2403,	5584,	11337,	14518	18 289	57,	2246,	16043,	18232
16 981	5350,	7843,	9138,	11631	18 301	788,	7548,	10753,	17513
16 993	4236,	6936,	10057,	12757	18 313	1583,	5148,	13165,	16730
17 029	3315,	6339,	10690,	13714	18 397	2880,	8515,	9882,	15517
17 041	1343,	6319,	10722,	15698	18 433	1182,	5349,	13084,	17251
17 053	1286,	1578,	15475,	15767	18 457	3750,	9071,	9386,	14707
17 077	5980,	7559,	9518,	11097	18 481	3872,	4205,	14276,	14609
17 137	3423,	3895,	13242,	13714	18 493	5194,	5330,	13163,	13299
17 209	1337,	7504,	9705,	15872	18 517	4656,	6057,	12460,	13861
17 257	7402,	8428,	8829,	9855	18 541	884,	5642,	12899,	17657
17 293	4006,	5996,	11297,	13287	18 553	618,	2852,	15701,	17935
17 317	4784,	5455,	11862,	12533	18 637	1316,	2450,	16187,	17321
17 341	954,	3181,	14160,	16387	18 661	6152,	9285,	9376,	12509
17 377	3319,	4822,	12555,	14058	18 757	3637,	5673,	13084,	15120
17 389	5871,	6288,	11101,	11518	18 793	1458,	2101,	16692,	17335
17 401	5534,	7078,	10323,	11867	18 913	2619,	5250,	13663,	16294
17 449	3203,	7240,	10209,	14246	18 973	8325,	8633,	10340,	10648
17 497	1558,	5020,	12477,	15939	19 009	367,	7096,	11913,	18642
17 509	220,	2308,	15201,	17289	19 069	1347,	8947,	10122,	17722
17 569	1058,	5098,	12471,	16511	19 081	7134,	9150,	9931,	11947
17 581	2674,	5582,	11999,	14907	19 141	3987,	7883,	11258,	15154
17 713	5785,	7768,	9945,	11928	19 213	2100,	5279,	13934,	17113
17 737	2028,	6096,	11641,	15709	19 237	5018,	7211,	12026,	14219
17 749	1143,	6553,	11196,	16606	19 249	656,	6074,	13175,	18593
17 761	7102,	7400,	10361,	10659	19 273	4071,	8924,	10349,	15202
17 881	2594,	6197,	11684,	15287	19 309	2511,	7213,	12096,	16798
17 929	2078,	3943,	13986,	15851	19 333	1992,	7114,	12219,	17341
17 977	6427,	8892,	9085,	11550	19 381	3052,	3626,	15755,	16329
17 989	2970,	5748,	12241,	15019	19 417	7111,	8462,	10955,	12306

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19 429	1931,	3592,	15837,	17498	21 121	5697,	6221,	14900,	15424
19 441	3421,	8081,	11360,	16020	21 157	8540,	10140,	11017,	12617
19 477	3452,	4542,	14935,	16025	21 169	8186,	9938,	11231,	12983
19 489	562,	2046,	17443,	18927	21 193	835,	10330,	10863,	20358
19 501	5598,	6253,	13248,	13903	21 277	3841,	4775,	16502,	17436
19 597	7318,	7458,	12139,	12279	21 313	2444,	2590,	18723,	18869
19 609	4921,	8105,	11504,	14688	21 397	1815,	10056,	11341,	19582
19 681	7042,	8281,	11400,	12639	21 433	1648,	2380,	19053,	19785
19 717	2820,	4286,	15431,	16897	21 481	1820,	9454,	12027,	19661
19 753	2278,	3824,	15929,	17475	21 493	8143,	9890,	11603,	13350
19 777	451,	7674,	12103,	19326	21 517	8259,	8587,	12930,	13258
19 801	807,	1006,	18795,	18994	21 529	2234,	5734,	15795,	19295
19 813	1703,	9668,	10145,	18110	21 577	6703,	7452,	14125,	14874
19 861	1745,	5338,	14523,	18116	21 589	5408,	6527,	15062,	16181
19 993	6811,	7770,	12223,	13182	21 601	4417,	5311,	16290,	17184
20 029	82,	9526,	10503,	19947	21 613	3119,	7761,	13852,	18494
20 089	4381,	7644,	12445,	15708	21 649	306,	10683,	10966,	21343
20 101	7245,	7910,	12191,	12856	21 661	1667,	6497,	15164,	19994
20 113	121,	1496,	18617,	19992	21 673	1748,	9857,	11816,	19925
20 149	1388,	7447,	12702,	18761	21 757	1022,	2065,	19692;	20735
20 161	3364,	3506,	16655,	16797	21 817	607,	1869,	19948,	21210
20 173	926,	7603,	12570,	19247	21 841	8192,	8401,	13440,	13649
20 233	8140,	8906,	11327,	12093	21 937	2793,	5875,	16062,	19144
20 269	4781,	5647,	14622,	15488	21 961	2640,	3003,	18958,	19321
20 341	1608,	5022,	15319,	18733	21 997	3162,	4014,	17983,	18835
20 353	1428,	8908,	11445,	18925	22 093	7690,	10696,	11397,	14403
20 389	754,	1974,	18415,	19635	22 129	1092,	10720,	11409,	21037
20 509	397,	2583,	17926,	20112	22 153	7001,	9651,	12502,	15152
20 521	8017,	8470,	12051,	12504	22 189	2381,	8788,	13401,	19808
20 533	5445,	5796,	14737,	15088	22 273	1355,	7512,	14761,	20918
20 593	12,	1716,	18877,	20581	22 369	5212,	7824,	14545,	17157
20 641	4037,	9551,	11090,	16604	22 381	8275,	9185,	13196,	14106
20 749	3021,	5474,	15275,	17728	22 441	7156,	8326,	14115,	15285
20 773	7811,	8973,	11800,	12962	22 453	4306,	5595,	16858,	18147
20 809	4451,	9944,	10865,	16358	22 501	10058,	10208,	12293,	12443
20 857	2161,	4044,	16813,	18696	22 549	2726,	9190,	13359,	19823
20 929	2334,	4349,	16580,	18595	22 573	7621,	8640,	13933,	14952
21 001	2138,	9420,	11581,	18863	22 621	7787,	8651,	13970,	14834
21 013	5187,	5392,	15621,	15826	22 669	3146,	6622,	16047,	19523
21 061	4363,	7849,	13212,	16698	22 717	6360,	10762,	11955,	16357

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22 741	1497,	8826,	13915,	21244	24 181	8824,	10597,	13584,	15357
22 777	586,	5325,	17452,	22191	24 229	578,	5156,	19073,	23651
22 861	4132,	6689,	16172,	18729	24 337	4288,	4444,	19893,	20049
22 921	1628,	8884,	14037,	21293	24 373	238,	2765,	21608,	24135
22 993	478,	1876,	21117,	22515	24 421	8152,	8373,	16048,	16269
23 017	4353,	6747,	16270,	18664	24 469	1390,	1954,	22515,	23079
23 029	3623,	5619,	17410,	19406	24 481	2529,	6960,	17521,	21952
23 041	1398,	7532,	15509,	21643	24 517	3804,	11672,	12845,	20713
23 053	3781,	10176,	12877,	19272	24 697	2115,	8606,	16091,	22582
23 173	2841,	11248,	11925,	20332	24 709	433,	11470,	13239,	24276
23 197	4344,	11465,	11732,	18853	24 733	2299,	9564,	15169,	22434
23 209	5386,	7123,	16086,	17823	24 781	5683,	6035,	18746,	19098
23 269	1158,	1708,	21561,	22111	24 793	651,	2704,	22089,	24142
23 293	416,	7783,	15510,	22877	24 841	2973,	8155,	16686,	21868
23 473	162,	2753,	20720,	23311	24 877	8000,	11857,	13020,	16877
23 497	5426,	9999,	13498,	18071	24 889	3911,	11054,	13835,	20978
23 509	3778,	10846,	12663,	19731	25 033	6690,	8842,	16191,	18343
23 557	782,	3886,	19671,	22775	25 057	600,	8060,	16997,	24457
23 581	4206,	8606,	14975,	19375	25 117	2965,	8666,	16451,	22152
23 593	10423,	11191,	12402,	13170	25 153	10691,	12253,	12900,	14462
23 629	5084,	5805,	17824,	18545	25 189	7979,	11327,	13862,	17210
23 677	3832,	10646,	13031,	19845	25 237	4186,	8760,	16477,	21051
23 689	1645,	9490,	14199,	22044	25 261	3760,	6846,	18415,	21501
23 761	4672,	10014,	13747,	19089	25 309	2964,	5832,	19477,	22345
23 773	3960,	9143,	14630,	19813	25 321	6347,	9826,	15495,	18974
23 833	931,	7987,	15846,	22902	25 357	2029,	9223,	16134,	23328
23 857	824,	4893,	18964,	23033	25 453	5356,	9015,	16438,	20097
23 869	3026,	8590,	15279,	20843	25 537	344,	1262,	24275,	25193
23 893	1754,	2847,	21046,	22139	25 561	8083,	9449,	16112,	17478
23 917	3154,	7166,	16751,	20763	25 609	2305,	10788,	14821,	23304
23 929	213,	3932,	19997,	23716	25 621	10494,	12581,	13040,	15127
23 977	2081,	3952,	20025,	21896	25 633	7586,	7944,	17689,	18047
24 001	6914,	9758,	14243,	17087	25 657	776,	3141,	22516,	24881
24 049	234,	5447,	18602,	23815	25 693	822,	5845,	19848,	24871
24 061	2273,	6309,	17752,	21788	25 717	673,	3057,	22660,	25044
24 097	3281,	6632,	17465,	20816	25 741	5236,	5629,	20112,	20505
24 109	551,	8751,	15358,	23558	25 801	2630,	10703,	15098,	23171
24 121	5375,	10519,	13602,	18746	25 849	487,	7484,	18365,	25362
24 133	6931,	8461,	15672,	17202	25 873	894,	7322,	18551,	24979
24 169	2412,	4439,	19730,	21757	25 933	4909,	5848,	20085,	21024

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25 969	2872,	7297,	18672,	23097	27 733	7499,	13554,	14179,	20234
25 981	4918,	10497,	15484,	21063	27 793	3897,	8273,	19520,	23896
26 017	10582,	12266,	13751,	15435	27 817	9873,	11346,	16471,	17944
26 029	6757,	11360,	14669,	19272	27 901	10731,	13944,	13957,	17170
26 041	3158,	6770,	19271,	22883	27 961	721,	12565,	15396,	27240
26 053	1626,	12674,	13379,	24427	27 997	5441,	7883,	20114,	22556
26 113	2009,	6525,	19588,	24104	28 057	6409,	10546,	17511,	21648
26 161	1227,	2324,	23837,	24934	28 069	3528,	12133,	15936,	24541
26 209	284,	646,	25563,	25925	28 081	6724,	10403,	17678,	21357
26 293	139,	3594,	22699,	26154	28 201	3147,	9015,	19186,	25054
26 317	7317,	7830,	18487,	19000	28 297	9951,	12239,	16058,	18346
26 437	1844,	4172,	22265,	24593	28 309	2773,	13690,	14619,	25536
26 449	2566,	7576,	18873,	23883	28 393	13,	2184,	26209,	28380
26 497	7124,	11638,	14859,	19373	28 429	2855,	9868,	18561,	25574
26 557	442,	12918,	13639,	26115	28 477	1845,	7208,	21269,	26632
26 641	8218,	10163,	16478,	18423	28 513	3161,	10247,	18266,	25352
26 701	6459,	10773,	15928,	20242	28 537	1709,	6913,	21624,	26828
26 713	622,	10522,	16191,	26091	28 549	4484,	9875,	18674,	24065
26 737	7772,	12966,	13771,	18965	28 573	567,	12044,	16529,	28006
26 821	2216,	7613,	19208,	24605	28 597	3677,	8415,	20182,	24920
26 833	7881,	8992,	17841,	18952	28 621	5750,	13534,	15087,	22871
26 881	12212,	12888,	13993,	14669	28 657	7254,	10457,	18200,	21403
26 893	12622,	12786,	14107,	14271	28 669	9183,	11442,	17227,	19486
26 953	7632,	13141,	13812,	19321	28 729	5118,	9436,	19293,	23611
27 061	916,	12142,	14919,	26145	28 753	1956,	14303,	14450,	26797
27 073	5774,	13077,	13996,	21299	28 789	9970,	12096,	16693,	18819
27 109	629,	3060,	24049,	26480	28 813	10759,	12169,	16644,	18054
27 241	3083,	9852,	17389,	24158	28 837	9897,	10434,	18403,	18940
27 253	7202,	13460,	13793,	20051	28 909	7675,	11541,	17368,	21234
27 277	5935,	13411,	13866,	21342	28 921	2057,	11051,	17870,	26864
27 337	3048,	3453,	23884,	24289	28 933	1262,	2178,	26755,	27671
27 361	4661,	7643,	19718,	22700	29 017	2789,	6617,	22400,	26228
27 397	788,	7336,	20061,	26609	29 077	2991,	5969,	23108,	26086
27 409	3667,	12019,	15390,	23742	29 101	2078,	3235,	25866,	27023
27 457	9278,	11580,	15877,	18179	29 137	12002,	13209,	15928,	17135
27 481	150,	5313,	22168,	27331	29 173	8915,	10249,	18924,	20258
27 529	2229,	11350,	16179,	25300	29 209	11138,	12795,	16414,	18071
27 541	7704,	11193,	16348,	19837	29 221	615,	8600,	20621,	28606
27 673	3169,	9431,	18242,	24504	29 269	4320,	13896,	15373,	24949
27 697	380,	3863,	23834,	27317	29 389	8170,	12011,	17378,	21219

p	y	y	y	y	p	y	y	y	y
29 401	2784,	11395,	18006,	26617	31 069	115,	1621,	29448,	30954
29 437	5275,	13661,	15776,	24162	31 081	245,	4567,	26514,	30836
29 473	5830,	9216,	20257,	23643	31 153	306,	8450,	22703,	30847
29 569	11425,	12045,	17524,	18144	31 177	400,	6781,	24396,	30777
29 581	5171,	5343,	24238,	24410	31 189	8987,	14038,	17151,	22202
29 629	1037,	9943,	19686,	28592	31 237	3998,	14134,	17103,	27239
29 641	4258,	12036,	17605,	25383	31 249	6131,	14888,	16361,	25118
29 761	8652,	11396,	18365,	21109	31 321	2476,	11777,	19544,	28845
29 833	11176,	14097,	15736,	18657	31 333	6260,	9495,	21838,	25073
29 881	2114,	9541,	20340,	27767	31 357	103,	4871,	26486,	31254
29 917	3412,	11460,	18457,	26505	31 393	10065,	13967,	17426,	21328
29 989	6254,	11916,	18073,	23735	31 477	5294,	8116,	23361,	26183
30 013	9686,	9931,	20082,	20327	31 489	928,	12996,	18493,	30561
30 097	1454,	3788,	26309,	28643	31 513	7197,	11761,	19752,	24316
30 109	12209,	12597,	17512,	17900	31 573	368,	13470,	18103,	31205
30 133	1503,	7017,	23116,	28630	31 657	8748,	12416,	19241,	22909
30 169	12197,	14455,	15714,	17972	31 729	9444,	11171,	20558,	22285
30 181	10996,	14440,	15741,	19185	31 741	8242,	11434,	20307,	23499
30 241	3812,	11622,	18619,	26429	31 849	4320,	7601,	24248,	27529
30 253	3668,	4726,	25527,	26585	31 873	2322,	3363,	28510,	29551
30 313	2449,	11437,	18876,	27864	31 957	12513,	15635,	16322,	19444
30 469	1867,	2807,	27662,	28602	31 981	3281,	14621,	17360,	28700
30 493	6634,	13031,	17462,	23859	32 029	3405,	12520,	19509,	28624
30 517	9703,	14760,	15757,	20814	32 077	2175,	7551,	24526,	29902
30 529	4135,	8461,	22068,	26394	32 089	10354,	11842,	20247,	21735
30 553	648,	7874,	22679,	29905	32 173	4902,	7213,	24960,	27271
30 577	3898,	11586,	18991,	26679	32 233	7642,	10996,	21237,	24591
30 637	401,	10085,	20552,	30236	32 257	8758,	10206,	22051,	23499
30 649	5189,	12386,	18263,	25460	32 341	4824,	14890,	17451,	27517
30 661	9681,	14762,	15899,	20980	32 353	1218,	15964,	16389,	31135
30 697	3098,	14318,	16379,	27599	32 377	602,	10380,	21997,	31775
30 757	5695,	13842,	16915,	25062	32 401	3106,	3286,	29115,	29295
30 781	1920,	10597,	20184,	28861	32 413	2784,	3225,	29188,	29629
30 817	3734,	7370,	23447,	27083	32 497	3912,	5574,	26923,	28585
30 829	4493,	8872,	21957,	26336	32 533	4010,	10198,	22335,	28523
30 841	3463,	5014,	25827,	27378	32 569	1391,	13627,	18942,	31178
30 853	9245,	10756,	20097,	21608	32 653	11317,	15924,	16729,	21336
30 937	5976,	10566,	20371,	24961	32 713	11803,	12777,	19936,	20910
30 949	4274,	14649,	16300,	26675	32 749	410,	16055,	16694,	32339
31 033	6646,	10156,	20877,	24387	32 797	1300,	6736,	26061,	31497

☞ *For solutions of* $(y^6+1)\div(y^2+1)\equiv 0$ (*mod* $p^\kappa > 10^4$), *see page* 70.

p	y	y	y	y	y	y	y	y
73	7,	17,	21,	30,	43,	52,	56,	66
97	4,	9,	24,	43,	54,	73,	88,	93
193	7,	12,	16,	55,	138,	177,	181,	186
241	2,	32,	113,	120,	121,	128,	209,	239
313	43,	131,	136,	145,	168,	177,	182,	270
337	54,	96,	156,	165,	172,	181,	241,	283
409	7,	38,	117,	183,	226,	292,	371,	402
433	8,	54,	133,	140,	293,	300,	379,	425
457	70,	111,	139,	217,	240,	318,	346,	387
577	9,	64,	195,	216,	361,	382,	513,	568
601	132,	214,	273,	295,	306,	328,	387,	469
673	4,	168,	232,	322,	351,	441,	505,	669
769	9,	164,	171,	211,	558,	598,	605,	760
937	23,	90,	163,	177,	760,	774,	847,	914
1 009	169,	203,	361,	450,	559,	648,	806,	840
1 033	135,	176,	407,	500,	533,	626,	857,	898
1 129	206,	231,	391,	422,	707,	738,	898,	923
1 153	324,	393,	399,	516,	637,	754,	760,	829
1 201	90,	253,	387,	394,	807,	814,	948,	1111
1 249	137,	209,	251,	547,	702,	998,	1040,	1112
1 297	61,	277,	398,	404,	893,	899,	1020,	1236
1 321	17,	218,	406,	544,	777,	915,	1103,	1304
1 489	67,	185,	200,	330,	1159,	1289,	1304,	1422
1 609	182,	255,	448,	610,	999,	1161,	1354,	1427
1 657	160,	399,	652,	756,	901,	1005,	1258,	1497
1 753	84,	290,	405,	480,	1273,	1348,	1463,	1669
1 777	121,	302,	406,	514,	1263,	1371,	1475,	1656
1 801	117,	347,	431,	846,	955,	1370,	1454,	1684
1 873	85,	410,	617,	836,	1037,	1256,	1463,	1788
1 993	463,	500,	570,	947,	1046,	1423,	1493,	1530
2 017	122,	248,	300,	316,	1701,	1717,	1769,	1895
2 089	358,	447,	531,	930,	1159,	1558,	1642,	1731
2 113	181,	537,	913,	1016,	1097,	1200,	1576,	1932
2 137	105,	160,	346,	975,	1162,	1791,	1977,	2032
2 161	157,	178,	234,	692,	1469,	1927,	1983,	2004
2 281	390,	696,	817,	899,	1382,	1464,	1585,	1891
2 377	134,	172,	408,	843,	1534,	1969,	2205,	2243
2 473	444,	498,	997,	1018,	1455,	1476,	1975,	2029
2 521	297,	908,	919,	1078,	1443,	1602,	1613,	2224
2 593	143,	272,	768,	969,	1624,	1825,	2321,	2450

p	y	y	y	y	y	y	y	y
2 617	513,	656,	755,	1121,	1496,	1862,	1961,	2104
2 689	132,	160,	521,	713,	1976,	2168,	2529,	2557
2 713	531,	591,	608,	1065,	1648,	2105,	2122,	2182
2 833	293,	981,	1262,	1402,	1431,	1571,	1852,	2540
2 857	560,	573,	852,	1072,	1785,	2005,	2284,	2297
2 953	876,	916,	1138,	1372,	1581,	1815,	2037,	2077
3 001	387,	853,	1276,	1437,	1564,	1725,	2148,	2614
3 049	155,	292,	449,	1495,	1554,	2600,	2757,	2894
3 121	212,	456,	1143,	1428,	1693,	1978,	2665,	2909
3 169	225,	239,	358,	1000,	2169,	2811,	2930,	2944
3 217	452,	762,	1032,	1085,	2132,	2185,	2455,	2765
3 313	216,	719,	1089,	1539,	1774,	2224,	2594,	3097
3 361	98,	128,	814,	926,	2435,	2547,	3233,	3263
3 433	480,	543,	1334,	1552,	1881,	2099,	2890,	2953
3 457	357,	395,	1164,	1346,	2111,	2293,	3062,	3100
3 529	255,	775,	1358,	1567,	1962,	2171,	2754,	3274
3 673	668,	821,	1794,	1831,	1842,	1879,	2852,	3005
3 697	973,	1243,	1502,	1839,	1858,	2195,	2454,	2724
3 769	917,	1326,	1418,	1623,	2146,	2351,	2443,	2852
3 793	306,	533,	610,	827,	2966,	3183,	3260,	3487
3 889	560,	1152,	1455,	1882,	2007,	2434,	2737,	3329
4 057	1013,	1307,	1622,	1760,	2297,	2435,	2750,	3044
4 129	704,	1657,	1682,	1695,	2434,	2447,	2472,	3425
4 153	1068,	1172,	1396,	1995,	2158,	2757,	2981,	3085
4 177	154,	631,	1549,	1980,	2197,	2628,	3546,	4023
4 201	835,	1454,	1561,	1717,	2484,	2640,	2747,	3366
4 273	258,	899,	1944,	2004,	2269,	2329,	3374,	4015
4 297	19,	1027,	1205,	1357,	2940,	3092,	3270,	4278
4 441	560,	1154,	1594,	1751,	2690,	2847,	3287,	3881
4 513	211,	385,	471,	1993,	2520,	4042,	4128,	4302
4 561	1306,	1596,	1739,	1912,	2649,	2822,	2965,	3255
4 657	129,	173,	361,	996,	3661,	4296,	4484,	4528
4 729	79,	137,	932,	2155,	2574,	3797,	4592,	4650
4 801	309,	1437,	1880,	1891,	2910,	2921,	3364,	4492
4 969	132,	293,	2069,	2221,	2748,	2900,	4676,	4837
4 993	485,	803,	1735,	2049,	2944,	3258,	4190,	4508
5 113	1021,	1873,	1918,	1930,	3183,	3195,	3240,	4092
5 209	322,	1320,	1614,	1828,	3381,	3595,	3889,	4887
5 233	124,	211,	820,	2023,	3210,	4413,	5022,	5109
5 281	97,	490,	1215,	1430,	3851,	4066,	4791,	5184

p	y	y	y	y	y	y	y	y
5 449	543,	621,	1518,	2007,	3442,	3931,	4828,	4906
5 521	510,	1613,	1841,	2759,	2762,	3680,	3908,	5011
5 569	483,	1153,	2163,	2500,	3069,	3406,	4416,	5086
5 641	526,	1401,	1984,	2287,	3354,	3657,	4240,	5115
5 689	70,	766,	894,	1270,	4419,	4795,	4923,	5619
5 737	35,	360,	749,	1967,	3770,	4988,	5377,	5702
5 857	74,	176,	2137,	2629,	3228,	3720,	5681,	5783
5 881	710,	1814,	1887,	2593,	3288,	3994,	4067,	5171
5 953	36,	1555,	1819,	2787,	3166,	4134,	4398,	5917
6 073	430,	1031,	1747,	3000,	3073,	4326,	5042,	5643
6 121	758,	806,	1074,	1329,	4792,	5047,	5315,	5363
6 217	190,	1125,	1407,	3056,	3161,	4810,	5092,	6027
6 337	333,	1903,	2241,	2873,	3464,	4096,	4434,	6004
6 361	1149,	1305,	1456,	1823,	4538,	4905,	5056,	5212
6 481	3,	243,	2160,	2187,	4294,	4321,	6238,	6478
6 529	72,	2267,	2779,	3167,	3362,	3750,	4262,	6457
6 553	511,	2161,	2257,	2902,	3651,	4296,	4392,	6042
6 577	1540,	1593,	1700,	2271,	4306,	4877,	4984,	5037
6 673	683,	850,	2820,	2894,	3779,	3853,	5823,	5990
6 793	943,	1901,	2795,	2873,	3920,	3998,	4892,	5850
6 841	149,	1056,	1091,	2690,	4151,	5750,	5785,	6692
6 961	355,	1001,	3178,	3255,	3706,	3783,	5960,	6606
7 057	139,	660,	1016,	2438,	4619,	6041,	6397,	6918
7 129	429,	479,	565,	1146,	5983,	6564,	6650,	6700
7 177	395,	1059,	1186,	2035,	5142,	5991,	6118,	6782
7 297	736,	2533,	2548,	2682,	4615,	4749,	4764,	6561
7 321	28,	1303,	3388,	3399,	3922,	3933,	6018,	7293
7 369	1423,	1588,	2408,	2594,	4775,	4961,	5781,	5946
7 393	306,	604,	805,	2048,	5345,	6588,	6789,	7087
7 417	1131,	2658,	3329,	3397,	4020,	4088,	4759,	6286
7 489	1731,	1931,	2704,	3378,	4111,	4785,	5558,	5758
7 537	145,	1365,	1916,	2495,	5042,	5621,	6172,	7392
7 561	1017,	1218,	1933,	2092,	5469,	5628,	6343,	6544
7 681	144,	1357,	2507,	3249,	4432,	5174,	6324,	7537
7 753	375,	1716,	3247,	3818,	3935,	4506,	6037,	7378
7 873	233,	642,	1932,	2009,	5864,	5941,	7231,	7640
7 993	230,	2273,	2771,	3927,	4066,	5222,	5720,	7763
8 017	570,	686,	783,	1083,	6934,	7234,	7331,	7447
8 089	518,	1309,	2764,	3924,	4165,	5325,	6780,	7571
8 161	496,	1596,	2260,	4048,	4113,	5901,	6565,	7665

p	y	y	y	y	y	y	y	y
8 209	746,	1637,	1710,	2740,	5469,	6499,	6572,	7463
8 233	815,	1091,	2165,	3567,	4666,	6068,	7142,	7418
8 329	478,	1272,	1553,	3116,	5213,	6776,	7057,	7851
8 353	1372,	2559,	4073,	4090,	4263,	4280,	5794,	6981
8 377	1214,	1476,	1504,	2077,	6300,	6873,	6901,	7163
8 521	514,	746,	1288,	1687,	6834,	7233,	7775,	8007
8 641	968,	1296,	2887,	3651,	4990,	5754,	7345,	7673
8 689	236,	405,	2484,	2606,	6083,	6205,	8284,	8453
8 713	559,	1886,	2036,	3231,	5482,	6677,	6827,	8154
8 737	865,	1346,	3515,	4005,	4732,	5222,	7391,	7872
8 761	464,	1436,	1873,	2549,	6212,	6888,	7325,	8297
8 929	48,	186,	2905,	3630,	5299,	6024,	8743,	8881
9 001	29,	131,	2130,	2483,	6518,	6871,	8870,	8972
9 049	747,	2215,	3513,	3926,	5123,	5536,	6834,	8302
9 241	1908,	2534,	3366,	4296,	4945,	5875,	6707,	7333
9 337	948,	1763,	2439,	3610,	5727,	6898,	7574,	8389
9 433	395,	1003,	1722,	4344,	5089,	7711,	8430,	9038
9 601	1540,	1895,	1905,	2500,	7101,	7696,	7706,	8061
9 649	95,	3083,	3546,	4469,	5180,	6103,	6566,	9554
9 697	624,	1168,	1868,	4460,	5237,	7829,	8529,	9073
9 721	917,	2417,	3634,	4807,	4914,	6087,	7304,	8804
9 769	321,	1279,	2952,	3819,	5950,	6817,	8490,	9448
9 817	1071,	2908,	3254,	4419,	5398,	6563,	6909,	8746

p^κ	y	y	y	y	y	y	y	y
73^2	786,	1417,	1818,	2430,	2899,	3511,	3912,	4543
97^2	412,	2809,	2822,	4031,	5378,	6587,	6600,	8997

p	y	y	y	y	y	y	y	y
10 009	3107,	3186,	3196,	3899,	6110,	6813,	6823,	6902
10 177	1460,	2665,	4193,	4970,	5207,	5984,	7512,	8717
10 273	871,	1111,	2737,	4706,	5567,	7536,	9162,	9402
10 321	2523,	2569,	2803,	3255,	7066,	7518,	7752,	7798
10 369	1339,	1572,	4465,	4554,	5815,	5904,	8797,	9030
10 513	138,	838,	1016,	4646,	5867,	9497,	9675,	10375
10 657	11,	1196,	4482,	4844,	5813,	6175,	9461,	10646
10 729	2669,	2834,	3741,	4322,	6407,	6988,	7895,	8060
10 753	954,	1021,	2491,	2812,	7941,	8262,	9732,	9799
10 993	831,	2286,	5265,	5333,	5660,	5728,	8707,	10162
11 113	2249,	2611,	4103,	4462,	6651,	7010,	8502,	8864
11 161	257,	304,	1592,	1970,	9191,	9569,	10857,	10904
11 257	544,	1622,	2089,	4449,	6808,	9168,	9635,	10713
11 329	415,	600,	3191,	4477,	6852,	8138,	10729,	10914
11 353	389,	536,	788,	2224,	9129,	10565,	10817,	10964
11 497	2023,	3638,	4484,	5502,	5995,	7013,	7859,	9474
11 593	941,	1628,	3967,	4237,	7356,	7626,	9965,	10652
11 617	1995,	2172,	2249,	5465,	6152,	9368,	9445,	9622
11 689	2545,	3473,	3746,	5585,	6104,	7943,	8216,	9144
11 833	1376,	3760,	3930,	5863,	5970,	7903,	8073,	10457
11 953	1295,	1899,	3249,	4469,	7484,	8704,	10054,	10658
12 049	801,	3189,	5264,	5569,	6480,	6785,	8860,	11248
12 073	145,	2347,	3179,	3497,	8576,	8894,	9726,	11928
12 097	474,	1812,	4721,	5381,	6716,	7376,	10285,	11623
12 241	472,	1143,	3553,	4830,	7411,	8688,	11098,	11769
12 289	117,	997,	3046,	5029,	7260,	9243,	11292,	12172
12 409	189,	1328,	2101,	2308,	10101,	10308,	11081,	12220
12 433	939,	1589,	3906,	5704,	6729,	8527,	10844,	11494
12 457	1429,	1669,	5396,	5971,	6486,	7061,	10788,	11028
12 553	1045,	2830,	3828,	4829,	7724,	8725,	9723,	11508
12 577	704,	4884,	5163,	5843,	6734,	7414,	7693,	11873
12 601	345,	806,	2228,	2392,	10209,	10373,	11795,	12256
12 697	647,	3098,	4180,	4496,	8201,	8517,	9599,	12050
12 721	1011,	1391,	4975,	5637,	7084,	7746,	11330,	11710
12 841	1481,	1999,	5289,	6231,	6610,	7552,	10842,	11360
12 889	2290,	3863,	5225,	6028,	6861,	7664,	9026,	10599
13 009	2256,	3604,	5448,	6249,	6760,	7561,	9405,	10753
13 033	803,	1255,	3292,	3311,	9722,	9741,	11778,	12230
13 177	890,	2088,	6322,	6355,	6822,	6855,	11089,	12287
13 249	1524,	1635,	3371,	6355,	6894,	9878,	11614,	11725

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13 297	1252,	1574,	4535,	6243,	7054,	8762,	11723,	12045
13 417	939,	5644,	5861,	6380,	7037,	7556,	7773,	12478
13 441	2027,	4648,	5265,	5665,	7776,	8176,	8793,	11414
13 513	482,	2383,	4446,	5328,	8185,	9067,	11130,	13031
13 537	1014,	4980,	6233,	6635,	6902,	7304,	8557,	12523
13 633	1068,	1146,	2770,	5270,	8363,	10863,	12487,	12565
13 681	362,	519,	3585,	4195,	9486,	10096,	13162,	13319
13 729	1401,	3420,	3621,	3655,	10074,	10108,	10309,	12328
13 873	932,	1581,	4927,	6274,	7599,	8946,	12292,	12941
13 921	2843,	3365,	4593,	4741,	9180,	9328,	10556,	11078
14 281	1547,	1560,	4385,	6582,	7699,	9896,	12721,	12734
14 401	229,	817,	1322,	2767,	11634,	13079,	13584,	14172
14 449	1360,	1882,	4737,	5429,	9020,	9712,	12567,	13089
14 593	5287,	5704,	5938,	5973,	8620,	8655,	8889,	9306
14 713	2752,	4400,	4651,	5982,	8731,	10062,	10313,	11961
14 737	261,	548,	1748,	6211,	8526,	12989,	14189,	14476
14 929	361,	4549,	5475,	5778,	9151,	9454,	10380,	14568
15 073	1230,	1818,	6164,	6525,	8548,	8909,	13255,	13843
15 121	706,	4466,	4562,	7012,	8109,	10559,	10655,	14415
15 193	2705,	3106,	5506,	5974,	9219,	9687,	12087,	12488
15 217	66,	3910,	4137,	5764,	9453,	11080,	11307,	15151
15 241	1023,	3182,	4488,	7181,	8060,	10753,	12059,	14218
15 289	1385,	2140,	4651,	4813,	10476,	10638,	13149,	13904
15 313	33,	464,	4635,	5775,	9538,	10678,	14849,	15280
15 361	1240,	3069,	3481,	4800,	10561,	11880,	12292,	14121
15 601	323,	483,	590,	7113,	8488,	15011,	15118,	15278
15 649	1251,	6805,	7516,	7685,	7964,	8133,	8844,	14398
15 817	1285,	1514,	6124,	6723,	9094,	9693,	14303,	14532
15 889	2731,	3165,	5271,	6481,	9408,	10618,	12724,	13158
15 913	684,	4230,	5278,	7561,	8352,	10635,	11683,	15229
15 937	1021,	1358,	2989,	3679,	12258,	12948,	14579,	14916
16 033	3219,	3404,	6465,	6985,	9048,	9568,	12629,	12814
16 057	744,	4247,	5864,	6712,	9345,	10193,	11810,	15313
16 249	2989,	3028,	3373,	7043,	9206,	12876,	13221,	13260
16 273	4291,	4762,	5266,	6648,	9625,	11007,	11511,	11982
16 369	2067,	5195,	5221,	6747,	9622,	11148,	11174,	14302
16 417	2183,	3136,	5439,	6731,	9686,	10978,	13281,	14234
16 561	3514,	3840,	5171,	5679,	10882,	11390,	12721,	13047
16 633	2917,	3273,	4171,	4566,	12067,	12462,	13360,	13716
16 657	2582,	3224,	4032,	8313,	8344,	12625,	13433,	14075

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16 729	1016,	4605,	6470,	7887,	8842,	10259,	12124,	15713
16 921	1804,	3236,	7565,	8144,	8777,	9356,	13685,	15117
16 993	3192,	5116,	7283,	8493,	8500,	9710,	11877,	13801
17 041	201,	2713,	3014,	6372,	10669,	14027,	14328,	16840
17 137	3942,	5985,	6647,	7940,	9197,	10490,	11152,	13195
17 209	942,	3197,	5615,	7328,	9881,	11594,	14012,	16267
17 257	3003,	6735,	7310,	7925,	9332,	9947,	10522,	14254
17 377	1053,	1352,	2969,	3482,	13895,	14408,	16025,	16324
17 401	5312,	5425,	5857,	6319,	11082,	11544,	11976,	12089
17 449	2460,	3643,	4852,	5029,	12420,	12597,	13806,	14989
17 497	4577,	5355,	6708,	7810,	9687,	10789,	12142,	12920
17 569	2595,	4746,	4893,	6061,	11508,	12676,	12823,	14974
17 713	1478,	2069,	3080,	6301,	11412,	14633,	15644,	16235
17 737	654,	2103,	4041,	8004,	9733,	13696,	15634,	17083
17 761	849,	4348,	6883,	8619,	9142,	10878,	13413,	16912
17 881	993,	2170,	2557,	3535,	14346,	15324,	15711,	16888
17 929	4841,	6322,	6752,	7751,	10178,	11177,	11607,	13088
17 977	638,	821,	7636,	8671,	9306,	10341,	17156,	17339
18 049	1126,	4376,	7050,	7939,	10110,	10999,	13673,	16923
18 097	7424,	7915,	8680,	8938,	9159,	9417,	10182,	10673
18 121	2211,	2483,	6696,	7028,	11093,	11425,	15638,	15910
18 169	2177,	2437,	3397,	4220,	13949,	14772,	15732,	15992
18 217	38,	2176,	2397,	8782,	9435,	15820,	16041,	18179
18 289	541,	1965,	2271,	8012,	10277,	16018,	16324,	17748
18 313	280,	1559,	3728,	4358,	13955,	14585,	16754,	18033
18 433	2426,	2776,	8016,	8174,	10259,	10417,	15657,	16007
18 457	1774,	4429,	5430,	7980,	10477,	13027,	14028,	16683
18 481	3847,	5658,	5760,	6843,	11638,	12721,	12823,	14634
18 553	2687,	5012,	8414,	9204,	9349,	10139,	13541,	15866
18 793	1532,	2418,	2711,	7640,	11153,	16082,	16375,	17261
18 913	371,	4210,	6797,	7086,	11827,	12116,	14703,	18542
19 009	1948,	3961,	7426,	9003,	10006,	11583,	15048,	17061
19 081	2403,	4679,	4914,	6138,	12943,	14167,	14402,	16678
19 249	1199,	2220,	2665,	9269,	9980,	16584,	17029,	18050
19 273	628,	5841,	6721,	8381,	10892,	12552,	13432,	18645
19 417	385,	4186,	4248,	5709,	13708,	15169,	15231,	19032
19 441	5663,	8476,	9554,	9576,	9865,	9887,	10965,	13778
19 489	196,	1471,	8253,	8360,	11129,	11236,	18018,	19293
19 609	2190,	4155,	5432,	7960,	11649,	14177,	15454,	17419
19 681	91,	2958,	7738,	8651,	11030,	11943,	16723,	19590

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19 753	805,	3239,	6387,	8375,	11378,	13366,	16514,	18948
19 777	436,	1134,	2332,	2417,	17360,	17445,	18643,	19341
19 801	821,	4971,	5716,	8827,	10974,	14085,	14830,	18980
19 993	923,	2974,	3005,	8751,	11242,	16988,	17019,	19070
20 089	1962,	2570,	6385,	8797,	11292,	13704,	17519,	18127
20 113	11,	147,	2326,	3657,	16456,	17787,	19966,	20102
20 161	58,	1738,	4748,	4760,	15401,	15413,	18423,	20103
20 233	305,	5108,	7759,	9166,	11067,	12474,	15125,	19928
20 353	5615,	9254,	9756,	10116,	10237,	10597,	11099,	14738
20 521	790,	2780,	7559,	9013,	11508,	12962,	17741,	19731
20 593	284,	574,	3408,	3480,	17113,	17185,	20019,	20309
20 641	1044,	1641,	5505,	5628,	15013,	15136,	19000,	19597
20 809	2986,	4441,	4606,	6265,	14544,	16203,	16368,	17823
20 857	1333,	2283,	2347,	7199,	13658,	18510,	18574,	19524
20 929	980,	6298,	7374,	7496,	13433,	13555,	14631,	19949
21 001	3226,	3448,	8387,	8860,	12141,	12614,	17553,	17775
21 121	594,	3757,	4658,	8056,	13065,	16463,	17364,	20527
21 169	180,	1421,	2175,	8350,	12819,	18994,	19748,	20989
21 193	2191,	5151,	5793,	6887,	14306,	15400,	16042,	19002
21 313	5532,	5544,	8003,	9779,	11534,	13310,	15769,	15781
21 433	2414,	6606,	8266,	9542,	11891,	13167,	14827,	19019
21 481	1273,	2494,	3088,	6589,	14892,	18393,	18987,	20208
21 529	1305,	3352,	5029,	9222,	12307,	16500,	18177,	20224
21 577	2005,	6382,	8645,	9976,	11601,	12932,	15195,	19572
21 601	2909,	3542,	8526,	8799,	12802,	13075,	18059,	18692
21 649	624,	1700,	1787,	5597,	16052,	19862,	19949,	21025
21 673	183,	5211,	6351,	10183,	11490,	15322,	16462,	21490
21 817	3919,	5121,	5901,	6514,	15303,	15916,	16696,	17898
21 841	1327,	4345,	6590,	9217,	12624,	15251,	17496,	20514
21 937	5738,	5805,	6419,	7660,	14277,	15518,	16132,	16199
21 961	139,	158,	6219,	8807,	13154,	15742,	21803,	21822
22 129	2224,	2765,	5582,	9836,	12293,	16547,	19364,	19905
22 153	281,	4336,	7007,	8552,	13601,	15146,	17817,	21872
22 273	1431,	6422,	6933,	8187,	14086,	15340,	15851,	20842
22 369	3881,	4015,	6196,	11165,	11204,	16173,	18354,	18488
22 441	2765,	3076,	3546,	8360,	14081,	18895,	19365,	19676
22 777	2222,	3803,	5810,	7084,	15693,	16967,	18974,	20555
22 921	5881,	6366,	9437,	9845,	13076,	13484,	16555,	17040
22 993	2459,	8201,	8509,	11268,	11725,	14484,	14792,	20534
23 017	981,	2059,	10089,	10195,	12822,	12928,	20958,	22036

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23 041	5254,	6744,	9597,	11310,	11731,	13444,	16297,	17787
23 209	1047,	7380,	7692,	8337,	14872,	15517,	15829,	22162
23 473	3661,	4295,	8400,	8816,	14657,	15073,	19178,	19812
23 497	1488,	4911,	5129,	9506,	13991,	18368,	18586,	22009
23 593	2600,	8557,	8605,	10671,	12922,	14988,	15036,	20993
23 689	3149,	4395,	4630,	7786,	15903,	19059,	19294,	20540
23 761	4123,	4555,	7350,	7515,	16246,	16411,	19206,	19638
23 833	1891,	1975,	3584,	6705,	17128,	20249,	21858,	21942
23 857	4597,	5335,	9365,	10949,	12908,	14492,	18522,	19260
23 929	154,	442,	1570,	7303,	16626,	22359,	23487,	23775
23 977	4258,	6354,	10592,	11347,	12630,	13385,	17623,	19719
24 001	652,	1951,	4413,	6211,	17790,	19588,	22050,	23349
24 049	4029,	8400,	10447,	10774,	13275,	13602,	15649,	20020
24 097	3484,	4391,	9026,	11936,	12161,	15071,	19706,	20613
24 121	780,	3680,	5065,	8234,	15887,	19056,	20441,	23341
24 169	4171,	4533,	6148,	9208,	14961,	18021,	19636,	19998
24 337	3305,	4503,	6318,	12128,	12209,	18019,	19834,	21032
24 481	3266,	3375,	8494,	9617,	14864,	15987,	21106,	21215
24 697	6566,	7116,	7376,	9867,	14830,	17321,	17581,	18131
24 793	52,	3377,	7584,	9059,	15734,	17209,	21416,	24741
24 841	6787,	6859,	9096,	9441,	15400,	15745,	17982,	18054
24 889	266,	3462,	8474,	10600,	14289,	16415,	21427,	24623
25 033	2771,	5338,	6125,	11391,	13642,	18908,	19695,	22262
25 057	909,	4042,	4420,	9896,	15161,	20637,	21015,	24148
25 153	583,	2628,	5044,	5091,	20062,	20109,	22525,	24570
25 321	2708,	5283,	8846,	8905,	16416,	16475,	20038,	22613
25 537	1196,	2669,	3802,	5501,	20036,	21735,	22868,	24341
25 561	3404,	6007,	8658,	10838,	14723,	16903,	19554,	22157
25 609	564,	4471,	6039,	10837,	14772,	19570,	21138,	25045
25 633	133,	3652,	5201,	9251,	16382,	20432,	21981,	25500
25 657	1929,	2486,	3937,	4861,	20796,	21720,	23171,	23728
25 801	2306,	8894,	10439,	12593,	13208,	15362,	16907,	23495
25 849	3422,	6067,	6111,	11265,	14584,	19738,	19782,	22427
25 873	5833,	7097,	7132,	8790,	17083,	18741,	18776,	20040
25 969	1154,	8660,	9443,	9744,	16225,	16526,	17309,	24815
26 017	2883,	4260,	8141,	10035,	15982,	17876,	21757,	23134
26 041	6965,	7201,	8983,	9665,	16376,	17058,	18840,	19076
26 113	1023,	1642,	8540,	9853,	16260,	17573,	24471,	25090
26 161	1344,	8431,	10297,	11242,	14919,	15864,	17730,	24817
26 209	1963,	2804,	2963,	7103,	19106,	23246,	23405,	24246

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26 449	3445,	5904,	9058,	11747,	14702,	17391,	20545,	23004
26 497	4907,	6631,	7407,	11941,	14556,	19090,	19866,	21590
26 641	1282,	1517,	9150,	12843,	13798,	17491,	25124,	25359
26 713	705,	2898,	11102,	12785,	13928,	15611,	23815,	26008
26 737	384,	4234,	6599,	10096,	16641,	20138,	22503,	26353
26 833	3905,	7836,	10637,	12783,	14050,	16196,	18997,	22928
26 881	5060,	6673,	6699,	12516,	14365,	20182,	20208,	21821
26 953	139,	3476,	6205,	7219,	19734,	20748,	23477,	26814
27 073	2898,	5054,	6992,	8863,	18210,	20081,	22019,	24175
27 241	6444,	6595,	8163,	10599,	16642,	19078,	20646,	20797
27 337	1004,	1552,	2199,	6539,	20798,	25138,	25785,	26333
27 361	1310,	4407,	6203,	8394,	18967,	21158,	22954,	26051
27 409	1305,	6847,	9422,	10970,	16439,	17987,	20562,	26104
27 457	6080,	6652,	7195,	7243,	20214,	20262,	20805,	21377
27 481	1684,	5271,	7983,	10496,	16985,	19498,	22210,	25797
27 529	3805,	3838,	6251,	10422,	17107,	21278,	23691,	23724
27 673	2788,	4204,	4278,	11763,	15910,	23395,	23469,	24885
27 697	5336,	6400,	12140,	12199,	15498,	15557,	21297,	22361
27 793	5909,	10294,	10419,	13024,	14769,	17374,	17499,	21884
27 817	1047,	7114,	9490,	10893,	16924,	18327,	20703,	26770
27 961	4312,	4905,	8138,	13419,	14542,	19823,	23056,	23649
28 057	5803,	6121,	9781,	12894,	15163,	18276,	21936,	22254
28 081	82,	4407,	7213,	10616,	17465,	20868,	23674,	27999
28 201	8467,	10102,	11233,	13803,	14398,	16968,	18099,	19734
28 297	2480,	9961,	11683,	13457,	14840,	16614,	18336,	25817
28 393	1580,	7314,	7853,	9903,	18490,	20540,	21079,	26813
28 513	5028,	9507,	10692,	11767,	16746,	17821,	19006,	23485
28 537	3986,	8269,	8744,	9892,	18645,	19793,	20268,	24551
28 657	6962,	7089,	8714,	12948,	15709,	19943,	21568,	21695
28 729	941,	2015,	5999,	8419,	20310,	22730,	26714,	27788
28 753	1662,	5362,	7145,	8435,	20318,	21608,	23391	27091
28 921	778,	1550,	7818,	8141,	20780,	21103,	27371,	28143
29 017	1710,	7532,	10402,	11962,	17055,	18615,	21485,	27307
29 137	3722,	4423,	5356,	6490,	22647,	23781,	24714,	25415
29 209	3562,	3951,	9750,	11725,	17484,	19459,	25258,	25647
29 401	1491,	5403,	9236,	12851,	16550,	20165,	23998,	27910
29 473	96,	307,	853,	7947,	21526,	28620,	29166,	29377
29 569	361,	12467,	13633,	14334,	15235,	15936,	17102,	29208
29 641	3737,	8064,	13135,	13670,	15971,	16506,	21577,	25904
29 761	2044,	2785,	6654,	10590,	19171,	23107,	26976,	27717

p	y	y	y	y	y	y	y	y
29 833	6737,	7270,	8835,	13050,	16783,	20998,	22563,	23096
29 881	706,	1566,	5561,	11155,	18726,	24320,	28315,	29175
30 097	648,	7582,	8078,	13330,	16767,	22019,	22515,	29449
30 169	1065,	4649,	8385,	14932,	15237,	21784,	25520,	29104
30 241	7094,	9502,	13865,	14982,	15259,	16376,	20739,	23147
30 313	2198,	4831,	8452,	12812,	17501,	21861,	25482,	28115
30 529	7286,	8036,	8795,	13308,	17221,	21734,	22493,	23243
30 553	525,	5078,	9195,	9705,	20848,	21358,	25475,	30028
30 577	475,	7499,	13647,	14157,	16420,	16930,	23078,	30102
30 649	1956,	9441,	10291,	14306,	16343,	20358,	21208,	28693
30 697	3764,	3988,	12303,	14968,	15729,	18394,	26709,	26933
30 817	1035,	2101,	12565,	13201,	17616,	18252,	28716,	29782
30 841	4185,	9095,	11509,	11710,	19131,	19332,	21746,	26656
30 937	1468,	1889,	3341,	13340,	17597,	27596,	29048,	29469
31 033	2205,	6854,	6965,	12333,	18700,	24068,	24179,	28828
31 081	995,	1466,	4873,	13802,	17279,	26208,	29615,	30086
31 153	266,	12063,	14406,	15490,	15663,	16747,	19090,	30887
31 177	20,	2912,	10912,	11231,	19946,	20265,	28265,	31157
31 249	296,	739,	2288,	3073,	28176,	28961,	30510,	30953
31 321	1007,	1128,	5359,	12348,	18973,	25962,	30193,	30314
31 393	7833,	8493,	11322,	12416,	18977,	20071,	22900,	23560
31 489	114,	7840,	9764,	11325,	20164,	21725,	23649,	31375
31 513	1201,	7137,	13576,	15341,	16172,	17937,	24376,	30312
31 657	3204,	7525,	10593,	11985,	19672,	21064,	24132,	28453
31 729	742,	2952,	4661,	11103,	20626,	27068,	28777,	30987
31 849	2442,	7421,	13746,	15665,	16184,	18103,	24428,	29407
31 873	5643,	8218,	9731,	12974,	18899,	22142,	23655,	26230
32 089	232,	5075,	12310,	15232,	16857,	19779,	27014,	31857
32 233	589,	2189,	7218,	9293,	22940,	25015,	30044,	31644
32 257	4594,	7170,	14113,	15314,	16943,	18144,	25087,	27663
32 353	6700,	7426,	7644,	13972,	18381,	24709,	24927,	25653
32 377	2993,	6471,	10302,	11326,	21051,	22075,	25906,	29384
32 401	317,	5110,	7742,	12572,	19829,	24659,	27291,	32084
32 497	2574,	11616,	13560,	16198,	16299,	18937,	20881,	29923
32 569	1410,	1640,	5530,	7356,	25213,	27039,	30929,	31159
32 713	5449,	7820,	8609,	10638,	22075,	24104,	24893,	27264
32 833	5691,	8227,	10476,	13881,	18952,	22357,	24606,	27142
33 049	1810,	9513,	13809,	14276,	18773,	19240,	23536,	31239
33 073	5496,	9301,	11905,	16025,	17048,	21168,	23772,	27577
33 289	1335,	2451,	8129,	16271,	17018,	25160,	30838,	31954

☞ *For solutions of* $(y^{12}+1)\div(y^4+1)\equiv 0 \pmod{p^\kappa > 10^4}$, *see page* 70.

Least Roots (y) *of* $y^{16}+1\equiv 0 \pmod p$.

p	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y
97	19,	20,	28,	30,	34,	42,	45,	46,	51,	52,	55,	63,	67,	69,	77,	78
193	8,	14,	23,	24,	42,	67,	69,	72,	121,	124,	126,	151,	169,	170,	179,	185
257	15,	17,	30,	34,	60,	68,	120,	121,	136,	137,	189,	197,	223,	227,	240,	242
353	6,	7,	10,	59,	67,	101,	106,	137,	216,	247,	252,	286,	294,	343,	346,	347
449	10,	22,	45,	99,	102,	127,	128,	221,	228,	321,	322,	347,	350,	404,	427,	439
577	33,	35,	127,	163,	177,	209,	215,	263,	314,	362,	368,	400,	414,	450,	542,	544
641	4,	10,	25,	64,	160,	241,	258,	282,	359,	383,	400,	481,	577,	616,	631,	637
673	107,	114,	118,	149,	154,	183,	239,	271,	402,	434,	490,	519,	524,	555,	559,	566
769	43,	144,	182,	251,	300,	304,	359,	377,	392,	410,	465,	469,	518,	587,	625,	726
929	170,	197,	228,	269,	273,	297,	388,	448,	481,	541,	632,	656,	660,	701,	732,	759

Least Roots (y) *of* $y^{32}+1\equiv 0 \pmod p$.

p	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y
193	11,	13,	20,	29,	33,	35,	39,	60,	68,	71,	74,	76,	87,	88,	89,	94
	182,	180,	173,	164,	160,	158,	154,	133,	125,	122,	119,	117,	106,	105,	104,	99
257	11,	22,	23,	35,	44,	46,	67,	70,	73,	81,	88,	92,	95,	111,	117,	123
	246,	235,	234,	222,	213,	211,	190,	187,	184,	176,	169,	165,	162,	146,	140,	134
449	24,	37,	52,	58,	71,	79,	84,	95,	108,	131,	155,	182,	188,	203,	209,	215
	425,	412,	397,	391,	378,	370,	365,	354,	341,	318,	294,	267,	261,	246,	240,	234
577	20,	37,	42,	67,	78,	83,	97,	123,	141,	146,	155,	202,	232,	258,	261,	266
	557,	540,	535,	510,	499,	494,	480,	454,	436,	431,	422,	375,	345,	319,	316,	311
641	2,	5,	8,	20,	32,	50,	77,	80,	125,	128,	129,	141,	159,	200,	308,	320
	639,	636,	633,	621,	609,	591,	564,	561,	516,	513,	512,	500,	482,	441,	333,	321
769	12,	25,	64,	85,	90,	94,	113,	123,	190,	197,	231,	245,	253,	289,	306,	324
	757,	744,	705,	684,	679,	675,	656,	646,	579,	572,	538,	524,	516,	480,	463,	445

Least Roots (y) *of*
$y^{64}+1 \equiv 0 \pmod p$.

p	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y
257	9,	13,	18,	21,	25,	26,	29,	31,	36,	42,	49,	50,	52,	57,	58,	59
	248,	244,	239,	236,	232,	231,	228,	226,	221,	215,	208,	207,	205,	200,	199,	198
	61,	62,	72,	79,	84,	89,	98,	99,	100,	104,	113,	114,	116,	118,	122,	124
	196,	195,	185,	178,	173,	168,	159,	158,	157,	153,	144,	143,	141,	139,	135,	133
641	21,	29,	31,	42,	58,	61,	62,	67,	84,	105,	116,	122,	124,	134,	145,	153
	620,	612,	610,	599,	583,	580,	579,	574,	557,	536,	525,	519,	517,	507,	496,	488
	155,	168,	177,	199,	210,	221,	232,	243,	244,	248,	268,	287,	290,	305,	306,	310
	486,	473,	464,	442,	431,	420,	409,	398,	397,	393,	373,	354,	351,	336,	335,	331
769	5,	8,	17,	18,	38,	49,	60,	82,	89,	93,	96,	125,	135,	141,	154,	181
	764,	761,	752,	751,	731,	720,	709,	687,	680,	676,	673,	644,	634,	628,	615,	588
	200,	204,	215,	216,	257,	273,	283,	285,	299,	310,	313,	319,	320,	344,	347,	383
	569,	565,	554,	553,	512,	496,	486,	484,	470,	459,	456,	450,	449,	425,	422,	386

Least Roots (y) *of*
$(y^{24}+1)\div(y^8+1) \equiv 0 \pmod p$.

p	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y
97	2,	3,	11,	25,	31,	32,	44,	48,	49,	53,	65,	66,	72,	86,	94,	95
193	4,	21,	28,	36,	46,	48,	59,	62,	131,	134,	145,	147,	157,	165,	172,	189
241	11,	19,	22,	38,	63,	65,	88,	89,	152,	153,	176,	178,	203,	219,	222,	230
337	38,	65,	105,	133,	138,	140,	153,	163,	174,	184,	197,	199,	204,	232,	272,	299
433	21,	45,	73,	77,	91,	138,	165,	172,	261,	268,	295,	342,	356,	360,	388,	412
577	3,	8,	19,	62,	72,	121,	192,	243,	334,	385,	456,	505,	515,	558,	569,	574
673	2,	21,	29,	32,	116,	128,	163,	336,	337,	510,	545,	557,	641,	644,	652,	671
769	3,	120,	186,	243,	250,	256,	277,	314,	455,	492,	513,	519,	526,	583,	649,	766

Least Roots (y) *of*
$(y^{48}+1) \div (y^{16}+1) \equiv 0 \mod p$.

p	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y
97	5,	7,	10,	13,	14,	15,	17,	21,	23,	26,	29,	37,	38,	39,	40,	41
	92,	90,	87,	84,	83,	82,	80,	76,	74,	71,	68,	60,	59,	58,	57,	56
193	2,	6,	18,	25,	31,	32,	54,	56,	65,	75,	83,	86,	92,	93,	95,	96
	191,	187,	175,	168,	162,	161,	139,	137,	128,	118,	110,	107,	101,	100,	98,	97
577	11,	46,	50,	59,	68,	88,	99,	105,	138,	150,	196,	204,	212,	262,	264,	280
	566,	531,	527,	518,	509,	489,	478,	472,	439,	427,	381,	373,	365,	315,	313,	297
673	57,	59,	77,	131,	195,	201,	214,	217,	228,	236,	245,	262,	283,	298,	307,	308
	616,	614,	596,	542,	478,	472,	459,	456,	445,	437,	428,	411,	390,	375,	366,	365
769	16,	48,	100,	129,	131,	143,	155,	223,	242,	308,	317,	337,	340,	362,	376,	382
	753,	721,	669,	640,	638,	626,	614,	546,	527,	461,	452,	432,	429,	407,	393,	387

Least Roots (y) *of*
$(y^{96}+1) \div (y^{32}+1) \equiv 0 \pmod p$.

p	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y
193	5,	10,	15,	17,	19,	22,	26,	30,	34,	37,	38,	40,	41,	44,	45,	47
	188,	183,	178,	176,	174,	171,	167,	163,	159,	156,	155,	153,	152,	149,	148,	146
	51,	52,	53,	57,	58,	61,	66,	70,	73,	77,	78,	79,	80,	82,	90,	91
	142,	141,	140,	136,	135,	132,	127,	123,	120,	116,	115,	114,	113,	111,	103,	102
577	14,	26,	29,	41,	47,	60,	86,	87,	111,	112,	115,	119,	125,	126,	139,	154
	563,	551,	548,	536,	530,	517,	491,	490,	466,	465,	462,	458,	452,	451,	438,	423
	160,	170,	180,	197,	199,	201,	206,	208,	220,	221,	234,	241,	244,	249,	281,	286
	417,	407,	397,	380,	378,	376,	371,	369,	357,	356,	343,	336,	333,	328,	296,	291
769	4,	10,	30,	34,	36,	41,	75,	76,	77,	98,	102,	108,	149,	160,	172,	178
	765,	759,	739,	735,	733,	728,	694,	693,	692,	671,	667,	661,	620,	609,	597,	591
	192,	193,	199,	203,	225,	228,	235,	248,	255,	270,	282,	294,	322,	338,	339,	369
	577,	576,	570,	566,	544,	541,	534,	521,	514,	499,	487,	475,	447,	431,	430,	400

FACTORISATION TABLES

OF

$$N = (x^2+y^2),\ (x^4+y^4),\ (x^8+y^8);\quad \frac{x^3\mp y^3}{x\mp y},\ \frac{x^6+y^6}{x^2+y^2},\ \frac{x^{12}+y^{12}}{x^4+y^4};\ \&c.$$

Explanation.

1. All factors shown are primes, or powers of primes [except when followed by a query (?)], and are usually printed in old-face type (*e.g.* 983).

[The powers of the *small* prime factors $\not> 11$ are entered thus :—
4, 8, 16, 32, &c. ; 9, 27, 81, 243, 729 ; 25, 125, 625 ; 49, 343, 2401 ; 121, 1331.]

The data (x, y, &c.) of numbers (N) to be factorised are usually printed in modern type (*e.g.* 983).

2. A query (?), on right, shows that the large factor ($> 9.10^6$) is of unknown composition.

3. A semicolon (;), on right, shows *complete* factorisation (into prime factors).

4. A full point (.), on right, shows that there are other (undetermined) factors.

5. A semicolon (;), in middle, separates algebraic factors.

6. A colon (:), in middle, separates the two Aurifeuillian or Diophantine co-factors (L, M).

7. The signs †, ‡, §, ¶—(when not referring to foot-notes)—show that the search for factors has been pushed to following limits (or a little further) :—

† to 1,000, ‡ to 10,000, § to 32,000, ¶ to 50,000.

8. When the last factor shown is followed by a query (?), the squares of the other factors shown have been tried as divisors.

9. In incomplete factorisations, when the last factor is not shown (see 4 above), the squares of the known factors have not been tried (except when possible from the above Congruence-Tables, pages 4, 22, 26, 37 ; 52, 70, 74, 85).

10. The column headed "Fig." shows the *number of figures* in the large number (N) to be factorised.

11. The initials, on right, indicate (according to a scheme in the Introduction) the names of original workers in the factorisations and in detection of High Primes ($> 9.10^6$).

All High Primes not specially marked (and also many of those marked) with initials are due to the Author ; also all High Primes $\not> 10^9$ are due to, or have been verified by, the Author.

12. Great use has been made of certain Binary, Ternary, &c., Canons (prepared by the author and Mr. H. J. Woodall jointly) giving the Residues of 2^x up to $x = 100$, 3^x and 5^x up to $x = 16$, 10^x up to $x = 12$, for all prime and powers of prime moduli $\not> 10,000$.

13. The phrase "High Numbers" means Numbers (N) $> 9.10^6$, or Numbers whose algebraic factors (if any) are $> 9.10^6$.

High Duan Chain, $N = Y^2+1 = Y'^2+1 = L.M > 6.10^{18}$.

$Y = y^2+y+1 = y'^2-y'+1 = Y'; \quad y'-y = 1.$

y	Y	$L = y^2+1$	$M = y'^2+1$	y'	Fig.
49 993	2 499 350 043	2.25.13.17.37.6113;	853.1069.2741;	49 994	19
4	2 499 450 031	853.1069.2741;	2.1249750013;	5	19
5	2 499 550 021	2.1249750013;	797.3136261;	6	19
6	2 499 650 013	797.3136261;	2.5.249970001;	7	19
7	2 499 750 007	2.5.249970001;	5.109.953.4813;	8	19
8	2 499 850 003	5.109.953.4813;	2.17509.71389;	9	19
9	2 499 950 001	2.17509.71389;	2500000001;	50 000	19
50 000	2 500 050 001	2500000001;	2.17.5197.14149;	1	19
1	2 500 150 003	2.17.5197.14149;	5.53.9434717;	2	19
2	2 500 250 007	5.53.9434717;	2.5.13.19233077;	3	19
3	2 500 350 013	2.5.13.19233077;	7561.330697;	4	19
4	2 500 450 021	7561.330697;	2.101.12378713;	5	19
5	2 500 550 031	2.101.12378713;	13.233.825553;	6	19

High Bin-Aurifeuillian Chain, $N = (y^4+4) = L.M; \quad [y = \omega]$.

$L = \{(y-1)^2+1\}; \quad M = \{(y+1)^2+1\}.$

y	L	M	Fig.
49 995	853.1069.2741;	797.3136261;	19
7	797.3136261;	5.109.953.4813;	19
9	5.109.953.4813;	2500000001;	19
50 001	2500000001;	5.53.9434717;	19
3	5.53.9434717;	7561.330697;	19
5	7561.330697;	13.233.825553;	19

High Bin-Aurifeuillian Chain, $\frac{1}{4}N = 4(\frac{1}{2}y)^4+1 = \frac{1}{2}L.\frac{1}{2}M; \quad [y = \epsilon]$.

$L = \{(y-1)^2+1\}; \quad M = \{(y+1)^2+1\}.$

y	$\frac{1}{2}L$	$\frac{1}{2}M$	Fig.
49 994	25.13.17.37.6113;	1249750013;	19
6	1249750013;	5.249970001;	19
8	5.249970001;	17509.71389;	19
50 000	17509.71389;	17.5197.14149;	19
2	17.5197.14149;	5.13.19233077;	19
4	5.13.19233077;	101.12378713;	19

High Irreducible Duans, $N = (x^{\alpha})^2 + (y^{\beta})^2 > 9.10^6$; $[x, y \not> 11]$.

[*Bin-Aurifeuillians, Quartans, and Octavans excluded.*]

$x^{2\alpha}+y^{2\beta}$	N
$3^2 + 2^{24}$	25·29·73·317 ;
$5^2 +$	13·41·31477 ;
$7^2 +$	5·3355453 ;
$11^2 +$	16777337 ;
$3^{10} +$	5·3367253 ;
$3^{14} +$	5·73·59069 ;
$3^2 + 2^{26}$	13·5162221 ;
$5^2 +$	1129·59441 ;
$7^2 +$	67108913 ;
$11^2 +$	5·13421797 ;
$3^6 +$	67109593 ;
$5^6 +$	109.615821 ;
$7^6 +$	109.616757 ;
$11^6 +$	25·509·5413 ;
$3^{10} +$	37·1815349 ;
$3^{14} +$	13·5530141 ;
$3^2 + 2^{28}$	5·5689·9437 ;
$5^2 +$	37·89·81517 ;
$7^2 +$	5·13·4129777 ;
$11^2 +$	‡
$3^6 +$	5·433·123989 ;
$5^6 +$	2081.129001 ;
$7^6 +$	5·281.191141 ;
$11^6 +$	101.2675317 ;
$3^{10} +$	5·53698901 ;
$3^2 + 2^{30}$	‡
$5^2 +$	29·37025581 ;
$7^2 +$	5849·183577 ;
$11^2 +$	5·214748389 ? ‡
$3^{14} +$	‡

$x^{2\alpha}+y^{2\beta}$	N
$3^2 + 2^{32}$	5·9629.89209 ;
$5^2 +$	733·5859437 ;
$7^2 +$	5· ‡
$11^2 +$	13· ‡
$3^6 +$	25· ‡
$5^6 +$	41·5573·18797 ;
$7^6 +$	5·137·6270197 ;
$3^{10} +$	5·3221.266689 ;
$3^{14} +$	5· ‡
$3^2 + 2^{34}$	37·313·1483453 ;
$5^2 +$	‡
$7^2 +$	113·152034241 ? ‡
$11^2 +$	5·181·353·53777 ;
$3^6 +$	‡
$5^6 +$	29·592409821 ? ‡
$7^6 +$	457·37592969 ? †
$3^{10} +$	13·829.1594129 ;
$3^{14} +$	2857·6014929 ;
$3^2 + 2^{36}$	5·193·557·127849 ;
$5^2 +$	13·809·6534133 ;
$7^2 +$	5· ‡
$11^2 +$	29. ‡
$3^{10} +$	5·29·5737·82609 ;
$3^{14} +$	5· ‡
$3^2 + 10^8$	149·671141 ;
$3^6 +$	29·73·47237 ;
$3^{10} +$	100059049 ;
$3^{14} +$	433·241993 ;
$7^2 +$	100000049 ; W
$11^2 +$	13·1049·7333 ;

$x^{2\alpha}+y^{2\beta}$	N
$2^2 + 5^{10}$	9765629 ;
$2^4 +$	797·12253 ;
$2^6 +$	1181.8269 ;
$2^8 +$	89·197·557 ;
$2^{12} +$	13·13·57809 ;
$2^{14} +$	1669.5861 ;
$2^{16} +$	373·26357 ;
$2^{18} +$	241.41609 ;
$2^{22} +$	3301.4229 ;
$2^{24} +$	13·2041757 ;
$2^{26} +$	641.119929 ;
$2^{28} +$	53·5249077 ;
$2^{32} +$	37·2237·52009 ;
$2^{34} +$	709·24244901 ;
$3^2 + 5^{10}$	2.29.137·1229 ;
$3^4 +$	2.37·131969 ;
$3^6 +$	2.13.149·2521 ;
$3^8 +$	2.41.119173 ;
$3^{12} +$	2.13·396041 ;
$3^{14} +$	2.277.26261 ;
$3^{16} +$	2.41.644053 ;
$7^2 +$	2.53·181.509 ;
$7^4 +$	2.877·5569 ;
$11^2 +$	2.109·44797 ;
$11^4 +$	2.4890133 ;
$5^2 + 3^{16}$	2.1481.14533 ;
$7^2 +$	2.5·13·137·2417 ;
$11^2 +$	2.21523421 ;
$5^6 +$	2.21531173 ;
$3^2 + 5^{12}$	2.101.269·4493 ;

High Associate Duans, $N_1, N, N_2 > 9\frac{1}{2}.10^{13}$.

$$N_1 = Y_1^2+1 = Y'.Y\,; \quad N = y^4+4 = Y'.Y''\,; \quad N_2 = Y_2^2+1 = Y.Y''.$$

$$Y_1 = (y^2-y+1)\,; \qquad y = \eta^r > 3.10^3\,; \qquad Y_2 = (y^2+y+1).$$

η^r	$Y' = (y-1)^2+1$	$Y = y^2+1$	$Y'' = (y+1)^2+1$	Fig.
3^8	13.1213.2729 ;	2.21523361 ; B	5.8611969 ;	16
3^9	125.17.182297 ;	2.5 ; 73 ; 530713 ;	9013.42989 ;	18
3^{10}	5.149.293.15973 ;	2.41 ; 42521761 ; B	18013.193577 ;	20
5^5	17.574081 ;	2.13 ; 41.9161 ;	97.100741 ;	14
5^6	244109377 ;	2.313 ; 390001 ;	461.529657 ;	17
5^7		2.13.234750601 ; B	673.9069349 ;	20
5^8	137.257.4333753 ;	2.2593.29423041 ; DeB	113.457.2954797 ;	23
6^5	2.1277.23669 ;	37 ; 241.6781 ;	2.5.89.67957 ;	16
6^6	2.521.2088953 ;	1297 ; 1678321 ;	2.25.157.277309 ;	19
7^5	241.1171957 ;	2.25 ; 5.281.4021 ;	5.29.1948337 ;	17
10^4	2.389.128509 ;	17.5882353 ;	2.5001001 ;	17
10^5	2. ‡	101 ; 3541.27961 ;	2.13.1597.240841 ;	21
10^6	2.41.109.111881629 ? ‡	73.137 ; 99990001 ; Lf,R	2.2357.212134493 ;	25
10^7	2.17.97.173.175268213 ? ‡	101 ; 29.281.121499449 ; Lf,R	2. ‡	29
10^8	2.13.53.557.809.3313.4861 ;	353.449.641.1409.69857 ;	2. ‡	33
11^4	2593.82657 ;	2.17.6304673 ;	5.181.236893 ;	17
12^4	2.214970113 ;	17.97 ; 260753 ;	2.5.41.1048837 ;	18

High Associate Duans, N_1, N, $N_2 > 89.10^{12}$.

$N_1 = Y_1^2+1 = Y'.Y$; $N = y^4+4 = Y'.Y''$; $N_2 = Y_2^2+1 = Y.Y''$;

$Y_1 = (y^2-y+1) > 9.10^6$; $y = \eta^\beta.2^\alpha > 3.10^3$; $Y_2 = (y^2+y+1) > 9.10^6$.

η^β 2^α	$Y' = (y-1)^2+1$	$Y = (y^2+1)$	$Y'' = (y+1)^2+1$	Fig.
3.2^{10}	2.4715521 ;	5.1887437 ;	2.5.13.17.4273 ;	14
2^{11}	2.25.757.997 ;	13.2903749 ;	2.18880513 ;	16
2^{12}	2.5.17.888061 ;	5.30198989 ;	2.181.417181 ;	17
2^{13}	2.13.13.29.61613 ;	1093.552589 ;	2.5.3037.19889 ;	18
2^{14}	2.17.71053553 ;	5.483183821 ;	2.5.241601741 ;	19
$3^2.2^9$	2.25.13.32653 ;	4513:5.941 ;	2.10621441 ;	15
2^{10}	2.1453.29221 ;	41.137.15121 ;	2.5.809.10501 ;	16
2^{11}	2.173.981797 ;	17.29.37:125.149 ;	2.5.61.653.853 ;	18
2^{12}	2.5.13.1601.6529 ;	281.1009.4793 ;	2.53.12821021 ;	19
$3^3.2^7$	2.17.109.3221 ;	1069.11173 ;	2.25.239017 ;	15
2^8	2.13.313.5869 ;	5.269.35521 ;	2.5.37.53.2437 ;	16
2^9	2.5.19107533 ;	577 ; 349 : 13.73 ;	2.17.5621489 ;	17
2^{10}	2.5.76435661 ;	5.6389.23929 ;	2.29.401.32869 ;	18
$3^4.2^6$	2.5.113.23773 ;	1409.19073 ;	2.13442113 ;	15
2^7	2.5.1093.9833 ;	25.409:10513 ;	2.13.4135237 ;	17
2^8	2.214970113 ;	17.97.260753 ;	2.5.41.1048837 ;	18
2^9	2.37.23241133 ;	5.8237:41761 ;	2.5.162000973 ;	19
$3^5.2^4$	2.5.1510877 ;	5.3023309 ;	2.17.444833 ;	15
2^5	2.1277.23669 ;	37 ; 241.6781 ;	2.5.89.67957 ;	16
2^6	2.17.7112753 ;	5.53.193.4729 ;	2.5.13.1860737 ;	17
2^7	2.5.7457.12973 ;	13.1873.39733 ;	2.29.16681397 ;	18
$3^6.2^3$	2.41.414641 ;	25.229:13.457 ;	2.5.3402389 ;	16
2^4	2.5.13602557 ;	1777.76561 ;	2.1889.36017 ;	17
2^5	2.5.13.137.30553 ;	29.797:5.17.277 ;	2.613.443917 ;	18
2^6	2.521.2088953 ;	1297 ; 1678321 ;	2.25.43537513 ;	19
$3^7.2$	2.5.13.17.17.509 ;	19131877 ;	2.9570313 ;	15
2^2	2.5.149.51349 ;	5.15305501 ;	2.38272501 ;	16
2^3	2.37.4136149 ;	61.5018197 ;	2.5.17.1800853 ;	17
2^4	2.13.29.1623833 ;	5.24488813 ;	2.25.1373.17837 ;	19
$3^8.2$	2.53.313.5189 ;	13.997:5.2657 ;	2.5.113.257.593 ;	17
2^2	2.25.2713.5077 ;	17.40514561 ;	2.344400013 ;	18
$3^9.2$	2.29.26717297 ;	397.3903481 ;	2.5.154976069 ;	19
5.2^{10}	2.97.293.641 ;	26214401 ;	2.17.29.26597 ;	15
2^{11}	2.13.13.310169 ;	104857601 ;	2.41.1279001 ;	17
2^{12}	2.829.252949 ;	13.32263877 ;	2.17.17.53.13693 ;	18
2^{13}	2.19081.43961 ;	29.389.148721 ;	2.838901761 :	19
$5^2.2^7$	2.661.7741 ;	3121:17.193 ;	2.5723201 ;	15
2^8	2.20473601 ;	40960001 ;	2.13.409.3853 ;	16
2^9	2.61.1342741 ;	12641:13.997 ;	2.29.41.68909 ;	17
2^{10}	2.3109.105389 ;	655360001 ;	2.327705601 ;	18
2^{11}	2.13.100820677 ;	17.41.73:51521 ;	2.137.509.18797 ;	19

High Associate Duans, N_1, N, $N_2 > 89.10^{12}$.

$N_1 = Y_1^2+1 = Y'.Y$; $N = y^4+4 = Y'.Y''$; $N_2 = Y_2^2+1 = Y.Y''$;

$Y_1 = (y^2-y+1) > 9.10^6$; $y = \eta^\beta.2^\alpha > 3.10^3$; $Y_2 = (y^2+y+1) > 9.10^6$.

η^β 2^α	$Y' = (y-1)^2+1$	$Y = y^2+1$	$Y'' = (y+1)^2+1$	Fig.
$5^3.2^5$	2.13.17.97.373 ;	109.229.641 ;	2.181.44221 ;	15
2^6	2.31992001 ;	401 ; 13.12277 ;	2.32008001 ;	16
2^7	2.41.3121561 ;	256000001 ;	2.17.7530353 ;	17
2^8	2.29.29.37.16453 ;	157.6522293 ;	2.13.73.709.761 ;	19
$5^4.2^3$	2.12495001 ;	13.13.29:5101 ;	2.37.337973 ;	15
2^4	2.389.128509 ;	17.5882353 ;	2.50010001 ;	17
2^5	2.13.41.457.821 ;	19801:20201 ;	2.569.351529 ;	18
2^6	2.269.349.8521 ;	1889.847009 ;	2.4657.171793 ;	19
$5^5.2$	2.19525001 ;	3529.11069 ;	2.73.267637 ;	16
2^2	2.17.4594853 ;	37.4222973 ;	2.13.1117.5381 ;	17
2^3	2.61.5122541 ;	241.2593361 ;	2.41.7622561 ;	18
2^4	2.17509.71389 ;	2500000001 ;	2.17.5197.14149 ;	19
$5^6.2$	2.3533.138197 ;	17.17.29.109.1069 ;	2.41.229.52009 ;	18
7.2^9	2.5.13.17.37.157 ;	29.233.1901 ;	2.6426113 ;	15
2^{10}	2.5.877.5857 ;	25.13.13.12161 ;	2.25697281 ;	16
2^{11}	2.17.73.82793 ;	113.1818769 ;	2.5.20554957 ;	17
2^{12}	2.389.1056589 ;	5.164416717 ;	2.5.13.1301.4861 ;	18
$7^2.2^6$	2.353.13921 ;	9834497 ;	2.5.881.1117 ;	14
2^7	2.13.1512517 ;	61.101:5.1277 ;	2.5.821.4793 ;	16
2^8	2.25.1153.2729 ;	3361.46817 ;	2.29.313.8669 ;	17
2^9	2.5.62935757 ;	5.4973:17.1489 ;	2.314728961 ;	18
2^{10}	2.13.41.577.4093 ;	2777.906601 ;	2.5.251773133 ;	19
$7^3.2^4$	2.5.17.177101 ;	5.6023629 ;	2.53.284237 ;	15
2^5	2.421.143053 ;	109.113.9781 ;	2.5.13.73.12697 ;	17
2^6	2.17.14171953 ;	5.157 ; 13.47221 ;	2.5.48193421 ;	18
$7^4.2$	2.11524801 ;	5.941:13.13.29 ;	2.5.2306881 ;	15
2^2	2.5.41.224921 ;	401.230017 ;	2.101.157.2909 ;	16
2^3	2.25.1109.6653 ;	19013:5.3881 ;	2.13.37.53.7237 ;	18
11.2^9	2.17.932593 ;	25.1268777 ;	2.5.3173069 ;	16
2^{10}	2.5.13.241.4049 ;	29.4375093 ;	2.63450113 ;	17
2^{11}	2.5.37.577.2377 ;	5.1621.62617 ;	2.17.14928113 ;	18
2^{12}	2.1014976513 ;	101.20099437 ;	2.125.1933.4201 ;	19
$11^2.2^5$	2.257.29153 ;	5.757:17.233 ;	2.5.137.10949 ;	15
2^6	2.25.13.92237 ;	59969537 ;	2.4721.6353 ;	16
2^7	2.5.23984717 ;	15313:5.13.241 ;	2.41.2925721 ;	17
2^8	2.37.53.244633 ;	10433.91969 ;	2.5.61.113.13921 ;	18
$11^3.2^2$	2.5.53.193.277 ;	29.977413 ;	2.13.17.64153 ;	15
2^3	2.5.11335861 ;	5.97 ; 157.1489 ;	2.109.137.3797 ;	17
2^4	2.4013.56501 ;	453519617 ;	2.5.17.1093.2441 ;	18
2^5	2.1013.895357 ;	5.362815693 ;	2.25.13.37.241.313 ;	19
$11^4.2$	2.13.137.240701 ;	113.257 : 25.1181 ;	2.5.41.2091449 ;	18

High Bin-Aurifeuillians, $N = X^4 + 4Y^4 = L.M > 10^{15}$; $[X = x^\alpha,\ Y = 2^\beta]$.

$L = (x^\alpha \sim 2^\beta)^2 + 2^{2\beta}$, $M = (x^\alpha + 2^\beta)^2 + 2^{2\beta}$.

N	L	M	N	L	M
$3^4 + 2^{50}$	5·17·233·1693;	37·337·2693;	$5^8 + 2^{54}$	13·173·59497;	134627953 ? ‡
3^8 +	5·13·515089;	41.820201;	5^{12} +	17·41·61·3109;	13·29·193·1873;
3^{12} +	33333977;	5·6755269;	5^{16} +	4349·28597;	13·11142181;
3^{16} +	5417·6073;	5·41·166949;	7^4 +	17·113·69809;	5·26866493 ?
5^4 +	73·459089;	17·1976201;	11^4 +	125·1072301;	17·7905769;
5^8 +	33350257;	29·373·3121;	$3^4 + 2^{58}$	89·6031153;	25·13·17·17·5717;
5^{12} +	32546057;	34594057;	3^8 +	53·10124077;	5·313·343237;
5^{16} +	53·113·4813;	39065057;	3^{12} +	5·107197381 ? ‡	537756377 ? ‡
7^4 +	149·224813;	25·13·103421;	3^{16} +	5·106844453 ? ‡	13·41502437;
11^4 +	73·373·1229;	5·53·126961;	5^4 +	41·53·61·4049;	17·97·325673;
$3^4 + 2^{54}$	5·61·401·1097;	134266889 ? ‡	5^8 +	37·14487901;	537690737 ? ‡
3^8 +	134070353 ? ‡	5·29·926657;	5^{12} +	532790537 ? ‡	2309·234293;
3^{12} +	157·617·1381;	25·13·17·24373;	5^{16} +	29·17820053;	457·701·1741;
3^{16} +	5·197·134921;	101·1342093;	7^4 +	5· †	†
5^4 +	13·10318141;	134299673 ? ‡	11^4 +	5·13·29·241·1181;	537231481 ?

Dimorph Bin-Aurifeuillians, $N = x^4 + 4y^4 = \xi^4 + 4\eta^4 = L.M$.

$\tau = \tau_0^4 + 2\upsilon_0^4$; $\upsilon = 2\tau_0\upsilon_0.t_0$; $t_0 = (\tau_0^4 \sim 2\upsilon_0^4)^{\frac{1}{2}}$; $t = (\tau^4 \sim 2\upsilon^4)^{\frac{1}{2}}$. $x = t,\ y = \tau\upsilon,\ \xi = \tau^2,\ \eta = \upsilon^2$.

$\tau_0\ \upsilon_0\ t_0$	τ	υ	t	x	y	ξ	η	L	M
1, 1, 1	3,	2,	7	7,	6	9,	4	37;	5·41;
3, 2, 7	113,	84,	7967	7967,	9492	12769,	7056	1553·59513;	5·277·389·733;
. . .	1,	13,	239	239,	13	1,	169	5·37·277;	41·1553;
1, 13, 239	57123,	6214,	t	t,	$\tau\upsilon$	τ^2,	υ^2		
. . .	1343,	1525,	t	2750257,	2048075	1803649,	2325625	59513·95457977 ?	389·733·78767173 ?

Dimorph Bin-Aurifeuillian Products.

$$(N_1 N_3 N_5 \dots N_{2r+1}) . N_\beta = (N_0 N_2 N_4 \dots N_{2r}) . N_\alpha ; \quad N_r = x^4 + 4y^4 = L_r . M_r .$$

i. $\quad N_r = (x_r) = x_r^4 + 4.2^4; \quad N_\alpha = \{\alpha\} = \alpha^4 + 4.1^4; \quad N_\beta = \{\beta\} = \beta^4 + 4.1^4.$

$x_0 = 4r^2 + 4r + 3, \quad x_1 = x_0 + 4, \quad x_2 = x_1 + 4, \quad \dots, \quad x_{r+1} = x_r + 4; \quad \alpha = 2r + 3, \quad \beta = 2r + 1.$

$$(x_1)(x_3)(x_5)\dots(x_{2r+1}) . \{\beta\} = (x_0)(x_2)(x_4)\dots(x_{2r}) . \{\alpha\}.$$

$r = 0 \qquad r = 1 \qquad r = 2 \qquad r = 3 \qquad$ &c.

$$\frac{(7)}{(3)} = \frac{\{3\}}{\{1\}}; \quad \frac{(15)(23)}{(11)(19)} = \frac{\{5\}}{\{3\}}; \quad \frac{(31)(39)(47)}{(27)(35)(43)} = \frac{\{7\}}{\{5\}}; \quad \frac{(55)(63)(71)(79)}{(51)(59)(67)(75)} = \frac{\{9\}}{\{7\}}; \quad \text{\&c.}$$

$$\frac{(7)}{(3)} . \frac{(15)(23)}{(11)(19)} . \frac{(31)(39)(47)}{(27)(35)(43)} \dots\dots \frac{(4r^2+4r+7)(4r^2+4r+15)\dots(4r^2+12r+7)}{(4r^2+4r+3)(4r^2+4r+11)\dots(4r^2+12r+3)} = \frac{\{2r+3\}}{\{1\}}$$

ii. $\quad N_\rho \doteq \{x_\rho\} = x_\rho^4 + 4.1^4; \quad N_\alpha = [\alpha] = 1^4 + 4\alpha^4; \quad N_\beta = [\beta] = 1^4 + 4\beta^4.$

$x_0 = 2\rho^2 + 1, \quad x_1 = x_0 + 2, \quad x_2 = x_1 + 2, \quad \dots, \quad x_{r+1} = x_r + 2; \quad r = 2\rho + 1, \quad \alpha = \rho + 2, \quad \beta = \rho.$

$$\{x_1\}\{x_3\}\{x_5\}\dots\{x_{2r+1}\} . [\beta] = \{x_0\}\{x_2\}\{x_4\}\dots\{x_{2r}\} . [\alpha].$$

$\rho = 0, \ r = 1 \qquad \rho = 2, \ r = 5 \qquad \rho = 4, \ r = 9 \qquad$ &c.

$$\frac{\{3\}\{7\}}{\{1\}\{5\}} = \frac{[2]}{[0]}; \quad \frac{\{11\}\{15\}\{19\}\{23\}\{27\}\{31\}}{\{9\}\{13\}\{17\}\{21\}\{25\}\{29\}} = \frac{[4]}{[2]}; \quad \frac{\{35\}\{39\}\{43\}\dots\{71\}}{\{33\}\{37\}\{41\}\dots\{69\}} = \frac{[6]}{[4]}; \quad \text{\&c.}$$

$$\frac{\{3\}\{7\}}{\{1\}\{5\}} . \frac{\{11\}\{15\}\{19\}\dots\{31\}}{\{9\}\{13\}\{17\}\dots\{29\}} . \frac{\{35\}\{39\}\dots}{\{33\}\{37\}\dots} \dots\dots \frac{\{2\rho^2+3\}\{2\rho^2+7\}\dots\{2\rho^2+8\rho+7\}}{\{2\rho^2+1\}\{2\rho^2+5\}\dots\{2\rho^2+8\rho+5\}} = \frac{[\rho+2]}{[0]}.$$

$\rho = 1, \ r = 3 \qquad \rho = 3, \ r = 7 \qquad \rho = 5, \ r = 11$

$$\frac{\{5\}\{9\}\{13\}\{17\}}{\{3\}\{7\}\{11\}\{15\}} = \frac{[3]}{[1]}; \quad \frac{\{21\}\{25\}\{29\}\dots\{49\}}{\{19\}\{23\}\{27\}\dots\{47\}} = \frac{[5]}{[3]}; \quad \frac{\{53\}\{57\}\dots\{97\}}{\{51\}\{55\}\dots\{95\}} = \frac{[7]}{[5]}.$$

$$\frac{\{5\}\{9\}\{13\}\{17\}}{\{3\}\{7\}\{11\}\{15\}} . \frac{\{21\}\{25\}\dots\{49\}}{\{19\}\{23\}\dots\{47\}} . \frac{\{53\}\dots}{\{51\}\dots} \dots\dots \frac{\{2\rho^2+3\}\{2\rho^2+7\}\dots\{2\rho^2+8\rho+7\}}{\{2\rho^2+1\}\{2\rho^2+5\}\dots\{2\rho^2+8\rho+3\}} = \frac{[\rho+2]}{[0]}.$$

Dimorph Bin-Aurifeuillian Products (continued).

$$(\mathrm{N}_1\mathrm{N}_3\mathrm{N}_5\ldots\mathrm{N}_{2r+1}).\mathrm{N}_\beta = (\mathrm{N}_0\mathrm{N}_2\mathrm{N}_4\ldots\mathrm{N}_{2r}).\mathrm{N}_\alpha\,; \quad \mathrm{N}_r = x_r^4 + y_r^4 = \mathrm{L}_r.\mathrm{M}_r.$$

iii. $$\mathrm{N}_\rho = [y_\rho] = 1^4 + 4y_\rho^4\,; \quad |\rho| = \tau_\rho^4 + 4v_\rho^4\,; \quad \tau_\rho^2 - 2v_\rho^2 = +1.$$

$$y_1 = \tau_{\rho-1}^2, \quad y_2 = y_1 + 1, \quad y_3 = y_2 + 1, \quad \ldots, \quad y_r = \tau_\rho^2 - 1.$$

$$[y_2][y_4][y_6]\ldots[y_r].|\rho-1| = [y_1][y_3][y_5]\ldots[y_{r-1}].|\rho|.$$

$$\rho = 1 \qquad\qquad \rho = 2 \qquad\qquad \rho = 3$$

$$\frac{[2][4][6][8]}{[1][3][5][7]} = \frac{|1|}{|0|}; \quad \frac{[10][12][14]\ldots[288]}{[9]\;[11][13]\ldots[287]} = \frac{|2|}{|1|}; \quad \frac{[290][292]\ldots[9800]}{[289][291]\ldots[9799]} = \frac{|3|}{|2|}.$$

$$\frac{[2][4][6][8]}{[1][3][5][7]}.\frac{[10][12]\ldots[288]}{[9]\;[11]\ldots[287]}.\frac{[290]\ldots}{[289]\ldots}.\frac{[\tau_{\rho-1}^2+1][\tau_{\rho-1}^2+3]\ldots[\tau_\rho^2-1]}{[\tau_{\rho-1}^2][\tau_{\rho-1}^2+2]\ldots[\tau_\rho^2-2]} = \frac{|\rho|}{|0|}.$$

iv. $$\mathrm{N}_\rho = [y_\rho] = 1^4 + 4y^4\,; \quad |\rho| = \tau_\rho'^4 + 4v_\rho'^4\,; \quad \tau_\rho'^2 - 2v_\rho'^2 = -1.$$

$$y_0 = \tau_{\rho-1}'^2, \quad y_1 = y_0 + 1, \quad y_2 = y_1 + 1, \quad \ldots, \quad y_r = \tau_\rho'^2.$$

$$[y_2][y_4][y_6]\ldots[y_r].|\rho-1| = [y_1][y_3][y_5]\ldots[y_{r-1}].|\rho|.$$

$$\rho = 2 \qquad\qquad \rho = 3$$

$$\frac{[3][5][7]\ldots[49]}{[2][4][6]\ldots[48]} = \frac{|2|}{|1|}; \quad \frac{[51][53][55]\ldots[1681]}{[50][52][54]\ldots[1680]} = \frac{|3|}{|2|}.$$

$$\frac{[3][5][7]\ldots[49]}{[2][4][6]\ldots[48]}.\frac{[51][53]\ldots[1681]}{[50][52]\ldots[1680]}\cdots\cdots\frac{[\tau_{\rho-1}'^2+2][\tau_{\rho-1}'^2+4]\ldots[\tau_\rho'^2]}{[\tau_{\rho-1}'^2+1][\tau_{\rho-1}'^2+3]\ldots[\tau_\rho'^2-1]} = \frac{|\rho|}{|1|}.$$

Compound Bin-Aurifeuillians, $\mathbf{N}_r = \mathrm{X}_r^4 + 4\mathrm{Y}_r^4$.

$$\mathrm{N}_r = x_r^4 + 4y_r^4 = \mathrm{L}_r.\mathrm{M}_r\,; \quad \mathbf{N}_1 = \mathrm{N}_1\,; \quad \mathbf{N}_r = \mathrm{N}_1.\mathrm{N}_2.\mathrm{N}_3\ldots\mathrm{N}_r.$$

r	1	2	3	4	Fig. in $\mathbf{N}_4$.
x_r, y_r	3, 2	5, 2	17, 15	161, 240	
L_r, M_r	5 : 29 ;	13:53 ;	229 : 1429 ;	63841 : 218401 ;	
X_r, Y_r	3, 2	17, 8	161, 289	141121, 25921	**21**
x_r, y_r	3, 4	3, 5	41, 40	1519, 720	
L_r, M_r	17:5·13 ;	29:89 ;	1601 : 8161 ;	1156801 :5531521 ;	
X_r, Y_r	3, 4	41, 9	1519, 1681	3344161, 2307361	27
x_r, y_r	5, 3	5, 4	7, 24	1201, 1200	
L_r, M_r	13: 73 ;	17:97 ;	5·173:29·53 ;	337·4273:7204801 ;	
X_r, Y_r	5, 3	7, 25	1201, 49	1437599, 1442401	26

Successive Pellians, $N_r = y_r^2 + 1 = D . x_r^2$.

[y_r in first line, x_r (factorised) in second line.]

D	r	y_r and x_r.
2	1	1 1 ;
	2	7 5 ;
	3	41 29 ;
	4	239 13·13 ;
	5	1393 5 ; 197 ;
	6	8119 5741 ;
	7	47321 33461 ;
	8	275807 5 ; 29 ; 5·269 ;
	9	1607521 137·8297 ;
	10	9369319 37·179057 ;
	11	54608393 5 ; 13·13 ; 45697 ;
	12	318281039 229·982789 ;
	13	1855077841 29 ; 1549·29201 ;
	14	10812186007 5 ; 197 ; 53·146449 ;
	15	63018038201 44560482149 ? ‡
	16	367296043199 61·1301·3272609 ;
	17	2140758220993 5 ; 5741 ; 52734529 ;
	18	12477253282759 29 ; 13·13 ; 1800193921 ? ‡
	19	72722761475561 593·86716286317 ? ‡
	20	423859315570607 5 ; 33461 ; 389·4605197 ;
	23	83922003724759193 25·29·269 ; 197·6481·238321 ;
5	1	2 1 ;
	2	38 17 ;
	3	682 5·61 ;
	4	12238 13·421 ;
	5	219602 17 ; 53·109 ;
	6	3940598 89·19801 ;
	7	70711162 233·135721 ;
	8	1268860318 57 ; 5·61 ; 109441 ;
10	1	3 1 ;
	2	117 37 ;
	3	4443 5·281 ;
	4	168717 53353 ;
	5	6406803 37 ; 17·3221 ;
13	1	18 5 ;
	2	23382 5 ; 1297 ;
	3	30349818 5 ; 5·109·3089 ;
17	1	4 1 ;
	2	268 5·13 ;
	3	17684 4289 ;
	4	1166876 283009 ;
26	1	5 1 ;
	2	515 101 ;
	3	52525 10301 ;
	4	5357035 197·5333 ;
	5	546365045 101 ; 37·53·541 ;
29	1	70 13 ;
	2	1372210 13 ; 17·1153 ;
37	1	6 1 ;
	2	882 5·29 ;
	3	128766 21169 ;
	4	18798954 13·237733 ;
	5	2744518518 5·29 ; 17·183041 ;
41	1	32 5 ;
	2	131168 5·17·241 ;
	3	537526432 25·3357901 ;
53	1	182 25 ;
	2	24114818 25 ; 37·3581 ;
58	1	99 13 ;
	2	3881493 13 ; 5·7841 ;

High Pellians, $(N) > 10^8$.

$N = (y^2 + 1) = D . x^2$; [x factorised].

y	D	x	y	D	x
11 782	701	5.89 ;	5 534 843	2.373	5.40529 ;
14 942	5.193	13.37 ;	8 118 568	857	25.11093 ;
18 018	1093	5.109 ;	8 890 182	109	25.34061 ;
20 457	2.13.29	5.149 ;	14 752 278	5.113	13.47741 ;
21 490	1229	613 ;	15 489 282	829	5.17.6329 ;
23 156	233	37.41 ;	18 245 310	709	13.52709 ;
25 382	1373	5.137 ;	21 019 276	1097	13.48817 ;
28 488	5.13.17	857 ;	24 314 110	461	17.29.2297 ;
29 718	61	5.761 ;	24 715 982	797	5.13.13469 ;
29 851	2.661	821 ;	27 628 256	1217	791969 ;
30 235	2.397	29.37 ;	33 995 032	17.73	5.17.11353 ;
38 899	2.709	1033 ;	41 009 716	617	17.97117 ;
42 801	2.13.53	1153 ;	54 610 269	2.389	17.41.53.53 ;
45 368	29.37	5.277 ;	70 600 734	1213	2027117 ;
69 051	2.269	13.229 ;	71 011 068	241	25.182969 ;
71 264	353	3793 ;	87 050 499	2.509	433.6301 ;
71 847	2.5.61	2909 ;	99 484 332	1489	5.233.2213 ;
84 906	997	2689 ;	126 862 368	313	5.17.29.2909 ;
104 092	5.277	2797 ;	128 377 240	521	733.7673 ;
113 582	149	5.1861 ;	153 352 043	2.733	5.801037 ;
114 669	2.17.37	53.61 ;	189 471 332	449	5.1788341 ;
174 293	2.277	5.1481 ;	218 623 878	5.137	13.29.22157 ;
217 318	1301	25.241 ;	348 345 108	5.173	233.50833 ;
241 326	877	29.281 ;	393 166 618	1013	5.29.85193 ;
328 173	2.5.97	41.257 ;	395 727 950	509	41.427813 ;
348 711	2.569	10337 ;	419 288 307	2.461	25.552341 ;
352 618	317	5.17.233 ;	731 069 390	941	17.37.37889 ;
409 557	2.149	25.13.73 ;	854 992 268	1193	5.1097.4513 ;
600 632	593	5.4933 ;	1 111 225 770	181	13.17.97.3853 ;
683 982	17.29	5.61.101 ;	2 291 286 382	653	5.13.1379461 ;
964 140	13.37	43961 ;	2 746 864 744	953	449.198173 ;
1 063 532	281	5.12689 ;	2 894 863 832	569	5.17.1427753 ;
1 262 101	2.541	17.37.61 ;	2 959 961 778	5.233	53.797.2053 ;
1 343 018	773	5.9661 ;	4 115 086 707	2.293	5.61.349.1597 ;
1 369 326	757	157.317 ;	5 767 329 724	1433	701.217337 ;
1 764 132	193	5.109.233 ;	6 547 100 182	1493	25.61.111109 ;
2 086 882	17.61	5.13.997 ;	7 376 748 868	977	5.13.3630817 ;
3 375 918	1429	5.53.337 ;	8 920 484 118	277	5.157.682777 ;
3 434 907	2.13.41	5.53.397 ;	20 478 302 982	397	5.17.37.173.1889 ;
4 832 118	157	5.13.17.349 ;	106 316 171 432	881	25.173.389.2129 ;
			930 015 700 509	2.701	113.181.1214393 ;

High Pellians, $N = Y^2+1 = D.X^2 > 15.10^{25}$; $Y = (4y^3+3y)$.

y	$Y = 4y^3+3y$	$D = y^2+1$	$X = (2y)^2+1$	Fig.
24 997	62 477 502 774 883	2.5.181.345221 ;	853.1069.2741 ;	28
8	62 485 001 274 962	5.8237.15173 ;	797.3136261 ;	28
9	62 492 500 374 993	2.61.5122541 ;	5.109.953.4183 ;	28
25 000	62 500 000 075 000	241.2593361 ;	2500000001 ;	28
1	62 507 500 375 007	2.41.7622561 ;	5.53.9434717 ;	28
2	62 515 001 275 038	5.125020001 ;	7561.330697 ;	28

High Pellians, $N = (Y^2+1) = D.X^2$; $Y = (4y^6+3y^2)$; $[y = \eta^r,\ \eta \not> 12]$.

$D = (y^4+1)$, $X = (4y^4+1) = L.M$; $L = \frac{1}{2}\left\{(2y-1)^2+1\right\}$, $M = \frac{1}{2}\left\{(2y+1)^2+1\right\}$.

η^r	Y	$D = \eta^{4r}+1$	L	M	Fig.
3	$4.\ 3^6+3.\ 3^2$	2.41 ;	13 ;	25 ;	7
3^2	$4.\ 3^{12}+3.\ 3^4$	2.17.193 ;	5.29 ;	181 ;	13
3^3	$4.\ 3^{18}+3.\ 3^6$	2.41 ; 6481 ;	5.281 ;	17.89 ;	19
3^4	$4.\ 3^{24}+3.\ 3^8$	2.21523361 ; B	13.997 ;	5.2657 ;	25
3^5	$4.\ 3^{30}+3.\ 3^{10}$	2.41 ; 42521761 ;	337.349 ;	5.37.641 ;	30
3^6	$4.\ 3^{36}+3.\ 3^{12}$	2.17.193 ; 97.577.769 ;	25.42457 ;	1064341 ;	36
5	$4.\ 5^6+3.\ 5^2$	2.313 ;	41 ;	61 ;	10
5^2	$4.\ 5^{12}+3.\ 5^4$	2.17.11489 ;	1201 ;	1301 ;	18
5^3	$4.\ 5^{18}+3.\ 5^6$	2.313 ; 390001 ;	29.1069 ;	17.17.109 ;	27
5^4	$4.\ 5^{24}+3.\ 5^8$	2.2593.29423041 ; De	53.14717 ;	782501 ;	35
5^5	$4.\ 5^{30}+3.\ 5^{10}$	2.313 ; 241.632133361 ;	19525001 ;	73.267637 ;	44
6	$4.\ 6^6+3.\ 6^2$	1297 ;	61 ;	5.17 ;	11
6^2	$4.\ 6^{12}+3.\ 6^4$	17.98801 ;	2521 ;	5.13.41 ;	20
6^3	$4.\ 6^{18}+3.\ 6^6$	1297 ; 1678321 ;	293.317 ;	5.18749 ;	30
6^4	$4.\ 6^{24}+3.\ 6^8$	353.1697.4709377 ;	3356641 ;	25.29.4637 ;	39
7	$4.\ 7^6+3.\ 7^2$	2.1201 ;	5.17 ;	113 ;	12
7^2	$4.\ 7^{12}+3.\ 7^4$	2.17.169553 ;	5.941 ;	13.13.29 ;	22
7^3	$4.\ 7^{18}+3.\ 7^6$	2.1201 ; 73.193.409 ;	234613 ;	5.109.433 ;	32
10	$4.10^6+3.10^2$	73.137 ;	181 ;	13.17 ;	14
10^2	$4.10^{12}+3.10^4$	17.5882353 ;	19801 ;	20201 ;	26
10^3	$4.10^{18}+3.10^6$	73.137 ; 99990001 ; Lo	277.7213 ;	2002001 ;	38
10^4	$4.10^{24}+3.10^8$	353.449.641.1409.69857 ;	13.41.457.821 ;	569.351529 ;	50
11	$4.11^6+3.11^2$	2.7321 ;	13.17 ;	5.53 ;	14
11^2	$4.11^{12}+3.11^4$	2.17.6304673 ;	113.257 ;	25.1181 ;	27
11^3	$4.11^{18}+3.11^6$	2.7321 ; 10657.20113 ;	3540461 ;	5.709157 ;	39
12	$4.12^6+3.12^2$	89.233 ;	5.53 ;	313 ;	15
12^2	$4.12^{12}+3.12^4$	17.97.260753 ;	5.8237 ;	41761 ;	28
12^3	$4.12^{18}+3.12^6$	89.233 ; 193.2227777 ;	17.109.3221 ;	25.239017 ;	41

High Pellians, $N = Y^2+1 = D.X^2$. $Y = (4y^3+3y)$.

y	$4y^3+3y$	$D = y^2+1$	$X = (2y)^2+1$	Fig.
3	$4.3^3+3.3$	2.5 ;	37 ;	5
3^3	$4.3^9+3.3^3$	2.5 ; 73 ;	2917 ;	10
3^5	$4.3^{15}+3.3^5$	2.5 ; 5.1181 ;	13.18169 ;	16
3^7	$4.3^{21}+3.3^7$	2.5 ; 29.16493 ;	19131877 ;	22
3^9	$4.3^{27}+3.3^9$	2.5 ; 73 ; 530713 ;	397.3903481 ;	27
5	$4.5^3+3.5$	2.13 ;	101 ;	6
5^3	$4.5^9+3.5^3$	2.13 ; 601 ;	62501 ;	14
5^5	$4.5^{15}+3.5^5$	2.13 ; 41.9161 ;	3529.11069 ;	23
5^7	$4.5^{21}+3.5^7$	2.13 ; 234750601 ; B	89.641.427949 ;	30
6	$4.6^3+3.6$	37 ;	5.29 ;	6
6^3	$4.6^9+3.6^3$	37 ; 13 : 97 ;	125.1493 ;	16
6^5	$4.6^{15}+3.6^5$	37 ; 241.6781 ;	5.53.193.4729 ;	25
7	$4.7^3+3.7$	2.25 ;	197 ;	7
7^3	$4.7^9+3.7^3$	2.25 ; 13.181 ;	470597 ;	17
7^5	$4.7^{15}+3.7^5$	2.25 ; 5.281.4021 ;		27

Pellian Bin-Aurifeuillian Chain (N_r).

$$N_r = x_r^4+4y_r^4 = L_r.M_r;\quad x_r^2-2y_r^2 = (-1)^r;\quad L_{r+1} = M_r = y_{2r+1}.$$

r =	0	1	2	3	4	5	6	7
x, y =	1, 0	1, 1	3, 2	7, 5	17, 12	41, 29	99, 70	239, 169
M =	1 ;	5 ;	29 ;	13.13 ;	5.197 ;	5741 ;	33461 ;	5.29 ; 5.269 ;

r =	8	9	10	11	12
x, y =	577, 408	1393, 985	3363, 2378	8119, 5741	19601, 13860
M =	137.8297 ;	37.179057 ;	5.13.13 ; 45697 ;	229.982789 ;	29 ; 1549.29201 ;

r =	13	14	15	16
x, y =	47321, 33461	114243, 80782	275807, 195025	665857, 470832
M =	5.197 ; 53.146449 ;	44560482149 ? ‡	61.1301.3272609 ;	5.5741 ; 52734529 ;

r =	17	18	19
x, y =	1607521, 1136689	3880899, 2744210	9369319, 6625109
M =	29.13.13 ; 1800193921 ? ‡	593.86716286317 ? ‡	5.33461 ; 389.4605197 ;

High Pellian Bin-Aurifeuillian Chain.

$$N_r = Y_r^2+1 = D_r.X_r^2;\quad D_r = 2^{2^r}+1 = X_{r-1},\quad D_{r+1} = 2^{2^{r+2}}+1 = X_r.$$

r =	11,	12,	13,	14,	15,	16,	17,	18,	19,	20,	21,	22,	23,	24,	25,	26
Fig. in N :	22,	23,	25,	27,	29,	31,	32,	34,	36,	38,	39,	41,	43,	45,	47,	49
r =	27,	28,	29,	30,	31,	32,	33,	34,	35,	36,	37,	38,	39,	40,	41,	42
Fig. in N :	50,	52,	54,	56,	58,	59,	61,	63,	65,	67,	69,	70,	72,	74,	76,	78

Factors of D_r, X_r all known (by Lucas's Tables) except for $X_{33} = D_{34}$, $X_{37} = D_{38}$, $X_{39} = D_{40}$.

Bin-Aurifeuillian Chains.

$$N_r = x_r^4 + 4y_r^4 = L_r . M_r\,; \quad x_r^2 - 2y_r^2 = (-1)^r . z_0 \quad [z_0 \text{ const.}].$$

$$x_{r-1} + 2y_{r-1} = x_r = x_{r+1} \sim 2y_{r+1}, \quad x_{r-1} + y_{r-1} = y_r = x_{r+1} \sim y_{r+1}$$

$$L_r = y_{r-1}^2 + y_r^2 = M_{r-1}, \quad M_r = y_r^2 + y_{r+1}^2 = L_{r+1}.$$

$z_0 = 7$	r	0	1	2	3	4	5	6	7	8	9
	x, y	3, 1	5, 4	13, 9	31, 22	75, 53	181, 128	437, 309	1055, 746	2547, 1801	6149, 4348
	L	5 ;	17 ;	97 ;	5·113 ;	37·89 ;	17·1129 ;	5·13·1721 ;	651997 ;	61·62297 ;	5·17·260573 ;
	M	17 ;	97 ;	5·113 ;	37·89 ;	17·1129 ;	5·13·1721 ;	651997 ;	61·62297 ;	5·17·260573 ;	†
	x, y	.	1, 2	5, 3	11, 8	27, 19	65, 46	157, 111	379, 268	915, 647	2209, 1562
	L	.	5 ;	13 ;	73 ;	25·17 ;	2477 ;	14437 ;	5·16829 ;	17·17·1697 ;	13·219881 ;
	M	.	13 ;	73 ;	25·17 ;	2477 ;	14437 ;	5·16829 ;	17·17·1697 ;	13·219881 ;	5·53·62869 ;

$z_0 = 17$	r	0	1	2	3	4	5	6	7	8
	x, y	5, 2	9, 7	23, 16	55, 39	133, 94	321, 227	775, 548	1871, 1323	4517, 3194
	L	13 ;	53 ;	5·61 ;	1777 ;	10357 ;	5·12073 ;	37·37·257 ;	13·233·677 ;	5·1373·1741 ;
	M	53 ;	5·61 ;	1777 ;	10357 ;	5·12073 ;	37·37·257 ;	13·233·677 ;	5·1373·1741 ;	†
	x, y	.	1, 3	7, 4	15, 11	37, 26	89, 63	215, 152	519, 367	1253, 886
	L	.	13 ;	25 ;	137 ;	797 ;	5·929 ;	27073 ;	157793 ;	5·13·14149 ;
	M	.	25 ;	137 ;	797 ;	5·929 ;	27073 ;	157793 ;	5·13·14149 ;	73·97·757 ;

High Pellian Chains, $N_r = Y_r^2 + 1 = D_r . X_r^2 > 10^{20}$.

$Y_r = (4y_r^3 + 3y_r)$, $D_r = y_r^2 + 1$, $D_{r+1} = (2y_r)^2 + 1 = X_r$; $y_{r+1} = 2y_r$, $y_r = 2^r . \eta^a$;

$[\eta = 3, 5, 7, 11]$.

r, η^a	$D_r = y_r^2 + 1$	Fig.
9, 3^1	61.38677 ;	21
10	5.1887437 ;	23
11	13.2903749 ;	24
12	5.30198989 ;	26
13	1093.552589 ;	28
14	5.483183821 ;	30
15	73.4969.26641 ;	32
16	5.29.173.1540949 ;	33
17	13.97.122616037 ;	35
18	3125.	37
19	37.37.1933.934861 ;	39
20	5.389.	40
8, 3^2	5308417 ;	22
9	4513 : 5.941 ;	24
10	41.137.15121 ;	25
11	17.29.37 : 125.149 ;	27
12	281.1009.4793 ;	29
13	5.14669 : 13.5701 ;	31
14	§	33
15	5.89.661 : 17.17393 ;	35
16	§	36
17	1178113 : 5.337.701 ;	38
18		
6, 3^3	5.29 ; 20593 ;	21
7	1069.11173 ;	23
8	5.269.35521 ;	25
9	577 ; 349 : 13.73 ;	27
10	5.6389.23929 ;	28
11	109.229.122497 ;	30
12	5.461 ; 5306113 ;	32
5, 3^4	2521 : 5.13.41 ;	22
6	1409.19073 ;	24
7	25.409 : 10513 ;	26
8	17.97.260753 ;	28
9	5.8237 : 41761 ;	29
10	§	31
11	165313 : 5.13.13.197 ;	33
12	17.2801.2311681 ;	35
13	662401 : 5.37.3593 ;	37
3, 3^5	73.51769 ;	21
4	5.3023309 ;	23
5	37 ; 241.6781 ;	25
6	5.53.193.4729 ;	27
7	13.1873.39733 ;	29
2, 3^6	8503057 ;	22
3	25.229 : 13.457 ;	24
4	1777.76561 ;	26
5, 3^6	29.797 : 5.17.277 ;	28
6	1297 ; 1678321 ;	30
7	293.317 : 5.18749 ,	32
0, 3^7	2.5 ; 29.16493 ;	22
1	19131877 ;	24
2	5.15305501 ;	25
3	61.5018197 ;	27
4	5.24488813 ;	29
0, 3^8	2.21523361 ; B	25
1	13.997 : 5.2657 ;	26
2	17.40514561 ;	28
3	5.10433 : 52813 ;	30
9, 5^1	2141.3061 ;	22
10	26214401 ;	24
11	104857601 ;	26
12	13.32263877 ;	28
13	29.389.148721 ;	29
14	1481.4531321 ;	31
15	3229.8313269 ;	33
16		35
17	2129.	37
18	13.41.53.653.93133 ;	38
19	89.2161.35730169 ;	40
6, 5^2	769.3329 ;	21
7	3121 : 17.193 ;	23
8	40960001 ; B	25
9	12641 : 13.997 ;	26
10	655360001 ;	28
11	17.41.73 : 51521 ;	30
12	§	32
13	204161 : 205441 ;	34
14	§	35
15	13.17.3701 : 820481 ;	37
4, 5^3	41.97561 ;	22
5	109.229.641 ;	23
6	401 ; 13.12277 ;	25
7	256000001 ;	27
8	157.6522293 ;	29
9	1601 ; 61.41941 ;	31
10		32
11	29.701.3223769 ;	34
12	37.173 ; 13.13.242329 ;	36
13	89.521.3181.7109 ;	38
14	41.	40
15	25601 ; 349.1009.1861 ;	41
16	1597.	43

High Pellian Chains, $N_r = Y_r^2 + 1 = D_r . X_r^2 > 10^{20}$.

$Y_r = (4y_r^3 + 3y_r)$, $D_r = y_r^2 + 1$, $D_{r+1} = (2y_r)^2 + 1 = X_r$; $y_{r+1} = 2y_r$, $y_r = 2^r . \eta^a$;

$[\eta = 3, 5, 7, 11]$.

$r,\ \eta^a$	$D_r = y_r^2 + 1$	Fig.
2, 5^4	97.64433 ;	22
3	13.13.29 : 5101 ;	24
4	17.5882353 ;	26
5	19801 : 20201 ;	28
6	1889 : 847009 ;	29
7	79601 : 37.41.53 ;	31
8	17. §	33
9	319201 : 13.24677 ;	35
10	§	37
11	1278401 : 757.1693 ;	38
12	17.17.113.337.641.929 ;	40
13	661.7741 : 5123201 ;	42
14		44
15	20473601 : 13.409.3853 ;	46
16	17.977. §	47
17	61.1342741 : 29.41.68909 ;	49
0, 5^5	2.13 ; 41.9161 ;	23
1	3529.11069 ;	24
2	37.4222973 ;	26
3	241.2593361 ;	28
4	2500000001 ;	30
5	101 ; 3541 : 27961 ;	32
8, 7^1	5.642253 ;	21
9	29.233.1901 ;	23
10	25.13.13.12161 ;	25
11	113.1818769 ;	27
12	5.164416717 ;	28
5, 7^2	17.89 : 125.13 ;	21
6	9834497 ;	23
7	61.101 : 5.1277.;	24
8	3361.46817 ;	26
9	5.4973 : 17.1489 ;	28
10	2777.906601 ;	30
11	5.13.29.53 : 100801 ;	32
12	41.73.337.39937 ;	34
13	97.4129 : 5.17.4733 ;	35
14	761.5441.155657 ;	37
15	181.8861 : 25.113.569 ;	39
16	89.	41
17	5.13.17.37.157 : 6426113 ;	43
3, 7^3	197 ; 37.1033 ;	22
4	5.6023629 ;	24
5	109.113.9781 ;	26
6	5.157 ; 13.47221 ;	28
7		30
8	5.617.2499269 ;	31
9	3137 ; 9831361 ;	33
10, 7^3	25.269.	35
11		37
12	5.13.193 ; 1741.90373 ;	39
0, 7^4	2.17.169553 ;	22
1	5.941 : 13.13.29 ;	24
2	401.230017 ;	26
3	19013 ; 5.3881 ;	27
4	17.5393 : 16097 ;	29
5	76441 ; 25.3089 ;	31
6	593.39818929 ;	33
7	5.37.1657 : 13.137.173 ;	35
8	17. §	36
9	5.41.53.113 : 1230881 ;	38
8, 11^1	13.609989 ;	22
9	25.1268777 ;	24
10	29.4375093 ;	26
11	5.1621.62617 ;	28
12	101.20099437 ;	30
4, 11^2	41.113.809 ;	21
5	5.757 : 17.233 ;	23
6	59969537 ;	25
7	15313 : 5.13.241 ;	27
8	10433.91969 ;	29
9	229.269 : 5.17.733 ;	30
10	§	32
11	5.73.677 : 181.1373 ;	34
12	401.881.695297 ;	36
13	25.17.17.137 : 13.29.2633 ;	38
14	41.97.	39
15	3962113 : 5.61.13009 ;	41
16	449.	43
17	17.932593 : 5.3173069 ;	45
1, 11^3	5.617.2297 ;	22
2	29.977413 ;	24
3	5.97 ; 157.1489 ;	26
4	453519617 ;	28
5	5.362815693 ;	29
0, 11^4	2.17.6304673 ;	27
1	25.113.257.1151 ;	29
2	§	30
3	5.41.569 : 337.349 ;	32
4	17. §	34
5	5.13.7193 : 29.16189 ;	36
6	577.5953.255617 ;	38
7	1872113 : 5.457.821 ;	39
8	17.241.3457.991873 ;	41
9	257.29153 : 5.137.10949 ;	43

Simple Quartans, $N = (y^4 + 1^4)$; [*y even*].

[All factors < 100,000 cast out.]

y	N	y	N	y	N
2	17;	102	5857.18481;	202	17.41.193.12377;
4	257;	104	17.1657.4153;	204	1731891457;
6	1297;	106	126247697;	206	17.521.203321;
8	17; 241;	108	1777.76561;	208	41.113.404009;
10	73.137;	110	17.17.506609;	210	1944810001;
12	89.233;	112	3361.46817;	212	17.118821361;
14	41.937;	114	1361.124097;	214	33601.62417;
16	65537;	116	353.512929;	216	1297; 1678321;
18	113.929;	118	193877777;	218	32353.69809;
20	160001;	120	41.5057561;	220	2342560001;
22	73.3209;	122	449.493393;	222	21001.115657;
24	331777;	124	73.3238649;	224	2777.906601;
26	17.26881;	126	41.89.69073;	226	337.7741121;
28	614657;	128	17; 15790321; L	228	2702336257;
30	241.3361;	130	97.2944433;	230	17.89.1849577;
32	17; 61681;	132	303595777;	232	41.1721.41057;
34	1336337;	134	17.17.1115633;	234	10321.290497;
36	17.98801;	136	73.233.20113;	236	17.193.945457;
38	41.50857;	138	17.21333761;	238	3208542737;
40	769.3329;	140	384160001;	240	17.6481.30113;
42	17.183041;	142	406586897;	242	3429742097;
44	41.113.809;	144	17.97.260753;	244	97.113.323377;
46	4477457;	146	7433.61129;	246	17.2153.100057;
48	5308417;	148	433.1108049;	248	3782742017;
50	97.64433;	150	41.193.63977;	250	457.8547593;
52	89.82153;	152	577.925121;	252	337.11966641;
54	8503057;	154	562448657;	254	4162314257;
56	9834497;	156	73.953.8513;	256	641.6700417;
58	2393.4729;	158	18433.33809;	258	97.929.49169;
60	17.281.2713;	160	655360001;	260	41.4513.24697;
62	761.19417;	162	17.40514561;	262	40177.117281;
64	257; 97.673;	164	723394817;	264	17.137.2085673;
66	17.409.2729;	166	89.8531833;	266	5006411537;
68	41.521497;	168	17.73.641897;	268	61961.83257;
70	17.353.4001;	170	457.1827593;	270	17.73.113.37897;
72	1409.19073;	172	17.51483121;	272	5473632257;
74	29986577;	174	916636177;	274	17.18089.18329;
76	17.569.3449;	176	10433.91969;	276	5802782977;
78	5449.6793;	178	17.41.137.10513;	278	5972816657;
80	40960001; B	180	1049760001;	280	17.361562353;
82	45212177;	182	113.617.15737;	282	73.86631049;
84	2089.23833;	184	193.5939009;	284	41.137.1158161;
86	7129.7673;	186	577.2074321;	286	2753.2430289;
88	59969537;	188	313.857.4657;	288	6879707137;
90	65610001; B	190	89.1753.8353;	290	41.8377.20593;
92	449.159553;	192	281.1009.4793;	292	569.1697.7529;
94	17.4592641;	194	1416468497;	294	3793.1969729;
96	41.137.15121;	196	17.5393.16097;	296	7676563457;
98	401.230017;	198	1536953617;	298	17.463891201;
100	17.5882353;	200	1889.847009;	300	8017.1010353;

Simple Quartans, $N = (y^4 + 1^4)$; [y *even*].
[All factors < 100,000 cast out.]

y	N	y	N	y	N
302	73.113947529;	402	1289.20260553;	502	17.281.13294121;
304	17.89.5644889;	404	53401.498857;	504	
306	67777.129361;	406	17.1598288641;	506	41.857.1865681;
308	17.313.1691257;	408	89.113.1601.1721;	508	17.55337.70793;
310	1601.5768401;	410	17.19961.83273;	510	257.8609.30577;
312	9475854337;	412	23321.1235497;	512	17; 241; 433.38737;
314	17.41.73.191057;	414		514	4073.17137129;
316	12697.785321;	416	17.73.24132457;	516	
318	313.641.50969;	418	46817.652081;	518	17.97.257.169889;
320		420	4457.6981593;	520	21737.3363673;
322	13297.808481;	422	337.94106561;	522	89.14449.57737;
324	97.113607841;	424	41.788278297;	524	1193.1801.35089;
326	673.16782449;	426		526	
328		428	73.6833.67273;	528	6961.11165137;
330	233.50897897;	430		530	41.34297.56113;
332	17.714666481;	432	14009.2486153;	532	97.825799841;
334		434	17.113.18468497;	534	593.137123009;
336	953.13374089;	436		536	17.41.193.613577;
338	17.97.1249.6337;	438	97.313.601.2017;	538	137.977.625913;
340		440	17.409.5390617;	540	
342	17.41.1433.13697;	442		542	17.601.8446441;
344	89.157341673;	444	17.17.134472673;	544	2969.29497513;
346	36017.397921;	446	401.98672257;	546	17.89.1993.29473;
348	17.7481.115321;	448	41.73.337.39937;	548	2017.44711201;
350	18433.814097;	450	17.257.2833.3313;	550	
352		452	97.137.241.13033;	552	17.5461442801;
354	433.36268129;	454	41.233.4447169;	554	
356	401.40054897;	456	2953.14641849;	556	
358	4673.3515089;	458	2161.20361377;	558	137.707646281;
360	15497.1083833;	460	73.613350137;	560	41.2398657561;
362	1193.14394409;	462	72169.631273;	562	73.1366540169;
364		464	1801.25737017;	564	521.194213177;
366	17.41.25744921;	466		566	
368	89.206063593;	468	17.17.165991393;	568	
370	137.281.486833;	470	113.431830177;	570	17.193.809.39769;
372	17.41.27475081;	472		572	2089.51244313;
374		474	17.241.12321041;	574	73.2473.601313;
376	17.1175716081;	476		576	17.2801.2311681;
378	409.49916473;	478	17.41.233.521.617;	578	257.10289.42209;
380	2129.9793969;	480	41113.1291177;	580	17.2377.2800489;
382	17.8513.147137;	482	89.606454393;	582	
384	35801.607337;	484	17.3227992561;	584	
386	35521.624977;	486	1097.50855561;	586	17.89.77938409;
388	1249.18145313;	488		588	41.193.1049.14401;
390	16217.1426553;	490	241.3761.63601;	590	353.4217.81401;
392	593.39818929;	492		592	3889.31582673;
394	22481.1071937;	494		594	73.1705386889;
396	41.599786777;	496	113.535609489;	596	769.6857.23929;
398	1033.24290249;	498		598	
400	17.1505882353;	500	13177.4743113;	600	

Simple Quartans, $N = (y^4 + 1^4)$; [*y even*].

[All factors < 100,000 cast out.]

y	N	y	N	y	N
602	233.563676649;	702		802	1033.400495049;
604	17.7828865521;	704	401.881.695297;	804	54601.7652857;
606	73.4561.405049;	706	17.	806	41.449.22925033;
608	2617.52216841;	708	73.3441994489;	808	17.4409.5686649;
610	17.8144612353;	710		810	
612	41.3421541657;	712	17.17.97.9167489;	812	137.3173244601;
614	17.8360352001;	714		814	17.313.82509577;
616		716	17.6073.2545657;	816	5657.78374441;
618	41.3557705897;	718	433.613775969;	818	17.
620	17.8691962353;	720	73.20873.176369;	820	
622	193.281.2759929;	722	17.113.141456017;	822	353.1293339569;
624		724	41.89.75297473;	824	17.26177.1035953;
626	761.201796057;	726	97.137.20905193;	826	97.1049.1481.3089;
628		728	3769.74524553;	828	
630	1609.97905289;	730		830	1217.389961553;
632	97.1644737441;	732		832	137.3497620921;
634	113.23473.60913;	734	75161.3861817;	834	41.41.287803777;
636	353.463504289;	736	6569.44669593;	836	521.937534777;
638	17.9746165761;	738		838	89.22433.247001;
640		740	17.73.241632361;	840	97.5132694433;
642	41.4143394217;	742		842	17.
644	17.137.73853993;	744	457.670464121;	844	
646	97.6961.257921;	746	17.	846	569.2129.422857;
648	17.	748		848	17.16057.1894393;
650	40841.4370761;	750	17.977.19050289;	850	
652	593.13217.23057;	752	41.73.409.261241;	852	17.42409.730889;
654	17.48017.224113;	754	281.1150215097;	854	73.7286326409;
656		756	17.617.31142473;	856	
658	45361.4132577;	758		858	17.41.1201.647401;
660	89.113.601.31393;	760		860	113.4840780177;
662	881.12721.17137;	762	1009.334140193;	862	20353.27126929;
664	457.8161.52121;	764	89.601.6369553;	864	41.77137.176201;
666	2657.74046641;	766	22769.15120673;	866	73.7704575369;
668	3121.63798737;	768		868	2081.11369.23993;
670	41.4914907561;	770	13841.25397761;	870	937.611416873;
672	17.1553.7724257;	772		872	9497.60880681;
674	12377.16673401;	774	17.8521.2477561;	874	
676	3617.57734881;	776	41.41.353.449.1361;	876	17.
678	17.	778		878	89.6677102713;
680	2897.73805233;	780	17.21817.998009;	880	
682	17.	782	41.9121014697;	882	17.
684	12953.16898729;	784	17.	884	409.1493089193;
686	2281.97089257;	786		886	17.73.113.4394249;
688	17.17.775275233;	788		888	41.
690		790	17.	890	2297.273148633;
692	1129.14057.14449;	792	1289.10009.30497;	892	17.
694	41.5657883817;	794	593.5897.113657;	894	
696	113.5233.396833;	796	233.1723043929;	896	761.5441.155657;
698		798		898	73.8908046729;
700	41.89.65798849;	800		900	337.401.4855073;

☞ *Continued on left of page* 119.

Simple Half-Quartans, $\frac{1}{2}N = \frac{1}{2}(y^4 + 1^4)$; [*y odd*].
[All factors < 100,000 cast out.]

y	½N	y	½N	y	½N
1	1;	101	89.584609;	201	593.1376257;
3	41;	103	56275441;	203	849090841;
5	313;	105	60775313;	205	883050313;
7	1201;	107	4201.15601;	207	457.2008793;
9	17.193;	109	41.1721441;	209	73.13068697;
11	7321;	111	17.337.13249;	211	241.4112281;
13	14281;	113	81523681;	213	17.60539593;
15	17.1489;	115	87450313;	215	89.12004217;
17	41761;	117	17.5511433;	217	13417.82633;
19	17.3833;	119	100266961;	219	17.41.881.1873;
21	97241;	121	17.6304673;	221	233.281.18217;
23	139921;	123	1153.99257;	223	17.1049.69337;
25	17.11489;	125	313.390001;	225	7121.179953;
27	41; 6481;	127	17.137.55849;	227	97.977.14009;
29	353641;	129	138461441;	229	17.73.1108001;
31	409.1129;	131	113.1303097;	231	1033.1378217;
33	97.6113;	133	3169.49369;	233	137.241.44633;
35	750313;	135	761.218233;	235	1321.1154353;
37	89.10529;	137	41.1409.3049;	237	353.4468777;
39	1156721;	139	617.302513;	239	809.1217.1657;
41	137.10313;	141	89.2220529;	241	73.97.238201;
43	17.193.521;	143	4561.45841;	243	41.42521761; B
45	401.5113;	145	17.13001489;	245	233.7731761;
47	97.25153;	147	97.137.17569;	247	17.1009.108497;
49	17.169553;	149	7001.35201;	249	41.241.194521;
51	73.46337;	151	17.15290753;	251	1984563001;
53	17.232073;	153	273990641;	253	17.257.468889;
55	41.111593;	155	17.17.998617;	255	89.23754217;
57	5278001;	157	113.2688377;	257	17.128307953;
59	17.593.601;	159	2521.126761;	259	2249930281;
61	6922921;	161	17.41.97.4969;	261	257.257.35129;
63	73.107897;	163	601.587281;	263	17.3041.46273;
65	8925313;	165	370600313;	265	2137.1153849;
67	937.10753;	167	41.9485321;	267	2541060761;
69	113.100297;	169	407865361;	269	30593.85577;
71	12705841; B	171	427518041;	271	241.2609.4289;
73	14199121; B	173	769.582409;	273	41.3089.21929;
75	1153.13721;	175	20129.23297;	275	2859570313;
77	17.89.11617;	177	881.557041;	277	569.5173409;
79	41.433.1097;	179	17.17.1776169;	279	89.34040569;
81	21523361; B	181	1777.301993;	281	17.183377633;
83	17.73.19121;	183	17977.31193;	283	353.9085337;
85	41.337.1889;	185	17.3041.11329;	285	433.2441.3121;
87	17.1684993;	187	6521.93761;	287	17.457.436649;
89	281.111641;	189	17.37529113;	289	18913.184417;
91	5297.6473;	191	41.16230041;	291	17.210907993;
93	17.2200153;	193	257.2699393;	293	25673.143537;
95	73.113.4937;	195	17.42526489;	295	113.33510401;
97	233.189977;	197	73.10316017;	297	17.7457.30689;
99	2617.18353;	199	784119601;	299	23633.169097;

Simple Half-Quartans, $\frac{1}{2}\mathrm{N} = \frac{1}{2}(y^4 + 1^4)$; [*y odd*].

[All factors < 100,000 cast out.]

y	$\frac{1}{2}$N	y	$\frac{1}{2}$N	y	$\frac{1}{2}$N
301	41.100104161;	401	137.1697.55609;	501	17.73.313.81097;
303	401.10509841;	403		503	
305	1913.2261801;	405		505	
307	33937.130873;	407	41.334629161;	507	137.1433.168281;
309	4558310681;	409	3769.3712249;	509	6353.5282777;
311	2897.1614593;	411	12041.1184881;	511	1481.23019641;
313	4798962481;	413	41.9049.39209;	513	
315	17.137.521.4057;	415		515	59929.586897;
317	5049019561;	417	17.889334833;	517	
319	89.58175849;	419		519	17.41.52048313;
321	17.113.257.10753;	421	97.137.1181969;	521	73.113.4466009;
323	641.8490281;	423	17.17.353.156913;	523	
325	17.41.1873.4273;	425	577.28271569;	525	17.2234386489;
327	449.12732529;	427	17.3889.251417;	527	33529.1150249;
329	16553.353897;	429	193.87748937;	529	17.3697.623009;
331	17.41.8610913;	431		531	9137.4350553;
333	6148185161;	433	17.89.11616697;	533	73.552784657;
335	2161.2914033;	435	97.617.299137;	535	17.97.24840737;
337	6448958881;	437	41.1129.393929;	537	
339	6603418121;	439	977.19007873;	539	
341	97.281.248033;	441	62897.300673;	541	449.95392169;
343	1201; 73.193.409;	443	81649.235849;	543	
345	2417.2930689;	445		545	28649.1539737;
347	673.10771417;	447	55889.357169;	547	41.113.9661777;
349	17.436337753;	449		549	97.468260633;
351	857.8855593;	451	17.34057.35729;	551	
353	7763701441;	453		553	17.2750563073;
355	17.41.73.97.1609;	455		555	67369.704177;
357	113.449.160073;	457	17.89.14414377;	557	25657.1875793;
359	17.11113.43961;	459	14633.1516657;	559	17.17657.162649;
361	15073.563377;	461	17.1049.1266337;	561	
363	8681534681;	463		563	17.233.337.37633;
365	17.522026489;	465	41.2273.250841;	565	
367	39089.232049;	467	17.1398906233;	567	
369	233.39785017;	469	353.68530937;	569	17.3082976033;
371	1321.7170721;	471		571	41.89.449.32441;
373	54217.178513;	473	281.4241.21001;	573	
375	73.135447881;	475	409.6329.9833;	575	12457.4387609;
377	193.52333297;	477		577	41.1351728281;
379	2657.3882713;	479		579	
381	34897.301913;	481	17921.1493441;	581	433.131579017;
383	17.41.113.136601;	483	1553.17522137;	583	113.5641.90617;
385	641.17137793;	485	17.1627376489;	585	1361.43026433;
387	73.1217.126241;	487		587	17.44953.77681;
389	17.23057.29209;	489	41.73.1753.5449;	589	137.337.1303409;
391	577.20253553;	491	17.1709413193;	591	8969.6801049;
393	17.89.7883177;	493	569.51909329;	593	17.27961.130073;
395	193.5737.10993;	495	17.41.43068329;	595	
397	1489.8341369;	497	89.1289.265921;	597	17.3736099273;
399	17.17.17.2579377;	499	401.7393.10457;	599	4153.15499417;

Simple Half-Quartans, $\frac{1}{2}N = \frac{1}{2}(y^4+1^4)$; [*y odd*].
[All factors < 100,000 cast out.]

y	$\frac{1}{2}N$	y	$\frac{1}{2}N$	y	$\frac{1}{2}N$
601	41.1591050761;	701		801	17.
603	17.3888573673;	703		803	7673.27093617;
605		705	17.7265701489;	805	
607		707	257.457.1063649;	807	17.3833.3254441;
609	113.673.904369;	709	2377.53152753;	809	97.113.769.25409;
611	89.1097.713737;	711	41.233.3001.4457;	811	
613		713		813	73.89.33621673;
615	97.3697.199457;	715	241.7561.71713;	815	193.1142991841;
617	953.5657.13441;	717	91457.1444873;	817	41.1009.5384969;
619	26113.2811097;	719		819	
621	17.313.13974721;	721		821	
623		723	17.8036635513;	823	41.97.761.75793;
625	2593.29423041; D,B	725		825	17.73.186643993;
627	17.457.9946609;	727	521.268083401;	827	34033.6872137;
629	41.97.2137.9209;	729	17.193; 97.577.769;	829	
631	17.313.14896841;	731	241.6577.90073;	831	17.1193.11756681;
633	3217.24953633;	733	17.17.499445449;	833	
635	73.89.1033.12113;	735	41.3559061593;	835	17.113.233.257.2113;
637	17.4842602393;	737	673.219192097;	837	62137.3949313;
639		739	17.7417.1182689;	839	93553.2648257;
641	81569.1034849;	741	41.6113.601457;	841	17.26209.561377;
643		743	97.2273.691121;	843	
645	569.6553.23209;	745		845	
647	73.1200229417;	747	113.4057.339601;	847	41.41.401.381761;
649		749	89.3329.531121;	849	409.635151689;
651	281.319585921;	751	313.508142377;	851	2801.93621401;
653	41.2689.824609;	753	241.3257.204793;	853	89.809.3676441;
655	17.1481.3655369;	755	4049.40124537;	855	
657		757	17.17.401.1416809;	857	
659	41.2299999841;	759	337.492387713;	859	17.
661	17.5897.952129;	761		861	569.1249.386641;
663	2113.45721937;	763	17.193.577.89513;	863	137.13049.155137;
665	17.4049.952129;	765	41.809.5162777;	865	17.
667	73.1355659057;	767	17.257.39606769;	867	4657.60665273;
669	857.1193.97961;	769		869	17.
671	17.929.6417937;	771	58313.3029857;	871	34457.8351513;
673	59417.1726313;	773	17.113.7489.12409;	873	2689.108003089;
675	89.137.8512841;	775	257.701849009;	875	17.41.6737.62417;
677		777	4129.4337.10177;	877	7057.41912953;
679	73.4969.292993;	779	5881.31308961;	879	
681		781	73.137.193.96377;	881	1553.13681.14177;
683	41.2653804721;	783	281.24113.27737;	883	
685		785	337.563402449;	885	2633.116490961;
687	233.478014457;	787	409.433.1083073;	887	1289.240110729;
689	17.6628236113;	789	89.32713.66553;	889	
691	69497.1640273;	791	17.25793.446401;	891	
693	241.4409.108529;	793	41.73.66062657;	893	17.
695	17.137.50088697;	795		895	
697	2593.45509137;	797	17.953.12452641;	897	641.504988801;
699	17.761.9226673;	799	617.3593.91921;	899	17.41.468571633;

☞ *Continued on right of page* 119.

Continued from page 115.

$N = (y^4+1^4)$; $[y = \epsilon]$.

y	N
902	89.59209.125617;
904	3257.205048201;
906	97.6946100401;
908	5393.126041329;
910	17.6833.5903441;
912	
914	
916	17.41.3793.266297;
918	137.5183822921;
920	17.97.233.1864553;
922	113.193.1913.17321;
924	
926	17.337.1433.89561;
928	
930	
932	281.4993.537769;
934	241.313.10088489;
936	
938	
940	41.5281.3605881;
942	89.769.11505017;
944	17.233.313.640529;
946	41.
948	113.35809.199601;
950	17.1321.36269593;
952	
954	17.
956	193.241.17957969;
958	6793.123993929;
960	17.1993.25068521;
962	
964	257.641.5242241;
966	
968	577.5953.255617;
970	41.1481.14579681;
972	241.3703804177;
974	193.58913.79153;
976	7177.126431801;
978	17.1217.2473.17881;
980	
982	
984	17.84377.653593;
986	
988	17.
990	449.2139412049;
992	
994	17.241.238275601;
996	
998	41.
1000	73.137; 99990001; Lo,R

[All factors < 100,000 cast out.]

Continued from page 118.

$\frac{1}{2}N = \frac{1}{2}(y^4+1^4)$; $[y = \omega]$.

y	½N
901	21121.15601081;
903	17.281.69593033;
905	41.41.2777.71849;
907	24097.14042233;
909	17.5113.3927361;
911	929.1249.296801;
913	
915	27809.12602857;
917	
919	
921	20201.17808841;
923	97.937.3992689;
925	5153.6257.11353;
927	17.73.89.89.37561;
929	41.57697.157433;
931	
933	17.2857.7800769;
935	3313.6217.18553;
937	17.97.233726369;
939	73.2953.1803209;
941	
943	17.353.65886001;
945	433.26161.35201;
947	929.2081.208009;
949	137.2960153873;
951	937.2729.159937;
953	3449.119577209;
955	7817.53203889;
957	41.
959	73.641.9037817;
961	17.
963	
965	
967	17.89.288959497;
969	137.3217692553;
971	17.73.4817.74353;
973	113.2297.1726561;
975	16273.27766481;
977	17.17.3881.406169;
979	1609.5417.52697;
981	41.
983	31649.14751089;
985	593.793707041;
987	41.56921.203321;
989	353.1355128457;
991	89.1913.2832433;
993	33961.14314841;
995	17.4481.6433369;
997	
999	113.521.673.12569;
1001	17.17.

Quartans, $N = (x^4 + y^4) \not> 9.10^6$; [$x$ and $y > 1$, y *even*].

x, y	N	x, y	N	x, y	N
3, 2	97;	51, 4	601.11257;	17, 10	41.2281;
5	641;	53, 4	17.41.11321;	19	140321;
7	2417;	5, 6	17.113;	21	204481;
9	6577;	7	3697;	23	289841;
11	14657;	11	15937;	27	73.7417;
13	17.41.41;	13	73.409;	29	17.42193;
15	89.569;	17	89.953;	31	17.89.617;
17	83537;	19	131617;	33	673.1777;
19	130337;	23	41.6857;	37	17.137.809;
21	17.17.673;	25	391921;	39	17.97.1409;
23	279857;	29	17.41681;	41	1193.2377;
25	113.3457;	31	17.54401;	43	3428801;
27	531457;	35	1501921;	47	281.17401;
29	73.9689;	37	17.110321;	49	401.14401;
31	97.9521;	41	433.6529;	51	6775201;
33	17.69761;	43	41.83417;	53, 10	7900481;
35	17.41.2153;	47	4880977;	5, 12	41.521;
37	1874177;	49	761.7577;	7	17.1361;
39	233.9929;	53, 6	7891777;	11	17.2081;
41	2825777;	3, 8	4177;	13	49297;
43	457.7481;	5	4721;	17	137.761;
45	4100641;	7	73.89;	19	151057;
47	17.41.7001;	9	10657;	23	17.17681;
49	5764817;	11	41.457;	25	411361;
51	6765217;	13	17.17.113;	29	728017;
53, 2	73.108089;	15	54721;	31	944257;
3, 4	337;	17	41.2137;	35	1521361;
5	881;	19	134417;	37	41.113.409;
7	2657;	21	17.11681;	41	17.167441;
9	17.401;	23	283937;	43	3439537;
11	14897;	25	394721;	47	73.67129;
13	28817;	27	97; 5521;	49	5785537;
15	17.41.73;	29	89.7993;	53, 12	7911217;
17	83777;	31	113.8209;	3, 14	137.281;
19	17.7681;	33	17.70001;	5	39041;
21	193.1009;	35	17.88513;	9	41.1097;
23	280097;	37	1878257;	11	17.3121;
25	17.22993;	39	569.4073;	13	66977;
27	137.3881;	41	401.7057;	15	89041;
29	41.17257;	43	73.46889;	17	121937;
31	577.1601;	45	4104721;	19	168737;
33	73.16249;	47	17.287281;	23	17.97.193;
35	97.15473;	49	5768897;	25	409.1049;
37	1874417;	51	6769297;	27	17.33521;
39	449.5153;	53, 8	7894577;	29	745697;
41	89.113.281;	3, 10	17.593;	31	961937;
43	17.201121;	7	12401;	33	1224337;
45	4100881;	9	16561;	37	1912577;
47	4879937;	11	41.601;	39	2351857;
49, 4	17.339121;	13, 10	38561;	41, 14	17.168481;

Quartans, $N = (x^4 + y^4) \not> 9.10^6$; [$x$ and $y > 1$, y *even*].

x, y	N	x, y	N	x, y	N
43, 14	3457217 ;	9, 20	166561 ;	25, 24	137.5273 ;
45	17.243473 ;	11	17.10273 ;	29	17.61121 ;
47	4918097 ;	13	193.977 ;	31	17.41.1801 ;
51	113.60209 ;	17	243521 ;	35	281.6521 ;
53, 14	7928897 ;	19	41.73.97 ;	37	17.17.17.449 ;
3, 16	65617 ;	21	113.3137 ;	41	3157537 ;
5	66161 ;	23	17.25873 ;	43	3750577 ;
7	41.1657 ;	27	17.89.457 ;	47	5211457 ;
9	17.4241 ;	29	867281 ;	49	41.241.617 ;
11	80177 ;	31	769.1409 ;	53, 24	8222257 ;
13	73.1289 ;	33	1345921 ;	3, 26	457057 ;
15	17.6833 ;	37	2034161 ;	5	41.11161 ;
17	149057 ;	39	2473441 ;	7	459377 ;
19	17.41.281 ;	41	17.175633 ;	9	463537 ;
21	260017 ;	43	3578801 ;	11	471617 ;
23	137.2521 ;	47	5039681 ;	15	97.5233 ;
25	17.26833 ;	49	953.6217 ;	17	89.6073 ;
27	596977 ;	51	6925201 ;	19	587297 ;
29	137.5641 ;	53, 20	8050481 ;	21	17.38321 ;
31	89.11113 ;	3, 22	89.2633 ;	23	736817 ;
33	1049.1193 ;	5	193.1217 ;	25	847601 ;
35	433.3617 ;	7	17.13921 ;	27	988417 ;
37	1249.1553 ;	9	281.857 ;	29	449.2593 ;
39	2378977 ;	13	89.2953 ;	31	601.2297 ;
41	233.12409 ;	15	284881 ;	33	17.241.401 ;
43	17.97.2113 ;	17	317777 ;	35	17.115153 ;
45	1009.4129 ;	19	193.1889 ;	37	41.56857 ;
47	1153.4289 ;	21	41.10457 ;	41	73.193.233 ;
49	17.193.1777 ;	23	17.30241 ;	43	1481.2617 ;
51	113.60449 ;	25	41.15241 ;	45	41.89.1249 ;
53, 16	17.468001 ;	27	17.73.617 ;	47	17.313921 ;
5, 18	105601 ;	29	941537 ;	49	433.14369 ;
7	107377 ;	31	233.4969 ;	51	7222177 ;
11	119617 ;	35	569.3049 ;	53, 26	1009.8273 ;
13	41.3257 ;	37	233.9049 ;	3, 28	17.36161 ;
17	233.809 ;	39	769.3313 ;	5	17.17.2129 ;
19	17.13841 ;	41	17.180001 ;	9	621217 ;
23	384817 ;	43	3653057 ;	11	113.5569 ;
25	17.29153 ;	45	17.254993 ;	13	643217 ;
29	812257 ;	47	97.52721 ;	15	577.1153 ;
31	73.73.193 ;	49	113.53089 ;	17	241.2897 ;
35	41.39161 ;	51	6999457 ;	19	744977 ;
37	1129.1753 ;	53, 22	577.14081 ;	23	41.21817 ;
41	2930737 ;	5, 24	17.19553 ;	25	761.1321 ;
43	17.17.89.137 ;	7	334177 ;	27	1146097 ;
47	41.121577 ;	11	346417 ;	29	17.77761 ;
49	17.449.769 ;	13	360337 ;	31	17.90481 ;
53, 18	17.137.3433 ;	17	73.5689 ;	33	1800577 ;
3, 20	160081 ;	19	462097 ;	37	17.281.521 ;
7, 20	17.41.233 ;	23, 24	193.3169 ;	39, 28	17.41.4201 ;

Quartans, $N = (x^4 + y^4) \not> 9.10^6$; [$x$ and $y > 1$, y *even*].

x, y	N	x, y	N	x, y	N
41, 28	73·47129 ;	11, 34	1350977 ;	35, 38	1697·2113 ;
43	41·98377 ;	13	1364897 ;	37	3959297 ;
45	4715281 ;	15	449·3089 ;	39	4398577 ;
47	433·12689 ;	19	1466657 ;	41	4910897 ;
51	97·76081 ;	21	41·37337 ;	43	17·569·569 ;
53, 28	8505137 ;	23	577·2801 ;	45	6185761 ;
7, 30	812401 ;	25	41·73·577 ;	47	6964817 ;
11	824641 ;	27	113·16529 ;	49	17·409·1129 ;
13	838561 ;	29	2043617 ;	51, 38	137·64601 ;
17	893521 ;	31	241·9377 ;	3, 40	17·41·3673 ;
19	17·55313 ;	33	2522257 ;	7	617·4153 ;
23	1089841 ;	35	2836961 ;	9	2566561 ;
29	977·1553 ;	37	89·36073 ;	11	137·18793 ;
31	41·42281 ;	39	3649777 ;	13	409·6329 ;
37	2684161 ;	41	4162097 ;	17	193·13697 ;
41	3635761 ;	43	4755137 ;	19	2690321 ;
43	17·248753 ;	45	5436961 ;	21	2754481 ;
47	89·63929 ;	47	113·55009 ;	23	2839841 ;
49	17·41·9433 ;	49, 34	7101137 ;	27	41·75401 ;
53, 30	17·511793 ;	5, 36	73·23017 ;	29	17·192193 ;
3, 32	41·25577 ;	7	1682017 ;	31	17·204913 ;
5	1049201 ;	11	73·23209 ;	33	89·42089 ;
7	1050977 ;	13	17·89·1129 ;	37	17·97·2689 ;
9	1055137 ;	17	1763137 ;	39	17·286673 ;
11	97·97·113 ;	19	1809937 ;	41	5385761 ;
13	17·63361 ;	23	1959457 ;	43	5978801 ;
15	241·4561 ;	25	2070241 ;	47	7439681 ;
17	857·1321 ;	29	41·58217 ;	49, 40	8324801 ;
19	1178897 ;	31	137·19001 ;	5, 42	3112321 ;
21	17·73121 ;	35	17·187073 ;	11	457·6841 ;
23	1328417 ;	37	3553777 ;	13	17·184721 ;
25	193·7457 ;	41	4505377 ;	17	3195217 ;
27	41·89·433 ;	43	97·52561 ;	19	3242017 ;
29	673·2609 ;	47	17·241·1601 ;	23	3391537 ;
31	1972097 ;	49, 36	353·21089 ;	25	73·47977 ;
33	17·131441 ;	3, 38	2085217 ;	29	3818977 ;
35	17·149953 ;	5	433·4817 ;	31	4035217 ;
37	2922737 ;	7	97·21521 ;	37	577·8641 ;
39	3362017 ;	9	17·41·3001 ;	41	89·66713 ;
41	3874337 ;	11	89·23593 ;	43	2393·2729 ;
43	4467377 ;	13	521·4057 ;	47, 42	17·470081 ;
45	73·70537 ;	15	17·73·1721 ;	3, 44	17·97·2273 ;
47	17·17·73·281 ;	17	2168657 ;	5	17·220513 ;
49	97·70241 ;	21	2279617 ;	7	449·8353 ;
51	7813777 ;	23	113·20929 ;	9	41·91577 ;
53, 32	8939057 ;	25	17·145633 ;	13	617·6121 ;
3, 34	1336417 ;	27	2616577 ;	15	113·33617 ;
5	1336961 ;	29	433·6449 ;	17	433·8849 ;
7	1338737 ;	31	137·21961 ;	19	73·53129 ;
9, 34	1342897 ;	33, 38	73·44809 ;	21, 44	3942577 ;

☞ *Continued on left of page* 125.

Half-Quartans, $\tfrac{1}{2}\mathrm{N} = \tfrac{1}{2}(x^4+y^4) \not> 9.10^6$; $[x$ and $y > 1$, xy *odd*$]$.

x, y	$\tfrac{1}{2}$N	x, y	$\tfrac{1}{2}$N	x, y	$\tfrac{1}{2}$N
5, 3	353 ;	27, 7	266921 ;	49, 11	41·70481 ;
7	17·73 ;	29	17·20873 ;	51	41·89·929 ;
11	17·433 ;	31	17·113·241 ;	53	3952561 ;
13	14321 ;	33	594161 ;	57	1321·4001 ;
17	41801 ;	37	17·97·569 ;	59	577·10513 ;
19	113·577 ;	39	17·68113 ;	61	6930241 ;
23	17·8233 ;	41	1414081 ;	63	17·463753 ;
25	195353 ;	43	1710601 ;	65, 11	17·97·5417 ;
29	353681 ;	45	1321·1553 ;	15, 13	17·17·137 ;
31	461801 ;	47	2441041 ;	17	56041 ;
35	750353 ;	51	3383801 ;	19	17·4673 ;
37	937121 ;	53	409·9649 ;	21	111521 ;
41	17·17·4889 ;	55	809·5657 ;	23	41·3761 ;
43	73·23417 ;	57	41·128761 ;	25	17·12329 ;
47	2439881 ;	59	97·62473 ;	27	280001 ;
49	2882441 ;	61	41·281·601 ;	29	97·3793 ;
53	89·97·457 ;	65, 7	17·73·7193 ;	31	476041 ;
55	953·4801 ;	11, 9	10601 ;	33	281·2161 ;
59	113·53617 ;	13	17·1033 ;	35	764593 ;
61	17·407233 ;	17	73·617 ;	37	951361 ;
65, 3	257·34729 ;	19	89·769 ;	41	97·14713 ;
7, 5	17·89 ;	23	89·1609 ;	43	17·41·2473 ;
9	3593 ;	25	198593 ;	45	761·2713 ;
11	17·449 ;	29	241·1481 ;	47	2454121 ;
13	14593 ;	31	465041 ;	49	17·170393 ;
17	42073 ;	35	17·97·457 ;	51	457·7433 ;
19	233·281 ;	37	940361 ;	53	17·89·2617 ;
21	97553 ;	41	1416161 ;	55	4589593 ;
23	17·73·113 ;	43	977·1753 ;	57	73·72497 ;
27	17·15649 ;	47	17·137·1049 ;	59	17·41·8713 ;
29	41·89·97 ;	49	113·25537 ;	61	257·26993 ;
31	462073 ;	53	3948521 ;	63, 13	281·28081 ;
33	593273 ;	55	17·41·6569 ;	17, 15	67073 ;
37	73·12841 ;	59	233·26017 ;	19	90473 ;
39	1157033 ;	61	2153·3217 ;	23	165233 ;
41	17·97·857 ;	65, 9	8928593 ;	29	378953 ;
43	1709713 ;	13, 11	21601 ;	31	487073 ;
47	2440153 ;	15	32633 ;	37	41·23473 ;
49	1009·2857 ;	17	49081 ;	41	601·2393 ;
51	1601·2113 ;	19	72481 ;	43	1734713 ;
53	41·96233 ;	21	104561 ;	47	17·145009 ;
57	17·310489 ;	23	73·2017 ;	49	2907713 ;
59	6058993 ;	25	97·2089 ;	53	233·17041 ;
61	17·407249 ;	27	137·1993 ;	59	6083993 ;
63, 5	257·30649 ;	29	17·17·1249 ;	61, 15	6948233 ;
9, 7	4481 ;	31	17·41·673 ;	19, 17	106921 ;
11	8521 ;	35	757633 ;	21	97·1433 ;
13	113·137 ;	37	17·73·761 ;	23	97·1873 ;
15	26513 ;	39	17·68473 ;	25	237073 ;
17	42961 ;	41	1420201 ;	27	307481 ;
19	66361 ;	43	89·19289 ;	29	233·1697 ;
23	141121 ;	45	2057633 ;	31	41·12281 ;
25, 7	41·4793	47, 11	2447161 ;	33, 17	41·113·137 ;

Half-Quartans, $\frac{1}{2}N = \frac{1}{2}(x^4 + y^4) \ngtr 9.10^6$; [$x$ and $y > 1$, xy *odd*].

x, y	$\frac{1}{2}N$	x, y	$\frac{1}{2}N$	x, y	$\frac{1}{2}N$
35, 17	792073 ;	33, 23	241.3041 ;	49, 29	3236041 ;
37	401.2441 ;	35	41.21713 ;	51	3736241 ;
39	1198481 ;	37	17.63353 ;	53	4298881 ;
41	193.7537 ;	39	17.89.857 ;	55	4928953 ;
43	113.15497 ;	41	569.2729 ;	57	17.257.1289 ;
45	2092073 ;	43	233.7937 ;	59	6412321 ;
47	2481601 ;	45	2190233 ;	61	17.428033 ;
49	41.73.977 ;	47	41.62921 ;	63, 29	8230121 ;
53	3987001 ;	49	521.5801 :	33, 31	1054721 ;
55	4617073 ;	51	3522521 ;	35	673.1801 ;
57	5319761 ;	53	521.7841 ;	37	1398841 ;
59	1777.3433 ;	55	4715233 ;	39	1618481 ;
61	809.8609 ;	57	241.22481 ;	41	17.110273 ;
63	89.88969 ;	59	6198601 ;	43	2171161 ;
65, 17	8967073 ;	61	761.9281 :	45	17.147769 ;
21, 19	17.41.233 ;	63, 23	17.471553 ;	47	2901601 ;
23	205081 ;	27, 25	241.1913 ;	49	3344161 :
25	41.6353 ;	29	548953 ;	51	281.13681 ;
27	409.809 ;	31	73.9001 ;	53	97.45433 ;
29	73.5737 ;	33	17.89.521 ;	55	73.69001 ;
31	457.1153 ;	37	1132393 ;	57	17.337633 ;
33	17.38713 ;	39	73.18521 ;	59	6520441 ;
35	17.47969 ;	41	241.6673 ;	61	17.337.1289 ;
37	1002241 ;	43	409.4657 ;	63, 31	8338241 ;
39	89.13729 ;	47	17.155009 ;	35, 33	73.18401 ;
41	1049.1409 ;	49	97.31729 ;	37	569.2689 ;
43	137.12953 ;	51	3577913 ;	41	2005841 ;
45	97.113.193 ;	53	929.4457 ;	43	17.135433 ;
47	17.147353 ;	57	5473313 ;	47	3032801 ;
49	2947561 ;	59	6253993 ;	49	17.89.2297 ;
51	1217.2833 ;	61	1217.5849 ;	53	17.266953 ;
53	73.137.401 ;	63, 25	41.196873 ;	59	17.391273 ;
55	17.17.16057 ;	29, 27	17.36433 ;	61, 33	233.32257 ;
59	6123841 ;	31	17.42793 ;	37, 35	1687393 ;
61	41.170441 ;	35	1016033 ;	39	41.193.241 ;
63	7941641 ;	37	17.70753 ;	41	2163193 ;
65, 19	593.15161 ;	41	1678601 ;	43	17.41.3529 ;
23, 21	237161 ;	43	1975121 ;	47	3190153 ;
25	17.17209 ;	47	2705561 ;	51	4132913 ;
29	450881 ;	49	3148121 ;	53	17.276209 ;
31	559001 ;	53	673.6257 ;	57	6028313 ;
37	601.1721 ;	55	113.42841 ;	59	17.41.9769 ;
41	1510121 ;	59	6324401 ;	61, 35	137.56009 ;
43	17.106273 ;	61, 27	1097.6553 ;	39, 37	2093801 ;
47	2537081 ;	31, 29	815401 ;	41	17.137.1009 ;
53	17.401.593 ;	33	113.8377 ;	43	953.2777 ;
55	4672553 ;	35	401.2753 ;	45	17.17.10337 ;
59	17.257.1409 ;	37	41.31481 ;	47	193.17497 ;
61, 21	7020161 ;	39	1510361 ;	49	3819481 ;
25, 23	281.1193 ;	41	17.103913 ;	51	4319681 ;
27	405641 ;	43	113.18257 ;	53	41.193.617 ;
29	17.29033 ;	45	17.41.3449 ;	55	89.241.257 ;
31, 23	17.35393 ;	47, 29	2793481 ;	57, 37	17.97.3769 ;

☞ *Continued on right of page* 125.

Continued from page 122.

$N = (x^4 + y^4) \not> 9.10^6$; $[x \text{ and } y > 1,\ y = \epsilon]$.

$x,\ y$	N
23, 44	137.29401 ;
25	401.10321 ;
27	4279537 ;
29	17.137.1913 ;
31	17.97.2833 ;
35	1129.4649 ;
37	17.330721 ;
39	17.356561 ;
41	409.16073 ;
43	7166897 ;
45	1289.6089 ;
47, 44	8627777 ;
3, 46	4477537 ;
5	4478081 ;
7	17.263521 ;
9	1153.3889 ;
11	17.89.2969 ;
13	4506017 ;
15	41.110441 ;
17	4560977 ;
19	4607777 ;
21	4671937 ;
25	1753.2777 ;
27	17.294641 ;
29	41.126457 ;
31	1433.3769 ;
33	5663377 ;
35	233.25657 ;
37	113.56209 ;
39	6790897 ;
41	17.353.1217 ;
43	353.22369 ;
45, 46	17.504593 ;
5, 48	5309041 ;
7	17.312401 ;
11	17.521.601 ;
13	1249.4273 ;
17	5391937 ;
19	449.12113 ;
23	17.328721 ;
25	41.97.1433 ;
29	6015697 ;
31	73.85369 ;
35	113.60257 ;
37	2609.2753 ;
41	17.478481 ;
43, 48	2473.3529 ;
3, 50	41.152441 ;
7	6252401 ;
9, 50	17.17.21649 ;

$x,\ y$	N
11, 50	73.85817 ;
13	6278561 ;
17	6333521 ;
19	17.89.4217 ;
21	6444481 ;
23	89.73369 ;
27	41.193.857 ;
29	929.7489 ;
31	593.12097 ;
33	7435921 ;
37	8124161 ;
39, 50	433.19777 ;
3, 52	1481.4937 ;
5	2273.3217 ;
7	929.7873 ;
9	17.73.5897 ;
11	7326257 ;
15	17.433073 ;
17	1721.4297 ;
19	17.97.4513 ;
21	7506097 ;
23	7591457 ;
25	17.453073 ;
27	7843057 ;
29	281.28537 ;
31	41.353.569 ;
33	41.207257 ;
35, 52	8812241 ;
5, 54	8503681 ;
7	17.500321 ;
11	17.501041 ;
13	8531617 ;
17	8586577 ;
19	8633377 ;
23	17.41.12601 ;
25, 54	89.99929 ;

☞ *These Tables* (*pp.* 120–125) *show all Quartans and Half-Quartans* $\not> 9.10^6$, (*with* x *and* $y > 1$).

Continued from page 124.

$\frac{1}{2}N = \frac{1}{2}(x^4 + y^4) \not> 9.10^6$; $[x \text{ and } y > 1,\ xy = \omega]$.

$x,\ y$	$\frac{1}{2}N$
59, 37	6995761 ;
61	17.401.1153 ;
63, 37	337.26153 ;
41, 39	17.151153 ;
43	2866121 ;
47	41.87721 ;
49	1777.2273 ;
53	5101961 ;
55	73.233.337 ;
59	7215401 ;
61, 39	17.475273 ;
43, 41	3122281 ;
45	73.47441 ;
47	73.89.593 ;
49	4295281 ;
51	4795481 ;
53	113.47417 ;
55	5988193 ;
57	6690881 ;
59	7471561 ;
61, 41	593.14057 ;
45, 43	3759713 ;
47	17.41.5953 ;
49	4591801 ;
51	281.18121 ;
53	5654641 ;
55	17.337.1097 ;
57	1193.5857 ;
59	7768081 ;
61, 43	97.88993 ;
47, 45	1889.2377 ;
49	4932713 ;
53	41.257.569 ;
59	113.71761 ;
61, 45	73.122921 ;
49, 47	17.337.929 ;
51	5822441 ;
53	17.375593 ;
55	113.62081 ;
57	1009.7649 ;
59, 47	17.41.89.137 ;
51, 49	6265001 ;
53	577.11833 ;
55	17.193.2273 ;
57	8160401 ;
59, 49	857.10433 ;
53, 51	257.28513 ;
55, 51	409.19457 ;
55, 53	17.501209 ;

High Irreducible Quartans, $N = (x^4 + y^4) > 9.10^6$.

[*Octavans excluded.*]

$x = \xi^r, \quad y = \eta^r; \quad [x, y \not> 11]$.

N	N
$2^{24} + 3^4$	193.86929 ;
$+ 5^4$	457.36713 ;
$+ 7^4$	313.53609 ;
$+ 11^4$	353.47569 ;
$2^{28} + 3^4$	‡ ‡
$+ 3^8$	‡ ‡
$+ 3^{12}$	‡ ‡
$+ 5^4$	‡ ‡
$+ 5^8$	1361.197521 ;
$+ 5^{16}$	17.3137.2866289 ;
$+ 7^4$	‡ ‡
$+ 7^8$	‡ ‡
$+ 11^4$	‡ ‡
$+ 11^8$	‡ ‡
$2^{32} + 3^4$	‡ ‡
$+ 3^{12}$	‡ ‡
$+ 5^4$	73.58835177 ;
$+ 5^{16}$	4801.32677121 ;
$+ 7^4$	193.22253729 ;
$+ 11^4$	41.73.1435009 ;
$2^{36} + 3^4$	89.10273.75161 ;
$+ 3^8$	2081.33022337 ;
$+ 5^4$	‡ ‡
$+ 5^8$	1697.40494913 ;
$+ 5^{16}$	17.2417.8386049 ;
$+ 7^4$	41.1049.1597793 ;
$+ 7^8$	‡ ‡
$+ 11^4$	6217.11053481 ;
$+ 11^8$	337.204551441 ? ‡ ‡
$2^{40} + 3^4$	41.2153.12455809 ;
$+ 3^{12}$	41.4513.5942249 ;
$+ 5^4$	‡ ‡
$+ 7^4$	‡ ‡
$+ 11^4$	‡ ‡
$3^{16} + 2^4$	17.2532161 ;
$+ 2^{12}$	17.2532401 ;
$+ 2^{20}$	17.2593841 ;
$+ 2^{28}$	17.17.1077793 ;
$+ 2^{36}$	17.4044854321 ? ‡ ‡
$+ 5^4$	2.4297.5009 ;
$+ 5^{12}$	2.137.1048129 ;
$+ 7^4$	2.73.294857 ;
$+ 11^4$	2.113.190537 ;
$5^{12} + 2^4$	1097.222553 ;
$+ 2^8$	‡ ‡
$+ 2^{16}$	449.543889 ;
$+ 2^{20}$	‡ ‡
$+ 2^{28}$	‡ ‡
$+ 2^{32}$	1657.2739353 ;
$+ 2^{40}$	‡ ‡
$+ 3^4$	2.17.89.80681 ;
$+ 3^8$	2.73.233.7177 ;
$+ 6^4$	41.5954681 ;
$+ 6^8$	241.1020001 ;
$+ 7^4$	2.7297.16729 ;
$+ 7^8$	2.73.1033.1657 ;
$+ 11^4$	2.401.304433 ;
$+ 11^8$	2.4153.55201 ;
$+ 12^4$	17.17.17.49697 ;
$10^8 + 3^4$	100000081 ; D
$+ 3^{12}$	‡ ‡
$+ 7^4$	113.884977 ;
$+ 11^4$	1489.67169 ;
$11^8 + 3^4$	2.577.185753 ;
$+ 5^4$	2.457.234529 ;
$+ 6^4$	41.5228297 ;
$+ 7^4$	2.521.205721 ;
$+ 10^4$	2137.100313 ;

‡ ‡ No divisors < 12,000.

Dimorph Quartans, $N = (x^4 + y^4) = (x'^4 + y'^4) = (L_1 . L_2) . (M_1 . M_2)$.

Ex.	t, u	x	y	x'	y'	L_1	L_2	M_1	M_2
1	2, 1	1203,	76	653,	1176	17 ;	8929 ;	2129 ;	6481 ;
2	2, 3	40465,	11888	2513,	40540	97 ;	428801 ;	73.2281 ;	390001 ;
3	4, 1	53935,	31494	52881,	35710	257 ;	521.2441 ;	89.937 ;	346561 ;
4	4, 3	419909,	31238	81659,	419762	337 ;	6026609 ;	257.10337 ;	73.193.409 ;
5	2, 5	444311,	345588	108201,	480032	641 ;	137.73721 ;	353.4177 ;	5576881 ;
6	1, 3	134,	133	59,	158	41 ;	569 ;	113 ;	241 ;
7	1, 5	1623,	3494	2338,	3351	313 ;	28057 ;	3217 ;	5521 ;
8	3, 5	17236,	6673	529,	17332	353 ;	73.1049 ;	51137 ;	97.673 ;
9	1, 7	6484,	32187	23109,	29812	1201 ;	41.9689 ;	73.601 ;	51361 ;
10	3, 7	84545,	59678	15322,	89345	17.73 ;	629081 ;	214481 ;	380881 ;
11	5, 7	287178,	67429	20773,	287394	17.89 ;	401.4457 ;	1049.1433 ;	1678321 ;
12	1, 9	7805,	174484	125516,	161405	17.193 ;	97.30113 ;	326561 ;	73.4057 ;
13	5, 9	875539,	400262	3106,	884947	3593 ;	137.52081 ;	233.17929 ;	337.16993 ;
14	?	103,	542	359,	514	17 ;			5082931681 ? †
15	$b = 3$	2903,	12231	10203,	10381		2.82361 ;		17.193.577.71993 ;
16	$b = 2$	2219449,	555617	1584749,	2061283		*		*

* The composition of these large numbers is not known.

$$N = (x^4 + y^4) = (X^4 - 4Y^4) = L.M.$$

$x = (\frac{1}{2}\eta^4 - \xi^4), \quad y = \xi\eta^3.$ $\qquad$ $x = (2\eta^4 - \xi^4), \quad y = 2\xi^3\eta.$

$X = (\frac{1}{2}\eta^4 + \xi^4), \quad Y = \xi^3\eta.$ $\qquad$ $X = (2\eta^4 + \xi^4), \quad Y = 2\xi\eta^3.$

ξ, η	x,	y	L	M	ξ, η	x,	y	L	M
1, 2	7,	8	73 ;	89 ;	1, 2	31,	4	577 ;	1601 ;
1, 4	127,	64	17·977 ;	16673 ;	1, 4	511,	8	17·13553 ;	295937 ;
1, 6	647,	216	353·1193 ;	421273 ;	1, 6	2591,	12	17·373553 ;	7096897 ;
1, 8	2048,	512	1289·3257 ;	4198529 ;	1, 8	8191,	16	65028097 ? †	97·137·5209 ;
3, 2	73,	24	2089 ;	17·809 ;	3, 2	49,	108	8161 ;	17377 ;
3, 4	47,	192	20353 ;	113·593 ;	3, 4	431,	216	56737 ;	17·73·521 ;
3, 8	1967,	1536	17·17·15361 ;	89·51977 ;	3, 8	8111,	432	49568161 ? †	87316897 ? †
5, 2	617,	40	17·16217 ;	521·1009 ;	5, 2	593,	500	418849 ;	444449 ;
5, 4	497,	320	113·593 ;	1067009 ;	5, 4	113,	1000	17·89·313 ;	2111969 ;
5, 6	23,	1080	137·3617 ;	2745529 ;	5, 6	1967,	1500	1017889 ;	73·257·1049 ;
5, 8	1423,	2560	5144929 ;	17·73·7369 ;					
7, 2	2393,	56	4862089 ;	6744473 ;	7, 2	2369,	1372	5894401 ;	17·89·3929 ;
7, 4	2273,	448	17·154769 ;	10160609 ? †					

Quartan Nexuses (N_1, N_2, N_3) *and Square 4-tan Products.*

$$\mathbf{A} = A^2 - 3B^2,\ \mathbf{B} = 2AB;\ x = A^2 + 3B^2;\ y_1 = \mathbf{A} + \mathbf{B},\ y_2 = \mathbf{A} \sim \mathbf{B},\ y_3 = 2\mathbf{B}.$$

$$N_1 = x^4 + y_1^4 = L_2 L_3;\quad N_2 = x^4 + y_2^4 = L_3 L_1;\quad N_3 = x^4 + y_3^4 = L_1 L_2;$$
$$N_1 N_2 N_3 = (L_1 L_2 L_3)^2.$$

A, B	x;	y_1,	y_2,	y_3	L_1	L_2	L_3
2, 1	7;	5,	3,	8	73;	89;	2.17;
1, 2	13;	15,	7,	8	113;	17.17;	2.137;
4, 1	19;	21,	5,	16	281;	17.41;	2.233;
1, 4	49;	55,	39,	16	1777;	17.193;	2.2273;
2, 3	31;	35,	11,	24	17.41;	1801;	2.673;
4, 3	43;	35,	13,	48	2473;	3529;	2.17.41;
1, 6	109;	119,	95,	24	9601;	14737;	2.11593;
5, 6	133;	143,	23,	120	14929;	34849;	2.17.617;
5, 2	37;	33,	7,	40	17.97;	2689;	2.569;
2, 5	79;	91,	51,	40	4201;	41.241;	2.5441;
5, 4	73;	63,	17,	80	6689;	10369;	2.2129;
4, 5	91;	99,	19,	80	6761;	17.953;	2.5081;
2, 7	151;	171,	115,	56	16361;	32377;	2.17.1249;
7, 2	61;	65,	9,	56	3217;	17.433;	2.2153;
4, 7	163;	187,	75,	112	18169;	47513;	2.20297;
7, 4	97;	57,	55,	112	15569;	17.929;	2.3137;
7, 6	157;	143,	25,	168	17.1697;	48673;	2.41.257;
8, 3	91;	85,	11,	96	9337;	41.401;	2.3673;
8, 1	67;	77,	45,	32	3049;	17.409;	2.41.97;
1, 8	193;	207,	175,	32	31649;	73.601;	2.17.2161;
8, 5	139;	91,	69,	160	97.313;	17.1993;	2.6521;
5, 8	217;	247,	87,	160	41.809;	257.337;	2.17.2017;
8, 7	211;	195,	29,	224	17.3001;	193.457;	2.19433;
7, 8	241;	255,	31,	224	51137;	115201;	2.32993;
52, 1	2707;	2805,	2597,	208	41.165553;	1753.4513;	2.7306217;
1, 30	2701;	2759,	2639,	120	17.410513;	7626481;	2.41.177761;

Solutions of

$$(\xi^4 + \eta^4 + \zeta^4)^2 = (x'^2 + y'^2 + z'^2)^2 = 2\,(x'^4 + y'^4 + z'^4) = 2\,(\xi^8 + \eta^8 + \zeta^8).$$

$$\xi^2 + \eta^2 = \zeta^2;\quad x' = \xi^2 - \xi\eta - \eta^2,\ \ y' = 2\xi\eta,\ \ z' = \xi^2 + \xi\eta - \eta^2.$$

ξ =	3,	5,	8,	7,	20,	12,	9,	28,	11,	16,	33,	48
η =	4,	12,	15,	24,	21,	35,	40,	45,	60,	63,	56,	55
ζ =	5,	13,	17,	25,	29,	37,	41,	53,	61,	65,	65,	73
x' =	5,	59,	41,	359,	379,	661,	1159,	19,	2819,	2705,	199,	1919
y' =	24,	120,	240,	336,	840,	840,	720,	2520,	1320,	2016,	3696,	5280
z' =	19,	179,	281,	695,	461,	1501,	1819,	2501,	4139,	4721,	3895,	3361

Associate Quartans, (H_n).

$$\mathrm{H}_n = \tfrac{1}{2}(x'^4 + y'^4) = 2^{2n} + 6.2^n + 1\,; \quad [x' = 2^{\frac{1}{2}n} - 1,\ y' = 2^{\frac{1}{2}n} + 1].$$

n	1	3	5	7	9	11
H	17 ;	113 ;	1217 ;	17.1009 ;	17.15601 ;	4206593 ;
n	2	4	6	8	10	12
x', y'	1, 3	3, 5	7, 9	15, 17	31, 33	63, 65
H	41 ;	353 ;	4481 ;	67073 ;	1054721 ;	16801793 ;

n	13	15	17
H	193.347969 ;	17.63172849 ;	17.97.10418833 ;
n	14	16	18
x', y'	127, 129	255, 257	511, 513
H	8009.33529 ;	593.7243441 ;	41.

☞ *For other Table of* H_n, *see foot of page* 133.

Criteria of $n = \omega$ *or* ϵ *in* $\mathrm{H}_n \equiv 2^{2n} + 6.2^n + 1 \equiv 0 \pmod{p}$.

$$p = \mathrm{a}^2 + \mathrm{b}^2 = \mathrm{c}^2 + 2\mathrm{d}^2 = 8\varpi + 1. \quad [2^{\xi} \equiv +1 \pmod{p}.]$$

$(2/p)_4$	$\bar{1}$,	$\bar{1}$,	+1,	+1	$(2/p)_8$	$\bar{1}$,	$\bar{1}$,	+1,	+1
d	2ω,	2ϵ,	2ω,	2ϵ	d	4ω,	4ϵ,	4ω,	4ϵ
$p-1$	8ϵ,	8ω,	8ω,	8ϵ	$p-1$	8ϵ,	8ϵ,	8ϵ,	8ϵ
b	4ω,	4ω,	4ϵ,	4ϵ	b	8ω,	8ω,	8ϵ,	8
ξ	ϵ,	ϵ,	ω, ϵ,	ω, ϵ	ξ	ϵ,	ϵ,	ω, ϵ,	ω, ϵ
n	ω,	ϵ,	. ,	inf.	n	ω,	ϵ,	. ,	inf.

$(2/p)_{16}$	$\bar{1}$,	$\bar{1}$,	$\bar{1}$,	$\bar{1}$	$(2/p)_{16}$	+1,	+1,	+1,	+1
d	8ω,	8ω,	8ϵ,	8ϵ	d	8ω,	8ω,	8ϵ,	8ϵ
$p-1$	16ϵ,	16ω,	16ϵ,	16ω	$p-1$	16ω,	16ϵ,	16ω,	16ϵ
b	16ω,	16ω,	16ω,	16ω	b	16ϵ,	16ϵ,	16ϵ,	16ϵ
ξ	ϵ,	ϵ,	ϵ,	ϵ	ξ	ω,	ω, ϵ,	ω,	ω, ϵ
n	ω,	. ,	ϵ,	.	n	ω & ϵ,	. ,	. ,	?

Factors (p) *of* $\mathrm{H}_n = 2^{2n} + 6.2^n + 1$. [*Complete to* $p \not> 1{,}400$.]

$$n = m\xi + n_0,\ n' = m'\xi + n_0'\,; \quad n + n' = \xi\,; \quad 2^{\xi} \equiv +1 \pmod{p}.$$

p	17	41	97	113	137	193	313	353	401	409	449	521	569	577	593
n_0	1	2	17	3	24	13	46	4	67	70	103	24	108	63	16
n_0'	7	18	31	25	44	83	110	84	133	134	121	236	176	81	132

p	761	769	809	857	929	977	1009	1129	1153	1201	1217	1297	1361,	&c.
n_0	54	41	134	60	181	155	7	122	84	86	5	35	49	
n_0'	326	393	270	368	213	333	497	442	204	214	147	613	631	

p	4481	4721	6353	8009
n_0, n_0'	6, 554	57, 238	52, 397	14, 3990

Base-Quartan $N_0 = x_0^4 + y_0^4 = h^4 + k^4$.

Ineffective Characteristics $[C = 0, \pm 1]$.

	a_0, c_0, e_0	x_0, y_0	C	a_0, c_0, e_0	x_0, y_0	C
ii.	$a_0 = h^2$	h, k	$C'' = 0$	$a_0 = k^2$	k, h	$C'' = 0$
iii.	$c_0 = h^2 - k^2$	h, k	$C''' = -1$	$c_0 = k^2 - h^2$	k, h	$C''' = -1$
iv.	$e_0 = h^2 + k^2$	h, k	$C^{iv} = +1$	$e_0 = -h^2 - k^2$	k, h	$C^{iv} = +1$

Equivalent and Reciprocal Characteristics (C_1, C_2).
(E), *Equivalent*, $C_1C_2 = -1$; (R), *Reciprocal*, $(C_1 - 1/C_1)(C_2 - 1/C_2) = 4$.

a_0	x_0, y_0	C''	c_0, e_0, P_0	x_0, y_0	C', C''', C^{iv}	R or E
h^2	k, h	$(h^2 - k^2)/h^2$	$c_0 = k^2 - h^2$	h, k	$C''' = (k^2 - 2h^2)/k^2$	R of C''
h^2	h, k	$(k^2 - h^2)/k^2$	$c_0 = h^2 - k^2$	k, h	$C''' = (h^2 - 2k^2)/h^2$	R of C''
$-h^2$	k, h	$-(h^2 + k^2)/h^2$	$e_0 = -h^2 - k^2$	h, k	$C^{iv} = -(2h^2 + k^2)/k^2$	R of C''
$-h^2$	h, k	$-(k^2 + h^2)/k^2$	$e_0 = -k^2 - h^2$	k, h	$C^{iv} = -(h^2 + 2k^2)/h^2$	R of C''
-1	$1, k$	$-2/k^2$	$P_0 = 1 + \frac{1}{2}k^4$	$1, k$	$C' = \frac{1}{2}h^2$	E of C''

Characteristics (C) *of Simple Quartans*, $N_4 = (1 + k^4)$.

		x_0, y_0	P_0 , Q_0	z_0 ,	C'	E, I, R
i.	1	$1, k$	$\frac{1}{2}k^4 + 1$, $\frac{1}{2}k^4$	$\frac{1}{2}k^3$,	$\frac{1}{2}k^2$	C_2''
	2	$1, k$	$-(\frac{1}{2}k^4 + 1)$, $\frac{1}{2}k^4$	$\frac{1}{2}k^3$,	$-(\frac{1}{2}k^4 - 2)/k^2$	
	3	$k, 1$	$\frac{1}{2}k^4 + 1$, $\frac{1}{2}k^4$	$\frac{1}{2}k^4$,	$\frac{1}{2}k^4 - k^2 + 1)$	
	4	$k, 1$	$-(\frac{1}{2}k^4 + 1)$, $\frac{1}{2}k^4$	$\frac{1}{2}k^4$,	$-(\frac{1}{2}k^4 + k^2 + 1)$	
		x_0, y_0	a_0 , b_0	z_0 ,	C''	
ii.	1	$1, k$	1 , k^2	k ,	0	I
	2	$1, k$	-1 , k^2	k ,	$-2/k^2$	E of C_1'
	3	$k, 1$	1 , k^2	k^2,	$1 - k^2$	R of C_1'''
	4	$k, 1$	-1 , k^2	k^2,	$-(1 + k^2)$	R of C_2^{iv}
	5	$1, k$	k^2 , 1	$1/k$,	$(k^2 - 1)/k^2$	R of C_4'''
	6	$1, k$	$-k^2$, 1	$1/k$,	$-(k^2 + 1)/k^2$	R of C_4^{iv}
	7	$k, 1$	k^2 , 1	1 ,	0	I
	8	$k, 1$	$-k^2$, 1	1 ,	$-2k^2$	
		x_0, y_0	c_0 , d_0	z_0''',	C'''	
iii.	1	$1, k$	$k^2 - 1$, k	1 ,	$(k^2 - 2)/k^2$	R of C_3''
	2	$1, k$	$-k^2 + 1$, k	1 ,	-1	I
	3	$k, 1$	$k^2 - 1$, k	k ,	-1	I
	4	$k, 1$	$-k^2 + 1$, k	k ,	$(-2k^2 + 1)$	R of C_5''
		x_0, y_0	e_0 , f_0	z_0^{iv},	C^{iv}	
iv.	1	$1, k$	$k^2 + 1$, k	1 ,	$+1$	I
	2	$1, k$	$-(k^2 + 1)$, k	1 ,	$-(k^2 + 2)/k^2$	R of C_4''
	3	$k, 1$	$k^2 + 1$, k	k ,	$+1$	I
	4	$k, 1$	$-(k^2 + 1)$, k	k ,	$-(2k^2 + 1)$	R of C_6''

[*Primary Sets.*]

$N_0 = 1^4 + 2^4 = 17.$ | $N_0 = 3^4 + 2^4 = 97.$

	Ref. No.	x_0, y_0	P_0, Q_0	z_0,	C'	R	x_0, y_0	P_0, Q_0	z_0,	C'	R
i.	1	1, 2	9, 8	4,	2	C_2''	3, 2	49, 48	24,	10	
	2	1, 2	$\bar{9}$, 8	4,	$-5/2$		3, 2	$\overline{49}$, 48	24,	$-29/2$	
	3	2, 1	9, 8	8,	5		2, 3	49, 48	16,	5	
	4	2, 1	$\bar{9}$, 8	8,	-13		2, 3	$\overline{49}$, 48	16,	$-53/9$	

		x_0, y_0	a_0, b_0	z_0,	C''	R	x_0, y_0	a_0, b_0	z_0,	C''	R
ii.	1	1, 2	1, 4	2,	0	I	3, 2	9, 4	2,	0	I
	2	1, 2	$\bar{1}$, 4	2,	$-1/2$	C_1'	3, 2	$\bar{9}$, 4	2,	$-9/2$	
	3	2, 1	1, 4	4,	-3	C_1'''	2, 3	9, 4	4/3,	5/9	C_2'''
	4	2, 1	$\bar{1}$, 4	4,	-5	C_2^{iv}	2, 3	$\bar{9}$, 4	4/3,	$-13/9$	C_2^{iv}
	5	1, 2	4, 1	1/2,	3/4	C_4'''	3, 2	4, 9	9/2,	$-5/4$	C_3'''
	6	1, 2	$\bar{4}$, 1	1/2,	$-5/4$	C_4^{iv}	3, 2	$\bar{4}$, 9	9/2,	$-13/4$	C_4^{iv}
	7	2, 1	4, 1	1,	0	I	2, 3	9, 4	4/3,	0	I
	8	2, 1	$\bar{4}$, 1	1,	-8		2, 3	$\bar{9}$, 4	4/3,	$-8/9$	

		x_0, y_0	c_0, d_0	z_0,	C'''	R	x_0, y_0	c_0, d_0	z_0,	C'''	R
iii.	1	1, 2	3, 2	1,	1/2	C_3''	3, 2	5, 6	3,	-1	I
	2	1, 2	$\bar{3}$, 2	1,	-1	I	3, 2	$\bar{5}$, 6	3,	$-7/2$	C_3''
	3	2, 1	3, 2	2,	-1	I	2, 3	5, 6	2,	1/9	C_5''
	4	2, 1	$\bar{3}$, 2	2,	-7	C_5''	2, 3	$\bar{5}$, 6	2,	-1	I

		x_0, y_0	e_0, f_0	z_0,	C^{iv}	R	x_0, y_0	e_0, f_0	z_0,	C^{iv}	R
iv.	1	1, 2	5, 2	1,	1	I	3, 2	13, 6	3,	1	I
	2	1, 2	$\bar{5}$, 2	1,	$-3/2$	C_4''	3, 2	$\overline{13}$, 6	3,	$-11/2$	C_4''
	3	2, 1	5, 2	2,	1	I	2, 3	13, 6	2,	1	I
	4	2, 1	$\bar{5}$, 2	2,	-9	C_6''	2, 3	$\overline{13}$, 6	2,	$-17/9$	C_6''

Factorisants from $N_0 = 17.$

	Ref. No.	x_0, y_0	P_0, Q_0	z_0',	C'	Factorisant.	Serial.
i.	1	1, 2	9, 8	4,	2	$(2x)^2+3y^2 = z^2$	x
	2	1, 2	$\bar{9}$, 8	4,	$-5/2$	$-5x^2+21(\frac{1}{2}y)^2 = z^2$	$x, x;\ z, z$
	3	2, 1	9, 8	8,	5	$10x^2+6(2y)^2 = z^2$	$x, x;\ y, y$
	4	2, 1	$\bar{9}$, 8	8,	-13	$-26x^2+42(2y)^2 = z^2$	$x, x;\ z, z$

		x_0, y_0	a_0, b_0	z_0'',	C''	Factorisant.	Serial.
ii.	3	2, 1	1, 4	4,	-3	$6x^2-2(2y)^2 = z^2$	$y,\ z$
	4	2, 1	$\bar{1}$, 4	4,	-5	$10x^2-6(2y)^2 = z^2$	$y, y;\ z, z$
	5	1, 2	4, 1	1/2,	3/4	$-\frac{3}{2}x^2+7(\frac{1}{4}y)^2 = z^2$	$x, x;\ z, z$
	6	1, 2	$\bar{4}$, 1	1/2,	$-5/4$	$\frac{5}{2}x^2-(\frac{3}{4}y)^2 = z^2$	$y, y;\ z, z$
	8	2, 1	$\bar{4}$, 8	1,	-8	$(4x)^2-7(3y)^2 = z^2$	z

$N_0 = 1921 = 17.113.$

Ref. No.	x_0, y_0	Primary. P_0, Q_0	$z'_0, \quad C'$	R	Secondary. P_0, Q_0	$z'_0, \quad C'$	R
i. 1	5, 6	961, 960	160, 26		65, 48	8, 10/9	
2	5, 6	$\overline{961}$, 960	160, −493/18		$\overline{65}$, 48	8, −5/2	
3	6, 5	961, 960	192, 37		65, 48	48/5, 29/25	
4	6, 5	$\overline{961}$, 960	192, −997/25		$\overline{65}$, 48	48/5, −101/25	
	x_0, y_0	a_0, b_0	$z''_0, \quad C''$	R	a_0, b_0	$z''_0, \quad C''$	R
ii. 1	5, 6	25, 36	6, 0	I	39, 20	10/3, 7/18	C_3^{iv}
2	5, 6	$\overline{25}$, 36	6, −25/18		$\overline{39}$, 20	10/3, −16/9	C_3'''
3	6, 5	25, 36	36/5, −11/25	C_1'''	39, 20	4, 3/25	
4	6, 5	$\overline{25}$, 36	36/5, −61/25	C_2^{iv}	$\overline{39}$, 20	4, −3	$C_1''' C_2^{iv}$
5	5, 6	36, 25	25/6, 11/36	C_4'''	20, 39	13/3, −5/36	
6	5, 6	$\overline{36}$, 25	25/6, −61/36	C_4^{iv}	$\overline{20}$, 39	13/3, −5/4	
7	6, 5	36, 25	5, 0	I	20, 39	39/5, −16/25	
8	6, 5	$\overline{36}$, 25	5, −72/25		$\overline{20}$, 39	39/5, −56/25	
	x_0, y_0	c_0, d_0	$z'''_0, \quad C'''$	R	c_0, d_0	$z'''_0, \quad C'''$	R
iii. 1	5, 6	11, 30	5, −7/18	C_3''	43, 6	1, 1/2	$C_4'' C_2^{iv}$
2	5, 6	$\overline{11}$, 30	5, −1	I	$\overline{43}$, 6	1, −17/9	
3	6, 5	11, 30	6, −1	I	43, 6	6/5, 7/25	C_2''
4	6, 5	$\overline{11}$, 30	6, −47/25	C_5''	$\overline{43}$, 6	6/5, −79/25	
	x_0, y_0	e_0, f_0	$z^{iv}_0, \quad C^{iv}$	R	e_0, f_0	$z^{iv}_0, \quad C^{iv}$	R
iv. 1	5, 6	61, 30	5, +1	I	47, 12	2, 11/18	
2	5, 6	$\overline{61}$, 30	5, −43/18	C_4''	$\overline{47}$, 12	2, −2	$C_4'' C_1'''$
3	6, 5	61, 30	6, +1	I	47, 12	12/5, 11/25	C_1^{iv}
4	6, 5	$\overline{61}$, 30	6, −97/25	C_6''	$\overline{47}$, 12	12/5, −83/25	

Associate Quartans, (H_n).

$$H_n = \tfrac{1}{2}(x'^4 + y'^4) = y^{4n} + 6y^{2n} + 1; \qquad [x' = y^n - 1,\ y' = y^n + 1].$$

n	$y = 6$: x', y'	$y = 6$: H	$y = 10$: x', y'	$y = 10$: H	$y = 12$: x', y'	$y = 12$: H
1	5, 7	17.89;	9, 11	10601;	11, 13	21601;
2	35, 37	1687393;	99, 101	100060001;	143, 145	41.10490393 ? †
3	215, 217	641.3396353;	999, 1001	17.5569.10562737;		

☞ *For Table of* H_n *with* $y = 2$, *see page* 130.

Simple Quartan Chains, $N_r = (1^4 + y_r^4) = L_r . M_r$.

$$y_{-1} = 0, \quad y_{r+1} = y_0^2 . y_r - y_{r-1};$$

$$L_0 = 1; \quad M_0 = 1^4 + y_0^4; \quad M_{r-1} = 1 + y_{r-1} y_r = L_r; \quad M_r = 1 + y_r y_{r+1} = L_{r+1}.$$

r	0	1	2	3	4	5
x, y	1, 2	1, 8	1, 30	1, 112	1, 418	1, 1560
M	17 ;	241 ;	3361 ;	46817 ;	652081 ;	313.29017 ;

r	6	7	8
x, y	1, 5822	1, 21728	1, 81090
M	17.89.83609 ;	7753.227257 ;	113.569.381673 ;

r	0	1	2	3	4
x, y	1, 3	1, 27	1, 240	1, 2133	1, 18957
M	2.41 ;	6481 ;	17.30113 ;	2.17.337.3529 ;	193.16548577 ;
x, y	1, 4	1, 64	1, 1020	1, 16256	
M	257 ;	97.673 ;	449.36929 ;	§	
x, y	1, 5	1, 125	1, 3120	1, 77875	
M	2.313 ;	390001 ;	17.113.126481 ;		
x, y	1, 6	1, 216	1, 7770	1, 279504	
M	1297 ;	1678321 ;	5521.393361 ;		
x, y	1, 7	1, 343	1, 16800		
M	2.1201 ;	73.193.409 ;	281.49195721 ;		
x, y	1, 8	1, 512	1, 32760		
M	17.241 ;	433.38737 ;	§		
x, y	1, 9	1, 729	1, 59040		
M	2.17.193 ;	97.577.769 ;	‡		
x, y	1, 10	1, 1000	1, 99990		
M	73.137 ;	99990001 ;	‡		

Simple Quartan Double Chain, $N_r = (1^4 + y_r^4) = L_r . M_r$.

$$N_0 = 1^4 + 1^4 = 2; \quad C' = -5/2; \quad z^2 - \tfrac{21}{4} y^2 = -\tfrac{5}{2}.1^2; \quad (\tfrac{5}{2})^2 - \tfrac{21}{4}.1^2 = +1.$$

$$y_{-1} = 0, \quad y_{r+1} = 5y_r - y_{r-1}, \quad z_{r+1} = 5z_r - z_{r-1};$$

$$L_r = 2, \quad M_{r-1} = \bar{1} + y_{r-1} y_r = L_r, \quad M_r = \bar{1} + y_r y_{r+1} = L_{r+1}.$$

	r	0	1	2	3	4	5	6	7
Ch. 1	x, y	1,1	1,2	1,9	1,43	1,206	1,987	1,4729	1,22658
	M	1 ;	17 ;	2.193 ;	17.521 ;	203321 ;	2.41.56921 ;	89.1203929 ;	‡
Ch. 2	x, y	1,1	1,3	1,14	1,67	1,321	1,1538	1,7369	1,35307
	M	2 ;	41 ;	937 ;	2.10753 ;	17.113.257 ;	641.17681 ;	2.17.7652273 ;	‡

Quartan Chains and Series.

i.—(2). $C' = -\frac{5}{2}$; $z^2 - \frac{21}{4}y^2 = -5x^2$; $(5/2)^2 - \frac{21}{4}.1^2 = +1$; $x = x_0 = 1$;

x-Chain 1°	x, y	1, 2	1, 9	1, 43	1, 206	1, 987	1, 4729	1, 22658
	$2z$	8	41	197	944	4523	21671	103832
	L	1;	17;	2.193;	17.521;	203321;	2.41.56921;	89.1203929;
	M	17;	2.193;	17.521;	203321;	2.41.56921;	89.1203929;	‡
x-Chain 2°	x, y	1, 1	1, 3	1, 14	1, 67	1, 321	1, 1538	1, 7369
	$2z$	1	13	64	307	1471	7048	33769
	L	1;	2;	41;	937;	2.10753;	17.113.257;	641.17681;
	M	2;	41;	937;	2.10753;	17.113.257;	641.17681;	2.17.7652273;

i.—(3). $C' = 5$; $z^2 - 24y^2 = 10x^2$; $5^2 - 24.1^2 = +1$; $x = x_0 = 2$;

x-Chain 1°	x, y	2,1	2,13	2,129	2,1277
	z	8	64	632	6256
	L	1;	17;	41.41;	257.641;
	M	17;	41.41;	257.641;	41.393721;

x-Chain 2°	x, y	2,3	2,31	2,307	2,3039
	z	16	152	1504	14888
	L	1;	97;	9521;	17.54881;
	M	97;	9521;	17.544881;	91422241;

i.—(3). $C' = 5$; $z^2 - 10x^2 = 24y^2$ †; $19^2 - 10.6^2 = +1$; $y = y_0 = 1$;

y-Series 1°	x, y	2, 1	86, 1	3266, 1
	z	8	272	10328
	L	1;	7129;	17.41.15289;
	M	17;	7673;	10677089;

y-Series 2°	x, y	10, 1	382, 1	14506, 1
	z	32	1208	45872
	L	73;	17.8513;	210378169;
	M	137;	147137;	210469913;

i.—(4). $C' = -13$; $z^2 - 168y^2 = -26x^2$; $13^2 - 168.1^2 = +1$; $x = x_0 = 2$;

x-Chain 1°	x, y	2,1	2,21	2,545	2,14149
	z	8	272	7064	183392
	L	1;	17;	17.673;	7711201;
	M	17;	17.673;	7711201;	

x-Chain 2°	x, y	2, 5	2, 129	2, 3349
	z	64	1672	43408
	L	1;	641;	41.41.257;
	M	641;	41.41.257;	4561.63841;

ii.—(3). $C'' = -3$; $z^2 - 6x^2 = -8y^2$; $5^2 - 6.2^2 = +1$; $y = y_0 = 1$;

y-Series.	x, y	2, 1	18, 1	178, 1	1762, 1	17442, 1
	z	4	44	436	4316	42724
	α, β	1, 0	7, 8	73, 72	719, 720	
	α, β	1, 4	23, 20	217, 220	2159, 2156	
	L	1;	113;	10513;	1035361;	
	M	17;	929;	17.41.137;	73.127529;	

ii.—(4). $C'' = -5$; $z^2 - 10x^2 = -24y^2$; $19^2 - 10.6^2 = +1$; $y = y_0 = 1$;

y-Series 1°.	x, y	2, 1	62, 1	2354, 1	89390, 1
	z	4	196	7444	282676
	α, β	1, 0	19, 20	745, 744	
	α, β	1, 4	101, 96	3719, 3724	
	L	1;	761;	1108561;	
	M	17;	19417;	17.1629361;	

y-Series 2°.	2, 1	14, 1	530, 1	20126, 1
	4	44	1676	63644
	1, 0	5, 4	167, 168	
	1, 4	19, 24	841, 836	
	1;	41;	56113;	
	17;	937;	41.34297;	

ii.—(5). $C''=3/4$; $z^2-\frac{7}{16}y^2=-\frac{3}{2}x^2$; $8^2-7.3^2=+1$; $x=x_0=1$;

x-Series 1°.	x, y	1, 2	1, 22	1, 350	1, 5578	*x*-Series 2°.	1, 2	1, 10	1, 158	1, 2518
	$2z$	1	29	463	7379		1	13	209	3331
	α, β	1, 0	3, 8	127, 48	765, 2024		1, 0	3, 8	127, 48	765, 2024
	α, β	1, 4	53, 20	319, 844	13451, 5084		1, 4	11, 4	65, 172	2741, 1036
	L	1;	73;	18433;	4681801;		1;	73;	18433;	4681801;
	M	17;	3209;	814097;	17.353.34457;		17;	137;	33809;	17.193.2617;

ii.—(6). $C''=-5/4$; $x^2-\frac{5}{2}x^2=-\frac{9}{16}y^2$; $19^2-10.6^2=+1$; $y=y_0=2$;

y-Series 1°.	x, y	1, 2	25, 2	949, 2	36037, 2	*y*-Series 2°.	1, 2	13, 2	493, 2	18721, 2
	$2z$	1	79	3001	113959		1	41	1559	59201
	α, β	1, 0	7, 8	501, 300			1, 0	5, 4	155, 156	
	α, β	1, 4	39, 44	1501, 1496			1, 4	21, 16	779, 784	
	L	1;	113;	313.577;			1;	41;	137.353;	
	M	17;	3457;	41.109537;			17;	17.41;	193.6329;	

2^{ic} *Partitions of Co-factors* (L, M) *of Quartans in Series.*

i.—(2).	L	*x*-Chain 1°.	1;	17;	2.193;	17.521;	203321;	*x*-Chain 2°.	1;	2;	41;	937;	2.10753;
	α, β		1, 0	1, 4	5, 19	91, 24	115, 436		1, 0	1, 1	5, 4	19, 24	91, 115
	γ, δ		1, 0	3, 2	12, 11	55, 54	261, 260		1, 0	0, 1	3, 4	17, 18	84, 85
	ϵ, ϕ		1, 0	5, 2	22, 7	107, 36	511, 170		1, 0	2, 1	7, 2	35, 12	166, 55
i.—(3).	L	*x*-Chain 1°.	1;	17;	41.41;	257.641;		*x*-Chain 2°.	1;	97;	9521;	17.54881;	
	α, β		1, 0	1, 4	9, 40	89, 396			1, 0	9, 4	89, 40	881, 396	
	γ, δ		1, 0	3, 2	23, 24	235, 234			1, 0	5, 6	57, 56	557, 558	
	ϵ, ϕ		1, 0	5, 2	57, 28	573, 286			1, 0	13, 6	137, 68	1365, 682	
ii.—(3).	L	*y*-Series.	1;	113;	10513;	1035361;							
	α, β		1, 0	7, 8	73, 72	719, 720							
	γ, δ		1, 0	9, 4	89, 36	881, 360							
	ϵ, ϕ		1, 0	11, 2	105, 16	1043, 162							
	M	*y*-Series.	17;	929;	17.41.137;	73.127529;							
	α, β		1, 4	23, 20	217, 220	2159, 2156							
	γ, δ		3, 2	27, 10	267, 110	2643, 1078							
	ϵ, ϕ		5, 2	31, 4	317, 50	3127, 484							
ii.—(4).	L	*y*-Series 1°.	1;	761;	1108561;			*y*-Series 2°.	1;	41;	56113;		
	α, β		1, 0	19, 20	745, 744				1, 0	5, 4	167, 168		
	γ, δ		1, 0	27, 4	1033, 144				1, 0	3, 4	121, 144		
	ϵ, ϕ		1, 0	31, 10	1177, 372				1, 0	7, 2	265, 84		
	M	*y*-Series 1°.	$\overline{17}$;	19417;	17.1629361;			*y*-Series 2°.	$\overline{17}$;	937;	41.34297;		
	α, β		$\bar{1}$, 4	101, 96	3719, 3724				$\bar{1}$, 4	19, 24	841, 836		
	γ, δ		3, 2	137, 18	5163, 722				3, 2	17, 18	603, 722		
	ϵ, ϕ		5, 2	155, 48	5885, 1862				5, 2	35, 12	1325, 418		
ii.—(5).	L	*x*-Series 1°.	1;	73;	18433;	4681801;		*x*-Series 2°.	1;	73;	18433;	4681801;	
	α, β		1, 0	3, 8	127, 48	765, 2024			1, 0	3, 8	127, 48	765, 2024	
	γ, δ		1, 0	1, 6	1, 96	1, 1530			1, 0	1, 6	1, 96	1, 1530	
	ϵ, ϕ		1, 0	9, 2	145, 36	2313, 578			1, 0	9, 2	145, 36	2313, 578	
	M	*x*-Series 1°.	17;	3209;	814097;	17.353.34957;		*x*-Series 2°.	17;	137;	33809;	17.193.2617;	
	α, β		1, 4	53, 20	319, 844	13451, 5084			1, 4	11, 4	65, 172	2741, 1036	
	γ, δ		3, 2	3, 40	3, 638	3, 10168			3, 2	3, 8	3, 130	3, 2072	
	ϵ, ϕ		5, 2	61, 16	965, 242	15373, 3844			5, 2	13, 4	197, 50	3133, 784	

2^{ic} *Partitions of Co-factors* (L, M) *of Quartans* (N) *in Series.*

Twin Chains i.—(2).

$\alpha'_r + \beta'_r = \alpha'_{r+1}$ or β'_{r+1} (alternately); $\beta'''_{r+1} - \alpha'''_{r+1} = \alpha'''_r$ or β'''_r (alternately).

(L'_r even), $\alpha'_r = \alpha'''_r$, $\beta'_r = \alpha'''_{r+1}$; ($L'_r$ even), $\alpha'''_r = \alpha'_{r-1}$, $\beta'''_r = \alpha'_r$.

$\beta'_r = \beta'''_r$ or β'''_{r+1} (if even).

α'_{r+1} or $\beta'_{r+1} = (-2C')(\alpha'_r$ or $\beta'_r) - (\alpha'_{r-1}$ or $\beta'_{r-1})$.

$\gamma' - \delta' = +1 = \delta''' - \gamma'''$.

$\epsilon' \sim 3\phi' = +1 = \epsilon''' \sim 3\phi'''$.

Twin Chains i.—(3).

$\alpha'_{r+1} = 2\beta'_r + \alpha'_r = \alpha'''_r$, $\alpha'''_{r+1} = 2\beta'''_r + \alpha'''_r$.

$\alpha'_{r+1} = 2C'\alpha'_r - \alpha'_{r-1} = \alpha'''_r$, $\beta'_{r+1} = 2C'\beta'_r - \beta'_{r-1} = \beta'''_{r+1}$.

$\gamma' \sim \delta' = +1 = \gamma''' \sim \delta'''$.

$\epsilon' - 2\phi' = +1 = \epsilon''' - 2\phi'''$.

Single Series ii.—(3).

$\alpha' \sim \beta' = +1$, $\alpha'' \sim \beta'' = +3$.

$\beta' = 2\delta'$; $\beta'' = 2\delta''$.

$\phi' = \gamma' - \alpha' = \epsilon' - \gamma'$; $\phi'' = \gamma'' - \alpha'' = \epsilon'' - \gamma''$.

$\phi' = \square$ or $2.\square$, $\phi'' = 2\square$ or $\square$ (alternately).

$\alpha'' = 3\alpha' \mp 2$, $\gamma'' = 3\gamma'$, $\epsilon'' = 3\epsilon' \pm 2$.

$2\phi'_{r+1} = \delta'_{r+1} - \delta'_r$, $2\phi''_{r+1} = \delta''_{r+1} - \delta''_r$.

$\alpha' = 2\delta' \mp 1$, $\alpha'' = 2\delta'' \pm 3$.

Twin Series ii.—(4).

$\alpha' \sim \beta' = +1 = \alpha''' \sim \beta'''$; $\alpha'' \sim \beta'' = +5 = \alpha^{iv} \sim \beta^{iv}$.

$\beta' = 2\phi'$, $\beta'' = 2\phi''$, $\beta''' = 2\phi'''$, $\beta^{iv} = 2\phi^{iv}$.

$\delta' = \frac{1}{2}(\gamma' - \alpha') = \frac{1}{3}(\epsilon' - \alpha') = \frac{1}{2}(\gamma''' + \alpha''') = \frac{1}{3}(\epsilon''' + \alpha''') = \delta'''$.

$\delta'' = \frac{1}{2}(\gamma'' - \alpha'') = \frac{1}{3}(\epsilon'' - \alpha'') = \frac{1}{2}(\gamma^{iv} + \alpha^{iv}) = \frac{1}{3}(\epsilon^{iv} + \alpha^{iv}) = \delta^{iv}$.

$\epsilon' = \gamma' + \delta'$, $\epsilon'' = \gamma'' + \delta''$, $\epsilon''' = \gamma''' + \delta'''$, $\epsilon^{iv} = \gamma^{iv} + \delta^{iv}$.

Twin Series ii.—(5).

$(\alpha', \beta', \gamma', \delta', \epsilon', \phi') = (\alpha''', \beta''', \gamma''', \delta''', \epsilon''', \phi''')$, respectively.

$\gamma' = \gamma''' = 1$, $\gamma'' = \gamma^{iv} = 3$.

$\epsilon' = 4\phi' + 1 = \epsilon'''$, $\epsilon'' = 4\phi'' - 3$, $\epsilon^{iv} = 4\phi^{iv} - 3$.

$\phi' = \phi''' = \square$ or $2.\square$, ϕ'' and $\phi^{iv} = 2\square$ or $\square$.

$\delta'_{r+1} = 16\delta'_r - \delta'_{r-1} = \delta'''_{r+1}$, $\delta''_{r+1} = 16\delta_r - \delta_{r-1}$, $\delta^{iv}_{r+1} = 16\delta^{iv}_r - \delta^{iv}_{r-1}$.

$\delta'_r = 2\alpha'_r$, $\delta''_r = 2\alpha''_r$, $\delta'''_r = 2\alpha'''_r$, $\delta^{iv}_r = 2\alpha^{iv}_r$, if r is *odd*.

$\delta'_r = 2\beta'_r$, $\delta''_r = 2\beta''_r$, $\delta'''_r = 2\beta'''_r$, $\delta^{iv}_r = 2\beta^{iv}_r$, if r is *even*.

$\phi'_{r-1} = 4\epsilon'_r - \phi'_{r-1}$, $\phi''_{r+1} = 4\epsilon''_r - \phi''_{r-1}$, $\phi^{iv}_{r+1} = 4\epsilon^{iv}_r - \phi^{iv}_{r-1}$.

Factorisants of Class i. *of Trinomial Quartans*, $N = (x^4 + 6x^2y^2 + y^4)$.

$x' = \frac{1}{2}(y-x)$, $y' = \frac{1}{2}(y+x)$, $N = 8(x'^4 + y'^4) = 8l.m$, $[xy = \omega,\ x'y' = \epsilon]$.

$x' = (y-x)$, $y' = (y+x)$, $N = \frac{1}{2}(x'^4 + y'^4) = L.M$, $[xy = \epsilon,\ x'y' = \omega]$.

Factorisants, $(2C'-6)x^2 + (C'^2-1)y^2 = z^2$.

Ex.	N_0	x_0, y_0	P_0 ,	Q_0	C' ,	z_0	Factorisant.	Serial.
1	8.17	3, 1	25/2,	17/2	7/2,	9/2	$x^2 + 5(\frac{3}{2}y)^2 = z^2$	x
2	4·34	1, 3	19 ,	15	2 ,	5	$-2x^2 + 3y^2 = z^2$	x
3	2.68	1, 3	−35,	33	−4,	11	$-14x^2 + 15y^2 = z^2$	x
4	41	1, 2	21 ,	20	5 ,	10	$(2x)^2 + 6(2y)^2 = z^2$	x
5	41	2, 1	21 ,	20	17 ,	20	$7(2x)^2 + 2(12y)^2 = z^2$	$x, x;\ y, y$
6	353	1, 4	177 ,	176	11 ,	44	$(4x)^2 + 30(2y)^2 = z^2$	x
7	17·73	5, 2	45 ,	28	5 ,	14	$(2x)^2 + 6(2y)^2 = z^2$	x, x

Examples from above Factorisants [Nos. 2 to 7].

Ex. 2. $C' = 2$; $z^2 - 3y^2 = -2$. $2^2 - 3.1^2 = +1$. $x = x_0 = 1$.

x, y, z	1, 3, 5	1, 11, 19	1, 41, 71	1, 153, 265	1, 571, 989
x', y'	1, 2	5, 6	20, 21	76, 77	285, 286
l	1 ;	17 ;	113 ;	3137 ;	21841 ;
m	17 ;	113 ;	3137 ;	21841 ;	608401 ;

x-Chain.

Ex. 3. $C' = -4$; $z^2 - 15y^2 = -14$. $4^2 - 15.1^2 = +1$. $x = x_0 = 1$.

x, y, z	1, 3, 11	1, 23, 89	1, 181, 701	1, 1425, 5519
x', y'	1, 2	11, 12	90, 91	712, 713
l	1 ;	17 ;	2081 ;	17·3793 ;
m	17 ;	2081 ;	17·3793 ;	7993537 ;

x-Chain.

Ex. 4. $C' = 5$; $(\frac{1}{2}z)^2 - 6y^2 = 1$. $5^2 - 6.2^2 = +1$. $x = x_0 = 1$.

x, y, z	1, 2, 10	1, 20, 98	1, 198, 970	1, 1960, 9602
x', y'	1, 3	19, 21	197, 199	1959, 1961
L	1 ;	41 ;	17.233 ;	388081 ;
M	41 ;	17·233 ;	388081 ;	38027921 ? †

x-Chain.

Ex. 5. $C' = 17$; $(\frac{1}{4}z)^2 - 7(\frac{1}{2}x)^2 = 2.3^2$. $8^2 - 7.3^2 = +1$. $y = y_0 = 1$.

x, y, z	2, 1, 20	46, 1, 244	734, 1, 3884	14, 1, 76	226, 1, 1196	3602, 1, 19060
x', y'	1, 3	45, 47	733, 735	13, 15	225, 227	3601, 3603
L	1 ;	1889 ;	534889 ;	137 ;	41.1217 ;	12955361 ? †
M	41 ;	2377 ;	17·137·233 ;	17·17 ;	52289 ;	12993481 ? †

Ex. 6. $C' = 11$; $(\frac{1}{4}z)^2 - 30(\frac{1}{2}y)^2 = 1$. $10^2 - 11.3^2 = +1$. $x = x_0 = 1$.

x, y, z	1, 4, 44	1, 88, 964	1, 1932, 21164
x', y'	3, 5	87, 89	1931, 1933
L	1 ;	353 ;	17·73·137 ;
M	353 ;	17·73·137 ;	113·725201 ;

x-Chain.

Ex. 7. $C' = 5$; $(\frac{1}{2}z)^2 - 6y^2 = 25$. $5^2 - 6.2^2 = +1$. $x = x_0 = 5$.

x, y, z	5, 2, 14	5, 24, 118	5, 238, 1166	5, 4, 22	5, 42, 206	5, 416, 2038
x', y'	3, 7	19, 29	233, 243	1, 9	37, 47	411, 421
L	17 ;	73 ;	5737 ;	17 ;	193 ;	17497 ;
M	73 ;	5737 ;	560753 ;	193 ;	17497 ;	577·2969 ;

High Simple Quartans, $N = (1^4 + y^4)$, $[y > 1{,}000]$.

y	N	y	N
1 001	2.17.17.1737034609 ? §	10 001	2.41.4129.29547107609 ? ‡
1 020	97.673 : 449.36929;	14 506	210378169 : 210469913;
1 538	17.113.257 : 641.17681;	16 256	449.36929: §
1 551	2.4217.4289.159977;	16 800	73.193.409 : 281.49195721;
1 560	652081 : 313.29017;	18 957	2.17.337.3529 : 193.16548577;
1 762	1035361 : 73.127529;	21 728	17.89.83609 : 7753.227257;
2 000	233. §	22 658	89.1203929: ‡
2 121	2.70001.144553441;	22 946	41.673.10729.25913.36137;
2 133	17.30113 : 2.17.337.3529;	32 760	433.38737: §
2 354	1108561 : 17.1629361;	35 307	2.17.7652273: ‡
2 413	2.23873.25793.27529;	59 040	97.577.769: ‡
2 518	461801 : 17.193.2617;	77 875	17.113.126481 : 2.
3 120	390001 : 17.113.126481;	81 090	7753.227257 : 113.569.381673;
3 266	17.41.15289 : 10677089;	92 564	41.337.457.4057.42073.68113;
4 729	2.41.56921 : 89.1203929;	99 990	99990001: ‡
5 578	4681801 : 17.353.34457;	99 999	2.89. §
5 822	313.29017 : 83609;	100 001	2. §
7 369	641.17681 : 2.17.7652273;		
7 453	2.6857.15889.14160017;		
7 770	1678321 : 5521.393361;		
9 999	2.337.13553.1094286241;		

High Quartans $N = (2^4 + y^4)$, $[y > 53]$.

y	N	y	N
129	41.41 : 257.641;	1 277	257.641 : 41.393721;
307	9521 : 17.544881;	3 039	17.544881 : 91422241;
493	137.353 : 193.6329;	3 349	41.41.257 : 4561.63841;
545	17.673 : 7711201;	14 149	7711201 : †
949	313.577 : 41.109537;		

End of Quartan Factorisations

Simple Octavans, $N = (y^8+1)$; [y even].
[All factors < 100,000 cast out.]

y	N	y	N
2	257;	102	
4	65537;	104	1201.
6	17.98801;	106	18049.
8	257; 97.673;	108	17.1361.2017.396622273;
10	17.5882353;	110	
12	17.97.260753;	112	17.1409.4673.221201713;
14	17.5393.16097;	114	17.881.
16	641.6700417;	116	17.
18	97.113607841;	118	
20	17.1505882353;	120	
22	17.	122	17.593.29537.164819521;
24	17.2801.2311681;	124	17.97.1249.98629.274993;
26	3617.57734881;	126	17.241.
28	17.	128	257; 5153.54410972897; La
30	337.401.4855073;	130	17.241.
32	257; 4278255361; La	132	
34	47441.37642417;	134	3697.
36	353.1697.4709377;	136	769.
38		138	8273.
40	17.17.113.337.641.929;	140	
42	113.	142	17.
44	17.241.3457.991873;	144	
46	17.929.1269398609;	146	17.337.
48	17.113.	148	17.97.113.
50	193.	150	17.
52	977.	152	
54	17.14593.291444977;	154	13457.
56	17.	156	17.1153.
58	17.	158	17.17.17.
60	353.	160	17.
62	17.1009.15217.836497;	162	
64	65537; 193.22253377; La	164	17.
66	37057.9715859521;	166	193.
68	881.	168	433.
70	97.	170	1153.
72		172	16369.
74	17.4481.69233.170497;	174	
76	241.	176	17.97.
78	17.113.	178	113.
80	17.977.91009.1109921;	180	17.14561.
82	17.593.	182	17.97.
84	673.	184	17.113.
86		186	97.113.
88	17.	188	
90	17.	190	17.193.1297.
92	17.	192	17.
94	593.	194	17.3793.
96	17.74209.5718266129;	196	193.
98	1249.	198	17.
100	353.449.641.1409.69857; Lo	200	

Simple Half-Octavans, $\frac{1}{2}N = \frac{1}{2}(y^8+1)$; [$y$ *odd*].

[All factors < 100,000 cast out.]

y	$\frac{1}{2}N$	y	$\frac{1}{2}N$
1	1 ;	101	929.
3	17.193 ;	103	10433.
5	17.11489 ;	105	17.97.2129.
7	17.169553 ;	107	17.3137.
9	21523361 ; B	109	17.97.
11	17.6304673 ;	111	241.
13	407865361 ;	113	17.3121.
15	7121.179953 ;	115	241.97.
17	18913.184417 ;	117	10289.95569.17855281 ;
19	15073.563377 ;	119	5153.
21	62897.300673 ;	121	51329.
23	17.3697.623009 ;	123	3889.
25	2593.29423041 ; D, B	125	17.11489 ;
27	17.193 ; 97.577.769 ;	127	14369.21713.108455953 ;
29	17.26209.561377 ;	129	17.193.257.
31	17.	131	17.17.7841.
33		133	17.401.
35	449.113.22191649 ;	135	
37	17.	137	
39	17.3457.45534289 ;	139	17.77681.
41	17.	141	17.3793.
43		143	17.193.1489.
45	17.	145	
47		147	17.401.
49	353.	149	23873.
51		151	433.1873.74561.2234849 ;
53		153	113.6529.
55		155	113.1409.2081.502761569 ;
57	17.769.3361.1268017 ;	157	1297.
59	337.	159	17.
61	17.12401.454677073 ;	161	113.673.6353.8513.54881 ;
63	17.1361.	163	17.
65	17.17.113.577.8455217 ;	165	17.241.
67	1153.2417.2593.28097 ;	167	17.97.43313.4235092033 ;
69		169	2657.
71	17.113.577.291295393 ;	171	577.
73	17.113.25057.8377153 ;	173	17.
75	17.17.	175	17.
77	449.5233.262965473 ;	177	17.
79	17.97.6481.70978049 ;	179	1009.15361.
81		181	17.2801.4289.22817.123601 ;
83	10289.16001.6840289 ;	183	433.1009.
85	97.881.	185	
87	37361.	187	9601.19441.50321.79601 ;
89	97.	189	1489.1777.
91	17.	191	113.337.
93		193	17.
95	17.	195	433.
97	17.	197	17.241.
99	17.1553.	199	17.401.
		257	193.1601.12097.15377.165553 ;

Irreducible Octavans, $N = (x^8 + y^8)$.

[All factors < 1,000 cast out.]

x, y	N	
3, 2	17.401;	
5, 2	17.22993;	
7, 2	17.339121;	
9, 2	449.95873;	
11, 2	17.241.52321;	
13, 2		
15, 2		
17, 2	113.61732369?	†
19, 2	241.1601.44017;	
3, 4	17.4241;	
5, 4	17.26833;	
7, 4	17.193.1777;	
9, 4	3041.14177;	
11, 4	17.17.97.7649;	
13, 4	7057.115601;	
15, 4		
17, 4		
19, 4		
5, 6	2070241;	
7, 6	353.21089;	
11, 6	97.2227201;	
13, 6	17.449.107089;	
17, 6	337.20704561?	†
19, 6	17.999131921?	†
3, 8	17.113.8737;	
5, 8	17.1009873;	
7, 8	17.1326001;	
9, 8	577.103681;	
11, 8	17.13596241;	
13, 8		
15, 8		
17, 8	337.2753.7537;	
19, 8	113.150445489?	‡
3, 10	100006561;	D
7, 10	353.299617;	
9, 10	17.1249.6737;	
11, 10	113.2781937;	
13, 10	17.6257.8609;	
17, 10	97.72945953?	†
19, 10	17.3361.298993;	
5, 12	1873.229777;	
7, 12		‡
11, 12	113.5702129;	
13, 12	17.73277201?	†
17, 12	977.7580081;	
19, 12	17.1024326161?	†
3, 14		
5, 14	113.13063537?	†
9, 14	17.1409.63409;	
11, 14		
13, 14	17.134795281?	†
15, 14	17.97.2449169;	
17, 14	1153.7330049;	
19, 14	17.401.2707841;	
3, 16	17.97.2604593;	
5, 16	17.113.2236001;	
7, 16	17.252984241?	‡
9, 16		‡
11, 16	17.241.1100641;	
13, 16		
15, 16		
17, 16	1009.11170193?	†
19, 16	241.88292657?	†
5, 18	17.648255953?	†
7, 18	17.433.1497857;	
11, 18	17.660842321?	†
13, 18	641.18464417?	†
17, 18		
19, 18	1361.20575697?	†
3, 20	433.59122417?	†
7, 20		
9, 20	17.113.929.14369;	
11, 20	433.59617457?	†
13, 20	17.1553866513?	†
17, 20		
19, 20	17.17.147347969?	†
3, 32	17.97.2604593;	
5, 32	17.113.2236001;	
7, 32	17.252984241?	†
11, 32	17.241.1100641;	
31, 16	334721:17.97.1553;	
25, 2		‡
25, 4	97.1573071713?	‡
25, 6	17.97.2113.43793;	
25, 8		‡
25, 16	4801.32677121;	

☞ *Continued on top of page* 143.

Irreducible Octavans (continued), $N = (x^8 + y^8)$.

[All factors < 1,000 cast out.]

x, y	N	x, y	N
5, 3	2.198593 ;	13, 9	2.
7, 3	2.113.25537 ;	17, 9	2.
11, 3	2.107182721 ;	19, 9	2.2161.3939521 ;
13, 3	2.17.113.241.881 ;	13, 11	2.17.401.75553 ;
17, 3	2.	15, 11	2.17.81683809 ? †
19, 3	2.17.499516753 ? †	17, 11	2.1217.2954033 ;
7, 5	2.97.31729 ;	19, 11	2.17.977.517729 ;
9, 5	2.17.1277569 ;	15, 13	2.4513.374321 ;
11, 5	2.1009.106417 ;	17, 13	2.
13, 5	2.17.24003569 ? †	19, 13	2.113.433.181889 ;
17, 5	2.769.593.7649 ;	17, 15	2.97.49168289 ? †
19, 5	2.17.499528049 ? †	19, 15	2.193.50638481 ? †
9, 7	2.17.17.84449 ;	19, 17	2.
11, 7	2.337.326593 ;	23, 19	2.17.641.4372513 ;
13, 7	2.17.97.249089 ;	25, 3	2.17.4487879329 ? †
15, 7	2.17.75548689 ? ‡	25, 7	2.17.4488048689 ? †
17, 7	2.	25, 9	2.97.786757409 ? †
19, 7	2.17.113.593.7457 ;	25, 11	2.17.337.13335857 ;
11, 9	2.17.97.78049 ;		

Simple Sexto-Decimans, $N = (y^{16} + 1^{16})$.

[All factors < 32,350 cast out.]

y	$N = (y^{16} + 1^{16})$; $[y = \epsilon]$	y	$\frac{1}{2}N = \frac{1}{2}(y^{16} + 1^{16})$; $[y = \omega]$
2	65537 ;	1	1 ;
4	641.6700417 ;	3	21523361 ;
6	353.1697.4709377 ;	5	2593.29423041 ;
8	65537 ; 193.22253377 ; Ll	7	353.47072139617?
10	353.449.641.1409.69857 ;	9	
12		11	
14	193.	13	2657.
16	274177.67280421310721 ; L	15	
18		17	257.
20	97.	19	97.
22	449.	21	1217.2689.31873.6857635489?
24	193.	23	193.
26		25	641.
28	97.	27	21523361 ;
30	97.257.	29	
32	65537 ;	31	1889.

16-mans, $N = (x^{16} + y^{16})$.

[All factors < 10,000 cast out.]

x, y	$N = (x^{16} + y^{16})$; $[xy = \epsilon]$	x, y	$\frac{1}{2}N = \frac{1}{2}(x^{16} + y^{16})$; $[xy = \omega]$
3, 2	3041.14177 ;	5, 3	97.786757409?
5, 2	97.1573071713?	7, 3	97.1713039877 13?
7, 2	97.449.8513.89633 ;	11, 3	
11, 2		7, 5	
3, 4		11, 5	
5, 4	4801.32677121 ;	11, 7	
7, 4			
11, 4	1249.36789218701793?		

Semi-Quartic Partitions of primes $p \ngtr 1{,}000$.

$p = (t_1^4 - 2u_1^2),\ (t_2^2 - 2u_2^4),\ (2u_3^4 - t_3^2),\ (2u_4^2 - t_4^4)$; $[p = 8\varpi \mp 1 = e^2 - 2f^2]$.

Blanks show that t^4 and $u^4 > 100^4$; dashes show partitions impossible.

$p = 8\varpi + 1$

p	t_1, u_1	t_2, u_2	u_3, t_3	u_4, t_4
1			1, 1	1, 1
17	—, —	7, 2	—, —	3, 1
41	—, —		3, 11	—, —
73	3, 2		—, —	—, —
89	7, 34	11, 2	—, —	—, —
97	—, —		—, —	7, 1
113	5, 16	25, 4	3, 7	
137	—, —	13, 2	3, 5	—, —
193	—, —	15, 2	—, —	
233	5, 14	4435, 56	—, —	—, —
241	—, —		—, —	11, 1
257		17, 2		13, 3
281	—, —		—, —	—, —
313	—, —		7, 67	—, —
337	5, 12		—, —	13, 1
353	7, 32		9, 113	
401	—, —		—, —	59, 9
409	—, —	21, 2	5, 29	—, —
433	—, —	55, 6	—, —	23, 5
449	—, —	31, 4	—, —	15, 1
457	—, —	93, 8		—, —
521	—, —		5, 27	
569	—, —	1109, 28		—, —
577		33, 4	7, 65	17, 1
593	5, 4	25, 2		
601	7, 30	9051, 80	—, —	—, —
617	5, 2		—, —	—, —
641	—, —		—, —	19, 3
673	—, —	815, 24	—, —	
761	—, —		15, 317	—, —
769	—, —		—, —	595, 29
809	—, —	29, 2	5, 21	—, —
857	—, —	37, 4		—, —
881				21, 1
929	—, —	31, 2	—, —	
937			—, —	—, —
953	—, —		13, 237	—, —

$p = 8\varpi - 1$

p	t_1, u_1	t_2, u_2	u_3, t_3	u_4, t_4
7	—, —	3, 1	2, 5	2, 1
23	—, —	5, 1	2, 3	18, 5
31	3, 5		2, 1	4, 1
47	5, 17	7, 1		8, 3
71	—, —		4, 21	6, 1
79	3, 1	9, 1		
103	—, —	115, 9	14, 277	442, 25
127		17, 3		8, 1
151	—, —		4, 19	86, 11
167	—, —	13, 1		58, 9
191	11, 85		6, 49	36, 7
199	—, —	19, 3		10, 1
223	7, 33	15, 1	4, 17	
239	13, 119	71, 7		120, 13
263	—, —		12, 203	
271	19, 255	39, 5	8, 89	
311	—, —			14, 3
359	—, —	19, 1		
367		23, 3		
383	5, 11		6, 47	
431		41, 5	4, 9	16, 3
439	—, —	21, 1		
463	5, 9	25, 3	4, 7	
479	7, 31	511, 19		
487	—, —		4, 5	38, 7
503	—, —		4, 3	
599	—, —	43, 5		
607	5, 3			
631	—, —			
647	—, —	173, 11		18, 1
719	7, 29			20, 3
727	—, —	27, 1		26, 5
743	—, —		6, 43	
751			16, 361	256, 19
823	—, —	75, 7		
839	—, —	29, 1		
863	11, 83			
887	—, —			22, 3
911			6, 41	
919	—, —			
967	—, —		8, 85	22, 1
977				23, 3
983	—, —			
991	27, 515			

Semi-Quartic Partitions of Composites $N \not> 1{,}000$.

$N = (t_1^4 - u_1^2),\ (t_2^2 - 2u_2^4),\ (2u_3^4 - t_3^2),\ (2u_4^2 - t_4^4)$; $[N = 8n \mp 1 = e^2 - 2f^2]$.

Blanks show that t^4 and $u^4 > 100^4$; dashes show partitions impossible.

N = 8n+1					N = 8n−1				
N	t_1, u_1	t_2, u_2	u_3, t_3	u_4, t_4	N	t_1, u_1	t_2, u_2	u_3, t_3	u_4, t_4
49	3, 4	9, 2		5, 1	119	—, —	11, 1	10, 141	10, 3
161			3, 1	9, 1	287	5, 13	17, 1	4, 15	12, 1
217		27, 4		—, —	343	—, - -		4, 13	22, 5
289	9, 56		5, 31		391	—, —		4, 11	14, 1
329		19, 2		—, —	511	9, 55	319, 15	4, 1	16, 1
497	5, 8	23, 2		17, 3	527	5, 7	23, 1		24, 5
529	11, 84	1273, 30		205, 17	623	5, 1	25, 1	8, 87	1432, 45
553	5, 6	205, 12		—, —	679	—, —	29, 3	10, 139	
697	15, 158	27, 2	—, —	—, —	791	—, —			
713	13, 118	35, 4		—, —	799		31, 3		20, 1
721			5, 23	19, 1	943	7, 27			28, 5
839		95, 8	21, 623	27, 5	959		31, 1		
889		59, 6	5, 19	—, —					
961			5, 17	41, 7					

Algebraic Semi-Quartic Partitions.

$$N_{iv} = (x^4 + y^4) = 2(x^2 \pm xy + y^4)^2 - (x \pm y)^4.$$

$$\tfrac{1}{2}N_{iv} = \tfrac{1}{2}(x^4 + y^4) = t^4 - 2\left\{\tfrac{1}{2}(x \pm y)^2\right\}^2, \text{ if } x^2 \pm xy + y^2 = t^2, \ [xy = \omega].$$

$$N_{viii} = (x^8 + y^8) = (x^4 + y^4)^2 - 2(xy)^4.$$

$$\tfrac{1}{2}N_{viii} = \tfrac{1}{2}(x^8 + y^8) = 2\left\{\tfrac{1}{2}(x^4 + y^4)\right\}^2 - (xy)^4.$$

$$N = t^4 \sim 2u^4 = t^4 \sim 2(u^2)^2 = (t^2)^2 \sim 2u^4.$$

$$N = 2(u^2 \mp t^2)^2 - t^4 = (2u^2 \mp t^2)^2 - 2u^4.$$

$$N = t^4 - 2u_1^2 = 2u_4^2 - t^4, \text{ if } t^4 = u_1^2 + u_4^2.$$

$$N_{iv} = (x^4 + y^4) = 2u^4 - (x \pm y)^4, \text{ if } x^2 \pm xy + y^2 = u^2.$$

Semi-Quartic Modular Forms.

$N = 40n + 1 = t_2^2 - 2u_2^4$ requires $u_2 = 10v$ and $t_2^2 \equiv N$, $(\text{mod } 2.10^4)$.

$N = 40n - 1 = 2u_3^4 - t_3^2$ requires $u_3 = 10v$ and $t_3^2 \equiv -N$, $(\text{mod } 2.10^4)$.

Impossible Semi-Quartic Forms.

$N = 16n + 9 \neq 2u_4^2 - t_4^4$; $\qquad N = 16n + 7 \neq t_1^4 - 2u_1^2$;

$N = a^2 + (4\omega)^2 \neq t_1^4 - 2u_1^2$; $\qquad N = c^2 + 2(2\omega)^2 \neq 2u_3^4 - t_3^2$.

$$N = t^2 - 2u_2^4 \neq 2u_3^4 - t^2, \quad [\text{or } t_2 \neq t_3].$$

Bin-Aurifeuillian Tree, [Base $N_1 = 5$].

$$N_r = (\xi_r^4 + 4\eta_r^4) = L_r . M_r ; \quad N'_{r+1} = L'_{r+1} . M'_{r+1}, \quad N''_{r+1} = L''_{r+1} . M''_{r+1}.$$

$$\xi'_{r+1} = (\xi_r + 2\eta_r) = \xi''_{r+1}, \quad \eta'_{r+1} = (\xi_r + \eta_r), \quad \eta''_{r+1} = \eta^r ; \quad [\xi_1 = 1, \ \eta_1 = 1, \ L_1 = 1, \ M_1 = 5].$$

Each M_r gives two N_{r+1} with same L_{r+1}; $M_r = L'_{r+1} = L''_{r+1} = (\xi_r + \eta_r)^2 + \eta_r^2$.

ξ_2, η_2	3, 1								3, 2							
L_2, M_2	5: 17;								5: 29;							
ξ_3, η_3	5, 1				5, 4				7, 5				7, 2			
L_3, M_3	17: 37;				17: 97;				29: 13·13;				29: 5·17;			
ξ_4, η_4	7, 1		7, 6		13, 9		13, 4		17, 5		17, 12		11, 9		11, 2	
L_4	37:		37:		97:		97:		13·13:		13·13:		5·17:		5·17:	
M_4	5·13;		5·41;		5·113;		5·61;		509;		5·197;		13·37;		173;	
ξ_5, η_5	9, 1	9, 8	19, 13	19, 6	31, 9	31, 22	21, 17	21, 4	27, 5	27, 22	41, 29	41, 12	29, 9	29, 20	15, 13	15, 2
L_5	5·13:	5·13:	5·41:	5·41:	5·113:	5·113:	5·61:	5·61:	509:	509:	5·197:	5·197:	13·37:	13·37:	173:	173:
M_5	$= (\xi_5^2 + 2\xi_5\eta_5 + 2\eta_5^2) = \{(\xi_5^2 + \eta_5^2) + \eta_5^2\} = L'_6 = L''_6$.															

Characteristics (C', C`) *and Factorisants of Bin-Aurifeuillians*; [$\mathrm{N} = x^4 + 4y^4$].

$$\text{I.}\quad C' = (\pm \mathrm{P}_0 - x_0^2) \div y_0^2\,; \qquad \textit{Factorist:}\ 2C'x^2 + (C'^2 - 4)y^2 = z^2.$$

$$\text{II.}\quad C\text{`} = (\pm \mathrm{P}_0 - 2y_0^2) \div x_0^2\,; \qquad \textit{Factorist:}\ 4C\text{`}y^2 + (C\text{`}^2 - 1)x^2 = z^2.$$

Ex.	N_0	x_0, y_0	P_0 , Q_0	C , z_0	Factorisant.	Serial.
1	$h^4 + 4k^4$	h, k	$h^2 + 2k^2$, $2hk$	2 , $2k$	Ineffective	
2	5	1, 1	−3 , 2	−4 , 2	$-2(2x)^2 + 3(2y)^2 = z^2$	x ; z, z
I. 3	65	1, 2	33 , 32	8 , 16	$(4x)^2 + 15(2y)^2 = z^2$	x
4	5·13	1, 2	−9 , 4	−5/2 , 2	$-5x^2 + (\frac{3}{2}y)^2 = z^2$	z, z
5	5·17	3, 1	−11 , 6	−20 ,	$-10(2x)^2 + 11(6y)^2 = z^2$	x, x ; z, z
1	$h^4 + 4k^4$	h, k	$h^2 + 2k^2$, $2hk$	1 , $2h$	Ineffective	
2	5	1, 1	−3 , 2	−5 , 2	$-5(2y)^2 + 6(2x)^2 = z^2$	y, y ; z, z
II. 3	65	1, 2	33 , 32	25 , 32	$(10y)^2 + 39(4x)^2 = z^2$	y, y ;
4	5·13	1, 2	−9 , 4	−17 , 4	$-17(2y)^2 + 2(12x)^2 = z^2$	y, y ; z, z
5	5·17	3, 1	−11 , 6	−13/9, 2	$-13(\frac{2}{3}y)^2 + 22(\frac{2}{3}x)^2 = z^2$	y, y ; z, z

Chain-Examples from above Factorisants.

$$\mathrm{N}_r = (x_r^4 + 4y_r^4) = \mathrm{L}_r.\mathrm{M}_r\,(\text{Aurifn.}) = \mathrm{L}'_r.\mathrm{M}'_r\,(\text{Dioph.}).$$

$$\mathrm{L} = l.\lambda,\ \ \mathrm{M} = m.\mu\,;\ \ \mathrm{L}' = l.\mu,\ \ \mathrm{M}' = \lambda.m\,;\ \ \mathrm{L}'_r = \mathrm{M}'_{r-1},\ \ \mathrm{M}'_r = \mathrm{L}'_{r+1}.$$

M'_r alone printed thus: $\mathrm{M}'_r = \lambda_r.{}_\wedge m_r$,
[the λ_r m_r out of L_r, M_r separated by the (‸)].

I.—2. $C' = -4$; $(\frac{1}{2}z)^2 - 3y^2 = -2.1^2$; $2^2 - 3.1^2 = +1$; $x = x_0 = 1$.

x-Chain.

r	0	1	2	3	4	5
x, y, z	1, 1, 2	1, 3, 10	1, 11, 38	1, 41, 142	1, 153, 530	1, 571, 1978
M′	1·‸5;	13·‸5;	17·‸53;	193·‸5·13;	241·‸25·29;	37·73·‸17·53;

r	6	7	8
x, y, z	1, 2131, 7382	1, 7953, 27550	1, 29681, 102818
M′	3361·‸5·2017;	37633·‸5·13·193;	46817·‸140453;

r	9	10	11
x, y, z	1, 110771, 383722	1, 413403, 1432070	1, 1542841, 5344558
M′	13.61.661·‸25·29·241;	652081·‸5·391249;	7300801; 17·37·53·73;

I.—3. $C' = 8$; $(\frac{1}{4}z)^2 - 15(\frac{1}{2}y)^2 = 1^2$; $4^2 - 15.1^2 = +1$; $x = x_0 = 1$.

x-Chain.

r	0	1	2	3	4
x, y, z	1, 2, 16	1, 16, 124	1, 126, 976	1, 992, 7684	1, 7810, 60496
M′	5·‸13;	37·‸109;	17·17·‸5·173;	2273·‸17·401;	29.617·‸13.4129;

r	5	6
x, y, z	1, 61488, 476284	1, 484094, 3749776
M′	140869·‸5·84521;	1109057·‸3327169;

Chain-Examples from Factorisants of Bin-Aurifeuillians continued).

II.—2. $C' = -5$; $(\frac{1}{2}z)^2 - 6x^2 = -5.1^2$; $5^2 - 6.2^2 = +1$; $y = y_0 = 1$.

y-Chain 1.	r	0	1	2	3	4
	y, x, z	1, 1, 2	1, 7, 34	1, 69, 338	1, 683, 3346	1, 6761, 33122
	M′	1.‸5;	37.‸13;	125.‸13.29;	61.61.‸17.73;	12281.‸5.7369;
	r	5	6			
	y, x, z	1, 66927, 327874	1, 662509, 3245618			
	M′	364717.‸61.1993;	5.233.1033.‸41.173.509;			
y-Chain 2.	r	0	1	2	3	4
	y, x, z	1, 1, 2	1, 3, 14	1, 29, 142	1, 287, 1406	1, 2841, 13818
	M′	1;	5.‸17;	157.‸53;	521.‸5.313;	113.137.‸13.397;
	r	5	6			
	y, x, z	1, 28123, 13774	1, 278389, 1363822			
	M′	5.17.601.‸13.11789;	29.52313.‸505693;			

End of Duan, Quartan, Octavan, &c., Factorisation Tables.

High Cuban Chains, $N = (Y^3-1) \div (Y-1) = Y'.Y'' > 6.10^{18}$.

$$Y = y^2; \quad y'-1 = y = y''+1;$$

$$Y' = y^2+y+1 = y'^2-y'+1; \quad Y'' = y'^2-y'+1 = y''^2+y''+1.$$

y	Y'	Y''	y'
49 991	7.223.16000993 ;	211.11844787 ;	49 992
2	211.11844787 ;	3.433.883.2179 ;	3
3	3.433.883.2179 ;	13.192265387 ;	4
4	13.192265387 ;	367.6810763 ;	5
5	367.6810763 ;	3.7.19.73.85819 ;	6
6	3.7.19.73.85819 ;	31.37.139.15679 ;	7
7	31.37.139.15679 ;	49.51017347 ;	8
8	49.51017347 ;	3.833316667 ;	9
9	3.833316667 ;	19.2269.57991 ;	50 000
50 000	19.2269.57991 ;	13.13.103.143629 ;	1
1	13.13.103.143629 ;	3.43.1291.15013 ;	2
2	3.43.1291.15013 ;	7.357192859 ;	3
3	7.357192859 ;	181.337.40993 ;	4
4	181.337.40993 ;	3.7.691.172321 ;	5
5	3.7.691.172321 ;	61.4153.9871 ;	6
6	61.4153.9871 ;	13.192365389 ;	7
7	13.192365389 ;	3.31.1723.15607 ;	8
8	3.31.1723.15607 ;	2500950091 ;	9

High Trin-Aurifeuillian Chains, $N = (Y^3+3^3) \div (Y+3) = L.M > 6.10^{18}$.

$Y = y^2; \; y''+1 = y = y'-1; \; [y \neq 3\eta]. \quad L = y''^2-y''+1, \; M = y'^2+y'+1.$

y	L	M
49 994	211.11844787 ;	367.6810763 ;
7	367.6810763 ;	49.51017347 ;
50 000	49.51017347 ;	13.13.103.143629 ;
3	13.13.103.143629 ;	181.337.40993 ;
6	181.337.40993 ;	13.192365389 ;
49 993	7.223.16000993 ;	13.192265387 ;
6	13.192265387 ;	31.37.139.15679 ;
9	31.37.139.15679 ;	19.2269.57991 ;
50 002	19.2269.57991 ;	7.357192859 ;
5	7.357192859 ;	61.4153.9871 ;
8	61.4153.9871 ;	2500950091 ;

High Trin-Aurifeuillian Chain, $\frac{1}{9}N = \frac{1}{9}(Y^3+3^3) \div (Y+3) = \frac{1}{3}L.\frac{1}{3}M > 6.10^{17}$.

$Y = 3\eta^2; \; y''+1 = y = y'-1; \; [y = 3\eta]. \quad L = y''^2-y''+1, \; M = y'^2+y'+1.$

y	$\frac{1}{3}$L	$\frac{1}{3}$M
49 995	433.883.2179 ;	7.19.73.85819 ;
8	7.19.73.85819 ;	833316667 ;
50 001	833316667 ;	43.1291.15013 ;
4	43.1291.15013 ;	7.691.172321 ;
7	7.691.172321 ;	31.1723.15607 ;

High Numbers, $\mathbf{N} = (Y^3+1) > 10^{19}$.

$$Y = 2^{2r}+1; \quad \mathbf{N} = N_i . N_{iii}; \quad N_i = (Y+1) = 2(2^{r-1}+1).$$

$$N_{iii} = \frac{Y^3+1}{Y+1} = \frac{(Y-1)^3-1}{(Y-1)-1} = \frac{2^{6r}-1}{2^{2r}-1} = X_1 . X_2; \quad X_1 = \frac{2^{3r}-1}{2^r-1}, \quad X_2 = \frac{2^{3r}+1}{2^r+1}.$$

r	11	13	15	17	19	21	23	25	27	29	31	33	35	37	39	41	43	45	47	49	
N_i											?			?	?		?		?	?	
X_1											?	?		?		?		?	?	?	
X_2										?	?			?	?	?	?		?	?	
Fig.	20	24	28	31	34	38	42	46	49	53	56	60	64	67	71	75	78	82	85	89	
r	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50	70
N_i												?	?		?		?	?			?
X_1																					
X_2											?		?		?		?		?		
Fig.	22	26	29	33	37	40	44	47	51	55	58	62	66	69	73	76	80	84	87	91	127

All factors of N_i, X_1, X_2 known from Lucas's Tables, except where queried thus (?).

High Irreducible Cubans, $N_1, N_2 > 9.10^6$.

$$N_1 = (x^3-y^3) \div (x-y), \quad N_2 = (x^3+y^3) \div (x+y); \quad x = \xi^\alpha, \ y = \eta^\beta; \quad [\xi, \eta \not> 11].$$

[6-*tans*, 9-*ans*, 12-*mans*, *&c., excluded.*]

x, y	N_1	N_2	x, y	N_1	N_2
2^{12}, 3	13.1291501 ;	3907 : 7.613 ;	2^{12}, 5	16797721 ;	3.7.61.103.127 ;
3^5	13.1371661 ;	7.373 : 6067 ;	5^5	3919.10039 ;	3.7.654421 ;
2^{13}, 3	43.1561243 ;	7.367.26113 ;	2^{13}, 5	3.13.769.2239 ;	19.757.4663 ;
3^2	31.2167183 ;	163.411259 ;	5^2	49.1373761 ;	3.43.313.1657 ;
3^3	547.123091 ;	7.9555487 ;	5^3	3.22716163 ;	7.13.726379 ;
3^4	7.601.16111 ;	19.67.52201 ;	5^4	127.577.991 ;	3.19.1094377 ;
3^5	69158569 ;	65177257 ;	5^5	3.13.151.17401 ;	7.37.197971 ;
2^{14}, 3	37.7256341 ;	13.1231 : 31.541 ;	2^{14}, 5	2311.116191 ;	7.38336223 ;
3^3	877.306589 ;	15259 : 7.13.193 ;	5^3	79.349.9811 ;	7.38057583 ;
3^5	349.780733 ;	13171 : 7.19.151 ;	5^5	19.17336899 ;	3.1069.70783 ;
2^{12}, 7	3.19.31.9511 ;	67.457.547 ;	2^{12}, 11	7.157.15307 ;	3.31.179917 ;
2^{13}, 7	1051.63907 ;	3.13.1719271 ;	2^{13}, 11	3.7.3199957 ;	67018873 ;
5^5, 2	3.13.43.5827 ;	343.37.769 ;	5^5, 3	103.94903 ;	9756259 ;
2^2	19.514639 ;	3.3251047 ;	3^2	9793831 ;	7.1391083 ;
2^3	3.151.21613 ;	7.571.2437 ;	3^3	7.1407247 ;	2791.3469 ;
2^4	9815881 ;	3.7.127.3643 ;	3^4	1039.9649 ;	661.14401 ;
2^6	79.126199 ;	3.7.455701 ;	7	997.9817 ;	3.13.433.577 ;
2^7	3.241.14083 ;	7.13.103099 ;	11	3.3266707 ;	13.748567 ;
2^8	10631161 ;	3.43.70009 ;	10^4, 3	673.148633 ;	31.313 : 10303 ;
2^9	3.199.19477 ;	7.103.11689 ;	3^3	13.7713133 ;	9127 : 49.223 ;
2^{11}	3.6786643 ;	13.19.127.241 ;	3^5	13.7883773 ;	19.397 : 7.43.43 ;
			10^5, 3	7.4597.310771 ;	13.43.17888551 ;

High Irreducible Cubans, $N_1, N_2 > 9.10^6$. $N_1 = (y^3-1) \div (y-1)$, $N_2 = (y^3+1) \div (y+1)$; $y = \xi^{\alpha}.\eta^{\beta}$; $[\xi, \eta \not> 11]$.

[*6-tans, 9-ans, 12-mans. &c., excluded.*]

y	N_1	N_2
$2 . 3^7$	19136251 ;	631.30313 ;
3^8	7.1609.15289 ;	19.43.210739 ;
3^9	67.23130169 ;	49.79.400321 ;
$2^2 . 3^7$	76536253 ;	31.277 : 7.19.67 ;
$2^3 . 3^7$	306127513 ;	7.97.450799 ;
3^8	7.31.12696049 ;	2754937657 ;
$2^4 . 3^5$	15120433 ;	19.199 : 7.571 ;
3^7	13.2221.42409 ;	37.937 : 35317 ;
$2^5 . 3^6$	7.79.984121 ;	739.736363 ;
$2^6 . 3^5$	241880257 ;	49.313 : 13.1213 ;
$2^7 . 3^3$	97.123169 ;	7.103.16561 ;
3^4	107505793 ;	1867.57571 ;
3^5	241.1483.2707 ;	7.138203959 ;
$2^8 . 3^3$	13.109.33721 ;	7.967 : 7057 ;
$2^9 . 3^2$	7.463.6553 ;	43.73.6763 ;
3^4	49.31.73.15511 ;	1719885313 ;
$2^{10} . 3$	373.25309 ;	13.229 : 3169 ;
3^3	9733.78541 ;	7.13.307.27361 ;
$2^{11} . 3$	19.1987099 ;	49.31.24847 ;
3^2	919.369703 ;	2689.126337 ;
$2^{12} . 3$	13.2797.4153 ;	12097 : 7.1783 ;
$2^{13} . 3$	97.283.22003 ;	603955201 ;
$2^{14} . 3$	61.39606037 ;	7.6967 : 49537 ;
$3 . 10^3$	7.757.1699 ;	13.613.1129 ;
10^4	13.4969.13933 ;	7.4243 : 157.193 ;

y	N_1	N_2
$2 . 5^5$	3.241.54037 ;	13.3004327 ;
5^6	7.139513393 ;	3.325510417 ;
$2^2 . 5^5$	151.229.4519 ;	3.7.67.111043 ;
$2^3 . 5^4$	7.79.103.439 ;	3.8331667 ;
5^5	3.208341667 ;	7.127.727.967 ;
$2^4 . 5^5$	19.2269.57991 ;	3.833316667 ;
$2^5 . 5^3$	3.13.410359 ;	49.326449 ;
5^4	31.12903871 ;	3.19.937.7489 ;
$2^7 . 5^2$	10243201 ;	3.19.179593 ;
5^3	3.85338667 ;	7.13.2813011 ;
$2^8 . 5^3$	109.9394789 ;	3.7.1129.43189 ;
$2^9 . 5^2$	7.3559.6577 ;	3.6067.9001 ;
$2^{10} . 5$	31.271.3121 ;	3.7.1248061 ;
5^3	3061.5352541 ;	3.7.67.367.31729 ;
$2^{11} . 5$	3.13.43.62533 ;	104847361 ;
5^2	7.127.823.3583 ;	3.3691.236737 ;
$2^{12} . 5$	419450801 ;	3.7.163.122527 ;
$2^{13} . 5$	3.559254187 ;	7.13.18436051 ;
5^2	313.1201.111577 ;	3. ¶
$7 . 10^3$	3.31.526957 ;	19.73.35323 ;
$7^2 . 10^3$	3.13.6151.10009 ;	2400951001 ;
$7^3 . 10$	3.37.97.1093 ;	61.192811 ;
10^2	3.392174767 ;	1176455701 ;
$7^4 . 10$	3.283.679039 ;	19081.30211 ;
$3^2 . 10^3$	81009001 ;	7.13.890011 ;
$3^5 . 10^2$	13.73.622249 ;	7.3433 : 24571 ;

y	N_1	N_2
$2 . 7^4$	23064007 ;	3.7684801 ;
7^5	13.86918047 ;	3.643.585727 ;
$2^3 . 7^4$	368966473 ;	3.73.601.2803 ;
$2^4 . 7^3$	3.193.52027 ;	30112657 ;
$2^5 . 7^3$	120483553 ;	3.13.919.3361 ;
$2^7 . 7^2$	39344257 ;	3.43.304897 ;
7^3	3.797216707 ;	37.64636549 ;
$2^9 . 7$	13.988357 ;	3.19.225289 ;
7^2	4153.151561 ;	3.19.67.97.1699 ;
$2^{10} . 7$	3.811.21121 ;	37.1388461 ;
$2^{11} . 7$	3361.61153 ;	3.13.5269399 ;
$2^{12} . 7$	3.193.1419883 ;	3571.230203 ;
$2 . 11^4$	857464807 ;	3.73.163.24019 ;
$2^2 . 11^3$	7.139.29137 ;	3.9446551 ;
$2^4 . 11^3$	49.211.43867 ;	3.151166107 ;
$2^5 . 11^2$	14996257 ;	3.241.20731 ;
11^3	3.7.86386717 ;	13.61.61.37501 ;
$2^7 . 11^2$	7.409.83791 ;	3.379.210961 ;
$2^9 . 11$	3.7.13.79.1471 ;	19.1669147 ;
11^2	7.241.577.3943 ;	3.19.139.484411 ;
$2^{10} . 11$	126888961 ;	3.1123.37657 ;
$2^{11} . 11$	3.7.24168253 ;	5281.96097 ;
$2^{12} . 11$	7.19.19.803359 ;	3.37.18288271 ;
$3^6 . 10$	31.613.2797 ;	7.13.13.44917 ;
$3^7 . 10$	7.68331253 ;	13.397.92671 ;

High Trin-Aurifeuillians,

$$N = (X^3 + Y^3) \div (X + Y) = L.M; \quad [X = x^2, \quad Y = 3y^2].$$

$$L = (X + Y - 3xy), \quad M = (X + Y + 3xy); \quad \mathbf{N} = (X + Y).N.$$

High Trin-Aurifeuillians, $x = 2^\alpha$, $y = 3^\beta$; $\mathbf{N} > 4.10^{21}$,

x, y	X + Y	X + Y	L	M	Fig.
$2^{12}, 1$	$2^{24} + 3$	1549.10831 ;	157.106783 ;	343.31.1579 ;	22
$2^{12}, 3^2$	$2^{24} + 3^5$	151.111109 ;	7.79.30139 ;	16876051 ;	22
$2^{13}, 1$	$2^{26} + 3$	7.9586981 ;	193.347587 ;	13.103.181.277 ;	24
$2^{13}, 3$	$2^{26} + 3^3$	5437.12343 ;	13.5156551 ;	7.127.75571 ;	24
$2^{13}, 3^2$	$2^{26} + 3^5$	13.1093.4723 ;	19.229.15373 ;	7.2011.4783 ;	24
$2^{14}, 1$	$2^{28} + 3$	268435459 ? ‡	7.38340901 ;	19.67.210907 ;	26
$2^{14}, 3$	$2^{28} + 3^3$	8779.30577 ;	7.38326861 ;	61.4402999 ;	26
$2^{14}, 3^2$	$2^{28} + 3^5$	7.38347957 ;	37.103.70321 ;	268878067 ? ‡	26

High Trin-Aurifeuillians, $x = 1$, $y = 2^\alpha.3^\beta$, &c.; $\mathbf{N} > 7.10^{21}$.

y	X + Y	X + Y	L	M	Fig.
2^{11}	$1 + 3 \ .2^{22}$	7.313.5743 ;	43.292483 ;	13.968389 ;	22
2^{12}	$3 \ .2^{24}$	61.825109 ;	859.58579 ;	7.7191991 ;	24
2^{13}	$3 \ .2^{26}$	13.1567.9883 ;	7.19.547.2767 ;	31.307.21157 ;	25
2^{14}	$3 \ .2^{28}$	7.37.139.22369 ;	805257217 ? ‡	157.619.8287 ;	27
$3 \ .2^{10}$	$1 + 3^3 \ .2^{20}$	28311553 ;	7.4043191 ;	28320769 ;	23
$3 \ .2^{11}$	$3^3 \ .2^{22}$	113246209 ? ‡	13.79.110251 ;	7.367.44089 ;	25
$3 \ .2^{12}$	$3^3 \ .2^{24}$	769 ; 7.103 : 19.43 ;	37.12241837 ;	937.483481 ;	26
$3^2.2^8$	$1 + 3^5 \ .2^{16}$	19.838171 ;	15918337 ;	7.151.15073 ;	22
$3^2.2^9$	$3^5 \ .2^{18}$	63700993 ;	7.13.283.2473 ;	1723.36979 ;	24
$3^2.2^{10}$	$3^5 \ .2^{20}$	49 ; 5200081 ;	31.61.134731 ;	37.6887341 ;	26
$3^3.2^7$	$1 + 3^7 \ .2^{14}$	13 ; 211.13063 ;	19.1885339 ;	49.43.17011 ;	23
$3^3.2^8$	$3^7 \ .2^{16}$	7.37.631.877 ;	7993.17929 ;	143347969 ? ‡	25
$3^3.2^9$	$3^7 \ .2^{18}$	573308929 ? ‡	7.673.121687 ;	13.733.60169 ;	27
$3^4.2^5$	$1 + 3^9 \ .2^{10}$	20155393 ;	7.2878231 ;	13.1551013 ;	22
$3^4.2^6$	$3^9 \ .2^{12}$	433 ; 397 : 7.67 ;	37.2178541 ;	19.4244059 ;	24
$3^4.2^7$	$3^9 \ .2^{14}$	31.229.45427 ;	79.4081711 ;	7.13.3544147 ;	26
$3^5.2^3$	$1 + 3^{11}.2^6$	11337409 ;	11331577 ;	7.13.31.4021 ;	22
$3^5.2^4$	$3^{11}.2^8$	7.6478519 ;	1543.29383 ;	37.1225981 ;	23
$3^5.2^5$	$3^{11}.2^{10}$	13.19.19.38653 ;	7.4507.5749 ;	781421857 ? ‡	25
$3^6.2^2$	$1 + 3^{13}.2^4$	7.37.98491 ;	25500421 ;	2137.11941 ;	23
$3^6.2^3$	$3^{13}.2^6$	102036673 ? ‡	13.43.182503 ;	7.709.20563 ;	25
$3^6.2^4$	$3^{13}.2^8$	2503.163063 ;	7.19.3068509 ;	31.13167151 ;	26
$3^7.2$	$1 + 3^{15}.2^2$	57395629 ;	7.13.630577 ;	57408751 ;	24

Pellian Trin-Aurifeuillian Chains, (N'_r, N_r).

$$\tau_r'^2 - 3\upsilon_r'^2 = \bar{2}; \quad \tau_r^2 - 3\upsilon_r^2 = +1.$$

i. $N'_r = (\tau_r'^6 + 3^3.\upsilon_r'^6) \div (\tau_r'^2 + 3\upsilon_r'^2) = L'_r.M'_r$; $M'_{r-1} = L'_r = \tau_{2r-2}$; $M'_r = L'_{r+1} = \tau_{2r}$.

r =	1	2	3	4	5	6	7
τ', υ' =	1, 1	5, 3	19, 11	71, 41	265, 153	989, 571	3691, 2131
L'_r =	1;	7;	97;	7·193;	31·607;	7·37441;	97·37633;
M'_r =	7;	97;	7·193;	31·607;	7·37441;	97·37633;	49·337·3079;

ii. $N_r = (\tau_r^6 + 3^3.\upsilon_r^6) \div (\tau_r^2 + 3\upsilon_r^2) = L_r.M_r$; $M_{r-1} = L_r = \frac{1}{2}\tau_{2r-1}$; $M_r = L_{r+1} = \frac{1}{2}\tau_{2r+1}$.

r =	0	1	2	3	4	5	6	7
τ, υ =	1, 0	2, 1	7, 4	26, 15	97, 56	362, 209	1351, 780	5042, 2911
L_r =	1;	1;	13;	181;	2521;	13·37·73;	489061;	6811741;
M_r =	1;	13;	181;	2521;	13·37·73;	489061;	6811741;	13.181; 61.661;

Ex. i. *continued.*		Ex. ii. *continued.*	
r =	8	r =	8
τ', υ' =	13775, 7953	τ, υ =	18817, 10864
L'_r =	49·337·3079;	L_r =	13.181; 61.661;
M'_r =	708158977 ? †	M_r =	1321442641 ? ‡

Pellian Trin-Aurifeuillian Chains.

$$N_r = (\tau_r^6 + 3^3.\upsilon_r^6) \div (\tau_r^2 + 3\upsilon_r^2) = L_r.M_r; \quad \tau_r^2 - 3\upsilon_r^2 = z = -11 \text{ or } +13.$$

$$L_r = (\tau_r^2 - 3\tau_r\upsilon_r + 3\upsilon_r^2), \quad M_r = (\tau_r^2 + 3\tau_r\upsilon_r + 3\upsilon_r^2); \quad M_r = L_{r+1}.$$

		r =	1	2	3	4	5	6	7
$z = -11$.	Chain 1.	τ, υ =	1, 2	8, 5	31, 18	116, 67	433, 250	1616, 933	6031, 3482
		L =	7;	19;	7·37;	3607;	7·7177;	43·16273;	7·19·127·577;
		M =	19;	7·37;	3607;	7·7177;	43·16273;	7·19·127·577;	
	Chain 2.	τ, υ =		4, 3	17, 10	64, 37	239, 138	892, 515	3329, 1922
		L =		7;	79;	7·157;	15307;	49·19·229;	2969479;
		M =		79;	7·157;	15307;	49·19·229;	2969479;	
$z = +13$.	Chain 1.	τ, υ =	4, 1	11, 6	40, 23	149, 86	556, 321	2075, 1198	7744, 4471
		L =	7;	31;	7·61;	19·313;	7·11833;	1153687;	7·2295541;
		M =	31;	7·61;	19·313;	7·11833;	1153687;	7·2295541;	
	Chain 2.	τ, υ =		5, 2	16, 9	59, 34	220, 127	821, 474	3064, 1769
		L =		7;	67;	49·19;	12967;	7·25801;	2515531;
		M =		67;	49·19;	12967;	7·25801;	2515531;	

Dimorph Trin-Aurifeuillian Products, $\Pi(\mathrm{N}') = \Pi(\mathrm{N}'')$.

$$\mathrm{N}_1\mathrm{N}_3\mathrm{N}_5\ldots\mathrm{N}_{2r+1}.\mathrm{N}_\beta = \mathrm{N}_0\mathrm{N}_2\mathrm{N}_4\ldots\mathrm{N}_{2r}.\mathrm{N}_\alpha\,; \quad \mathrm{N}_r = \frac{x_r^6+3^3y_r^6}{x_r^2+3y_r^2} = \mathrm{L}_r.\mathrm{M}_r.$$

i. $\mathrm{N}_r = [y_r] = \dfrac{2^6+3^3y_r^6}{2^2+3y_r^2} = \mathrm{L}_r.\mathrm{M}_r\,;\ \mathrm{N}_\alpha = |\rho| = \dfrac{\tau_\rho^6+3^3v_\rho^6}{\tau_\rho^2+3v_\rho^2}\,;\ \tau_\rho^2-3v_\rho^2 = +1\,;\ \mathrm{N}_\beta = |\rho-1|.$

$y_0 = 1,\ y_1 = y_0+2,\ y_2 = y_1+2,\ \ldots,\ y_{r+1} = y_r+2\,;\quad y_{2r+1} = \tau_\rho v_\rho - 1\,;\quad \mathrm{M}_r = \mathrm{L}_{r+1}.$

$\rho = 2,\ y_{2r+1} = 7.4-1$ | $\rho = 3,\ y_{2r+1} = 26.15-1$ | $\rho = 4,\ y_{2r+1} = 97.56-1$

$\dfrac{[3]\,[7]\,[11]\ldots[27]}{[1]\,[5]\,[9]\ldots[25]} = \dfrac{|2|}{|1|}\,;$ | $\dfrac{[31]\,[35]\ldots[389]}{[29]\,[33]\ldots[387]} = \dfrac{|3|}{|2|}\,;$ | $\dfrac{[393]\,[397]\ldots[5431]}{[391]\,[395]\ldots[5429]} = \dfrac{|4|}{|3|}.$

$$\frac{[3]\,[7]\,[11]\ldots[\tau_{\rho-1}.v_{\rho-1}-1]}{[1]\,[5]\,[9]\ldots[\tau_{\rho-1}.v_{\rho-1}-3]}\cdot\frac{[\tau_{\rho-1}.v_{\rho-1}+3]\ldots[\tau_\rho.v_\rho-1]}{[\tau_{\rho-1}.v_{\rho-1}+1]\ldots[\tau_\rho.v_\rho-3]} = \frac{|\rho|}{|1|}.$$

ii. $\mathrm{N}_r = (x_r) = \dfrac{x_r^6+3^3.2^6}{x_r^2+3.2^2} = \mathrm{L}_r.\mathrm{M}_r\,;\ \mathrm{N}_\alpha = |\rho| = \dfrac{\tau_\rho'^6+3^3v_\rho'^6}{\tau_\rho'^2+3v_\rho'^2}\,;\ \tau_\rho'^2-3v_\rho'^2 = 2\,;\ \mathrm{N}_\beta = |\rho-1|.$

$x_1 = 5,\ x_2 = x_1+6,\ x_3 = x_2+6,\ \ldots,\ x_{r+1} = x_r+6\,;\quad x_{2r} = \tau_\rho'^2-2\,;\quad \mathrm{M}_r = \mathrm{L}_{r+1}.$

$\rho = 2,\ x_{2r} = 5^2-2$ | $\rho = 3,\ x_{2r} = 19^2-2$ | $\rho = 4,\ x_{2r} = 71^2-2$ &c.

$\dfrac{(11)\,(23)}{(5)\,(17)} = \dfrac{|2|}{|1|}\,;$ | $\dfrac{(35)\,(47)\ldots(359)}{(29)\,(41)\ldots(353)} = \dfrac{|3|}{|2|}\,;$ | $\dfrac{(371)\,(383)\ldots(5039)}{(365)\,(377)\ldots(5033)} = \dfrac{|4|}{|3|}\,;$ &c.

$$\frac{(11)\,(23)\,(35)\ldots(\tau_{\rho-1}^2-2)}{(5)\,(17)\,(29)\ldots(\tau_{\rho-1}^2-8)}\cdot\frac{(\tau_{\rho-1}^2+10)\,(\tau_{\rho-1}^2+22)\ldots(\tau_\rho^2-2)}{(\tau_{\rho-1}^2+4)\,(\tau_{\rho-1}^2+16)\ldots(\tau_\rho^2-8)} = \frac{|\rho|}{|1|}.$$

Compound Trin-Aurifeuillians, $\mathbf{N} = (\mathrm{X}^3+\mathrm{Y}^3)\div(\mathrm{X}+\mathrm{Y})\,;\quad [\mathrm{X} = \Xi^2,\ \mathrm{Y} = 3\mathrm{H}^2].$

$\mathrm{N}_r = (x_r^3+y_r^3)\div(x_r+y_r) = \mathrm{L}_r.\mathrm{M}_r,\ \ [x_r = \xi_r^2,\ y_r = 3\eta_r^2]\,;\quad \mathbf{N} = \mathrm{N}_1\mathrm{N}_2 = (\mathrm{L}_1\mathrm{M}_1)\,(\mathrm{L}_2\mathrm{M}_2).$

$$\mathrm{L}_r = \xi_r^2-3\xi_r\eta_r+3\eta_r^2,\quad \mathrm{M}_r = \xi_r^2+3\xi_r\eta_r+3\eta_r^2.$$

	$m,\ n$	$\xi_1\ ,\ \eta_1$	$\mathrm{L}_1\quad \mathrm{M}_1$	$\xi_2\ ,\ \eta_2$	$\mathrm{L}_2\quad \mathrm{M}_2$	$\Xi\ ,\ \mathrm{H}$	Fig.
i.	3, 2	5, 4	13 : 7.19;	13, 4	61 : 373;	73, 48	8
	3, 4	7, 8	73 : 409;	25, 8	7.31 : 13.109;	241, 192	10
	1, 54	2915, 36	73.127.883 :	2917, 36	8197741 :	8501123, 3888	28
			49.179917;		613.14401;		
ii.	1, 2	4, 3	7 . 79;	4, 5	31 : 151;	43, 16	7
	1, 4	8, 15	379 : 7.157;	8, 17	523 : 13.103;	219, 144	10
	1, 40	80, 1599	79.92317 :	80, 1601	43.97.1753 :	7676803, 6400	28
			7.127.9067;		31.61.4273;		
iii.	3, 2	5, 12	277 . 49.13;	5, 13	337 : 727;	457, 25	11
	3, 4	7, 24	19.67 : 2281;	7, 25	1399 : 31.79;	1777, 49	14
	2, 33	1085, 132	799837 :	1085, 1093	1203457 :	1229497, 1085^2	26
			883.1879;		271.30697;		
iv.	1, 2	4, 1	7 : 31;	5, 1	13 : 43;	19, 3	6
	1, 4	8, 5	19 : 7.37;	17, 5	109 : 619;	139, 75	9
	1, 44	88, 645	7.13.79.151 :	1937, 645	409.3061 :	1255819,	26
			181.7879;		97.90187;	1248075	

Trin-Aurifeuillian Tree, [Base $N_1 = 7$].

$N_r = (\xi_r^4 - 3\xi_r^2\eta_r^2 + 9\eta_r^4) = L_r.M_r$; N_r gives five $N_{r+1} = L_{r+1}.M_{r+1}$, [same L in each].

Each of five $L_{r+1} = M_r = (\xi_r^2 + 3\xi_r\eta_r + 3\eta_r^2) = (A_r^2 + 3B_r^2)$.

[$\xi_1 = 1$, $\eta_1 = 1$, $L_1 = 1$, $M_1 = 7$.]

Formulæ 1 to 7 give the five values of ξ_{r+1}, η_{r+1}; [two of Nos. 4 to 7 giving − values fail].

		1	2	3	4	5	6	7
	ξ_{r+1}	3B + A	2A	A + 3B	3B − A	2A	A − 3B	3B − A
	η_{r+1}	2B	A + B	A + B	2B	A − B	A − B	B − A
$r = 2$	ξ_2, η_2	5, 2	4, 3	5, 3	1, 2	4, 1	.	.
	L_2, M_2	7:67;	7:79;	7:97;	7:19;	7:31;	.	.
$r = 3$	ξ_3, η_3	11, 2	16, 9	11, 9	.	16, 7	5, 7	.
	L_3, M_3	67:199;	67:49.19;	67:661;	.	67:739;	67:277;	.
	ξ_3, η_3	17, 10	4, 7	17, 7	13, 10	.	.	13, 3
	L_3, M_3	79:7.157;	79:13.19;	79:13.61;	79:859;	.	.	79:313;
	ξ_3, η_3	19, 8	14, 11	19, 11	5, 8	14, 3	.	.
	L_3, M_3	97:1009;	97:1021;	97:7.193;	97:337;	97:349;	.	.
	ξ_3, η_3	7, 2	8, 5	7, 5	.	8, 3	1, 3	.
	L_3, M_3	19:103;	19:7.37;	19:229;	.	19:163;	19:37;	.
	ξ_3, η_3	11, 6	4, 5	11, 5	7, 6	.	.	7, 1
	L_3, M_3	31:7.61;	31:151;	31:19.19;	31.283;	.	.	31:73;

Characteristics (C', $C^{\backprime}$) *and Factorisants of Trin-Aurifeuillians;*

[$N = x^4 - 3x^2y^2 + 9y^4$].

I. $C' = (\pm P_0 - x_0^2) \div y_0^2$; *Factorist:* $(2C' + 3)x^2 + (C'^2 - 9)y^2 = z^2$.

II. $C^{\backprime} = (\pm P_0 - 3y_0^2) \div x_0^2$; *Factorist:* $(6C^{\backprime} + 3)y^2 + (C^{\backprime 2} - 1)x^2 = z^2$.

Ex.	N_0	x_0, y_0	P_0, Q_0	C, z_0	Factorisant.	Serial.
1	N_0	h, k	$h^2 + 3k^2$, $3hk$	3, $3h$	Ineffective.	
2	7	1, 1	−4, 3	−5, 3	$-7x^2 + (4y)^2 = z^2$	z, z
I. 3	13	2, 1	−7, 6	−11, 6	$-19x^2 + 7(4y)^2 = z^2$	x, x; z, z
4	133	1, 2	67, 66	33/2, 33	$(6x)^2 + 13(9y)^2 = z^2$	x, x;
5	7.19	1, 2	−13, 6	−7/2, 3	$-(2x)^2 + 13(\frac{1}{2}y)^2 = z^2$	x, x; z, z
1	N_0	h, k	$h^2 + 3k^2$, $3hk$	1, $3k$	Ineffective.	
2	7	1, 1	−4, 3	−7, 3	$-39y^2 + 3(4x)^2 = z^2$	y, y; z, z
II. 3	13	2, 1	−7, 6	−5/2, 3	$-3(2y)^2 + 21(\frac{1}{2}x)^2 = z^2$	y, y; z, z
4	133	1, 2	67, 66	55, 66	$37(3y)^2 + 21(12x)^2 = z^2$	y, y; x, x
5	7.19	1, 2	−13, 6	−25, 6	$-3(7y)^2 + 39(4x)^2 = z^2$	y, y; z, z

Examples from above Factorisants (*page* 155).

$$N_r = (x_r^4 - 3x_r^2 y_r^2 + 9y_r^4) = L_r . M_r \text{ (Aurifn.)} = L'_r . M'_r \text{ (Dioph.)}.$$

$$L = l\lambda,\ \ M = m\mu\,;\ \ \ L' = l.\mu,\ \ M' = \lambda.m\,;$$

L'_r, M'_r printed thus: $L'_r = l_r.{}_\wedge\mu_r$, $M'_r = \lambda_r.{}_\wedge m_r$, [the factors out of L_r, M_r separated by the (${}_\wedge$)].

I.—2. $C' = -5$; $z^2 + 7x^2 = 4y^2$; $x = tu$, $y = \frac{1}{8}(t^2 + 7u^2)$, $z = \frac{1}{2}(t^2 - 7u^2)$.

t, u	1, 1	101, 1	1, 101	1, 343
x, y, z	1, 1, 3	101, 1276, 5097	101, 8926, 35703	343, 102943, 411771
L′	1;	601.‸2707;	31.571.‸7.643;	51343.‸13.15877;
M′	7;	13.577.‸1951;	13.13.79.‸43.1249;	619.997.‸154543;

I.—5. $C' = -7/2$; $z^2 - 13(\frac{1}{2}y)^2 = -4.1^2$; $649^2 - 13.180^2 = +1$; $x = x_0 = 1$.

Series 1.			Series 2.			
r	0	1	r	0	1	2
x, y, z	1, 2, 3	1, 2378, 4287	x, y, z	1, 2, 3	1, 218, 393	1, 282962, 510117
L′	7;	859.‸11173;	L′	7;	1021.‸79;	102241.‸7.189877;
M′	19;	19.1039.‸49.31;	M′	19;	139.‸49.37;	2349367.‸127.1423;

II.—2. $C' = -7$; $(4x)^2 - 3(\frac{1}{3}z)^2 = 13.1^2$; $7^2 - 3.4^2 = +1$; $y = y_0 = 1$.

y-Chain 1.

r	0	1	2	3	4
y, x, z	1, 1, 3	1, 10, 69	1, 139, 963	1, 1936, 13423	1, 26965, 186819
L′	1;	1.‸7;	73.‸19;	7.37.‸1039;	14449.‸3613;
M′	1.‸7;	73.‸19;	7.37.‸1039;	14449.‸3613;	67.751.‸7.28753;

r	5
y, x, z	1, 375574, 2602053
L′	67.751.‸7.28753;
M′	1627.1723.‸700831;

y-Chain 2.

r	0	1	2	3	4
y, x, z	1, 1, 3	1, 4, 27	1, 55, 381	1, 766, 5307	1, 10669, 73917
L′	1.‸7;	1;	7.‸31;	409.‸103;	1429.‸5719;
M′	1;	7.‸31;	409.‸103;	1429.‸5719;	79633.‸43.463;

r	5
y, x, z	1, 148600, 1029531
L′	79633.‸43.463;
M′	7.7.5659.‸1109167;

End of Cuban Factorisation Tables.

Simple Sextans, $N = (y^6 + 1^6) \div (y^2 + 1^2)$.
[All divisors < 100,000 cast out.]

y	N	y	N	y	N
1	1;	51	37.182773;	101	61.109.15649;
2	1:13;	52	7308913;	102	13.8325601;
3	73;	53	1153.6841;	103	37.97.31357;
4	241;	54	2089:13.313;	104	116975041;
5	601;	55	9147601; B	105	121539601;
6	13:97;	56	9831361;	106	13.9710497;
7	13:181;	57	577.18289;	107	157.834829;
8	37:109;	58	13.870241;	108	3049.44617;
9	6481;	59	13.931837;	109	141146281;
10	9901;	60	37.241.1453;	110	13.2437.4621;
11	13.1117;	61	13842121; B	111	13.11676517;
12	20593;	62	14772493; B	112	1741.90373;
13	28393;	63	13.193.6277;	113	97.733.2293;
14	37.1033;	64	433.38737;	114	168883021;
15	13.3877;	65	17846401; B	115	13.433.31069;
16	97.673;	66	37.512713;	116	1381.131101;
17	83233;	67	13.1549741;	117	109.337.5101;
18	229:457;	68	109.196117;	118	193863853;
19	13.13.769;	69	373.60757;	119	13.37.416881;
20	13.12277;	70	73.328837;	120	601.345001;
21	61.3181;	71	13.1954357;	121	10657.20113;
22	157.1489;	72	13.337:6133;	122	73.3034501;
23	37.7549;	73	3037.9349;	123	13.17605501;
24	349:13.73;	74	2377.12613;	124	13.18185077;
25	390001;	75	4657.6793;	125	37.6597973;
26	181:2521;	76	13.73.35149;	126	252031501;
27	530713;	77	829.42397;	127	457.569209;
28	13.47221;	78	757.48889;	128	14449:13.1429;
29	37.61.313;	79	3169.12289;	129	1657.167113;
30	809101;	80	13.13.242329;	130	193.1479757;
31	922561;	81	97.577.769;	131	294482761;
32	13; 61:1321;	82	37.61.20029;	132	13.23352181;
33	13.91141;	83	47451433;	133	3313.94441;
34	1069.1249;	84	13.3829237;	134	37.8713513;
35	277.5413;	85	13.4014877;	135	157.2115493;
36	1678321;	86	4357.12553;	136	13.4129.6373;
37	13.144061;	87	1153.49681;	137	13.2473.10957;
38	2083693;	88	37.1620589;	138	3709.97777;
39	2311921;	89	13.13.229.1621;	139	14197.26293;
40	61.41941;	90	61.1075441;	140	37.229.45337;
41	13.109.1993;	91	97.709.997;	141	13.4201.7237;
42	673.4621;	92	71630833;	142	613.663241;
43	3416953;	93	13.61.94321;	143	61.73.93901;
44	1753.2137;	94	78066061;	144	193.2227777;
45	13.37.8521;	95	277.294013;	145	13.34002277;
46	13.344257;	96	7177:11833;	146	1789.253969;
47	1213.4021;	97	2521:13.37.73;	147	4729.98737;
48	5306113;	98	8317:13.853;	148	2689.178417;
49	73.193.409;	99	96049801;	149	13.73.519349;
50	13.157:3061;	100	99990001; Lo,R	150	13.13.109:27481;

Simple Sextans, $N = (y^6 + 1^6) \div (y^2 + 1^2)$.

[All divisors < 100,000 cast out.]

y	N	y	N	y	N
151	61.8522341;	201	13.125553877;	251	37.9601.11173;
152	4357.122509;	202	13.128071201;	252	997.997.4057;
153	9601.57073;	203	1021.1117.1489;	253	13.315160621;
154	13.61.709237;	204	61.28390981;	254	13.320173057;
155	181.3188821;	205	7717.228853;	255	1621.2608381;
156	37.16005853;	206	13.1297.106801;	256	193.22253377; La
157	397.1530349;	207	157.181.64609;	257	4362404353;
158	13.47936641;	208	37.3037.16657;	258	13.13.1801.14557;
159	421.1518061;	209	1907986081;	259	109.313.131893;
160	349.1009.1861;	210	13.97.109.14149;	260	4569692401;
161	10861.61861;	211	229.8655349;	261	4640402521;
162	23473:13.37.61;	212	61.33113413;	262	13.421.860941;
163	13.54298861;	213	23929.86017;	263	4784281393;
164	3109.232669;	214	13.37.4036141;	264	157.30939253;
165	757.979093;	215	13.61.2694457;	265	61.337.239893;
166	759305581;	216	73.541:55117;	266	13.385103137;
167	13.59828341;	217	409.5421337;	267	13.37.1873.5641;
168	796565953;	218	5101.442753;	268	73.541.130621;
169	815702161;	219	13.176939197;	269	5236041961;
170	73.1297.8821:	220	337.397.17509;	270	5314337101;
171	13.37.1021.1741;	221	1861.1281781;	271	13.8353.49669;
172	875183473;	222	73.33272101;	272	5473558273;
173	373.2401381;	223	13.61.3118441;	273	37.61.541.4549;
174	181.5064121;	224	2517580801;	274	3313.1701277;
175	13.1237.58321;	225	2562840001;	275	13.97.4535341;
176	13.73806277;	226	109.2857.8377;	276	61.95126341;
177	37.109.397.613;	227	13.204245101;	277	1933.3045661;
178	97.2269.4561;	228	13.207868021;	278	5972739373;
179	157.6538813;	229	2750006041;	279	13.466087957;
180	13.4933.16369;	230	37.75631273;	280	13.9133.51769;
181	241.4453321;	231	2847342961;	281	3001.2077561;
182	277.3960889;	232	13.6037.36913;	282	37.170918569;
183	1121479633;	233	421.7000573;	283	1069.6000157;
184	13.88168837;	234	24049.124669;	284	13.61.313.26209;
185	1171316401;	235	853.3575317;	285	97.1597.42589;
186	6073.197077;	236	13.37.6449041;	286	109.61380769;
187	709.1724677;	237	241.13090873;	287	4597.1475869;
188	13.13.97.181.421;	238	24373.131641;	288	13.5869:37.2437;
189	13.349.281233;	239	3262751521;	289	73.1321.72337;
190	1303173901;	240	13.397.733.877;	290	3301.2142601;
191	2713.490537;	241	13.1093.237409;	291	7170787081;
192	409.3322537;	242	193.277:64153;	292	13.157.3561913;
193	13.37.2884513;	243	73;47763361;	293	13.566920381;
194	1416430861;	244	3544475761;	294	74929:99709;
195	73.19806337;	245	13.37.241.31081;	295	73.2593.40009;
196	1475750641;	246	661.5540281;	296	7676475841;
197	13.3217.36013;	247	229.16253437;	297	13.97.1669.3697;
198	1536914413;	248	3457.1094209;	298	1129.6984997;
199	37.42383773;	249	13.13.22745929;	299	7992449401;
200	97.373:44221;	250	1381.2828521;	300	8099910001;

Simple Sextans, $\mathrm{N} = (y^6 + 1^6) \div (y^2 + 1^2)$.

[All divisors < 100,000 cast out.]

y	N	y	N	y	N
301	13.241.2619997;	351	3469.4375429;	401	13.30637.64921;
302	8318078413;	352	661.23225533;	402	3769.6929077;
303	3181.2649733;	353	13.1194406021;	403	11497.2294209;
304	37.373.618841;	354	6421.2445721;	404	97.2953.93001;
305	13.665658277;	355	181.87746821;	405	13.2069541277;
306	13.21649.31153;	356	37.37.1201.9769;	406	61.8221.54181;
307	97.1201.76249;	357	13.13.96113137;	407	
308	8999083633;	358	13.1263529441;	408	937.29573209;
309	661.1009.13669;	359	2029.8186389;	409	13.337.6387301;
310	13.37.19199821;	360	409.41066089;	410	13.32749.66373;
311	9354855121;	361	4297.3952393;	411	2797.10201693;
312	277.34208509;	362	13.37.73:489061;	412	
313	9597826993;	363		413	
314	13.747774817;	364	157.111815653;	414	13.73.30955129;
315	10141.970861;	365	12049.1473049;	415	37.661.1212793;
316	73.136590697;	366	13.1380313537;	416	23293.1285717;
317	2221.4546573;	367	313.3049.19009;	417	
318	13.229.433.7933;	368	73.73.109.31573;	418	13.13.13.13895449;
319	13.13.37.109.15193;	369	181.229.433.1033;	419	14869.2072869;
320		370	13.5701.252877;	420	3733.8335597;
321	1933.5492677;	371	13.1457300557;	421	37.157.1609.3361;
322	1093.9835561;	372	97.277.712717;	422	13.241.10122481;
323	13.193.829.5233;	373	349.55463437;	423	13.2462723701;
324		374		424	
325	37.349.541.1597;	375	13.1521173077;	425	1777.18359713;
326	61.185155441;	376		426	
327	13.879515701;	377	6709.3010957;	427	13.13.196708177;
328		378	37.551775529;	428	109.307854997;
329		379	13.7669.206953;	429	
330	457.25949893;	380	27697.752833;	430	37.923995273;
331	13.923346397;	381	5857.3597673;	431	13.2654381797;
332	13.934555381;	382	97.97.2263117;	432	9721.3582793;
333	937.13123009;	383	13.6961.237781;	433	24709.1422637;
334	61.204010321;	384	37.3517:13.12853;	434	
335	109.115544389;	385	7789.2820709;	435	13.73.193.195493;
336	13.157.181.34501;	386	3793.5852797;	436	13.37.613.122557;
337	61.193.1095541;	387	61.367714813;	437	
338	105769:123397;	388	13.181.9631681;	438	
339		389	73.3433.91369;	439	457.81271753;
340	13.2137.481021;	390		440	13.229.12590113;
341	37.73.5005981;	391	877.26650453;	441	73.518118697;
342	313.43707541;	392	143053:13.12697;	442	373.3853.26557;
343		393	37.644711869;	443	
344	13.25537.42181;	394	97.248433613;	444	13.109.27425833;
345	13.61.17864857;	395	61.109.3661249;	445	74941.523261;
346		396	13.9001.210157;	446	13729.2882029;
347	37.391843429;	397	13.20509.93169;	447	
348		398	61.411338833;	448	13.61.50796841;
349	13.3253.350809;	399	37.684994573;	449	13.157.193.103177
350		400	31177.821113;	450	189421:216481;

Simple Sextans, $N = (y^6 + 1^6) \div (x^2 + 1^2)$.

[All divisors < 100,000 cast out.]

y	N	y	N	y	N
451	19777.2091913;	501	13.97.49961341;	551	1249.3061.24109;
452	37.1128105949;	502		552	13.38449.185749;
453	13.3239271421;	503		553	13.109.65997769;
454	337.126064093;	504	37.109.15998977;	554	397.4993.47521;
455		505	13.5002884277;	555	
456	61.708806101;	506	829.79076209;	556	8377.11407993;
457	13.3355207381;	507		557	13.337.1453.15121;
458	37.409.2907601;	508	73.912284521;	558	
459	61.277.2626873;	509	13.61.349.242533;	559	
460	3457.12951793;	510	37.1828425673;	560	73.1347186937;
461	13.3474227917;	511		561	13.13789.552553;
462	13.73.48006457;	512	246241:279073;	562	19489.5118637;
463		513	13.5197.1025113;	563	37.2715379189;
464	5413.8563117;	514	13.73.73550329;	564	12253.8257957;
465	8941.5229061;	515		565	13.37321.210037;
466	13.1129.3212953;	516	193.1237.296941;	566	13.97.81385921;
467	37.61.2389.8821;	517	61.181.1669.3877;	567	28573.3617221;
468		518	13.5538269281;	568	
469	97.498789913;	519	277.261931693;	569	37.181.15651913;
470	13.1873.2004049;	520	61.577.2077333;	570	13.61.133114357;
471	613.80282557;	521	157.469299013;	571	
472		522	13.709.8055469;	572	313.342009721;
473	37.37.36562777;	523		573	373.289006981;
474	13.3883006177;	524		574	13.397.21033541;
475	13.457.8568661;	525		575	66361.1647241;
476	229.224176669;	526	13.13.37.12242017;	576	97.1134793633;
477	109.474946957;	527	13.5933316901;	577	1777.62375569;
478	241.9421.22993;	528	61.1274102293;	578	13.24229:37.61.157;
479	13.97.3361.12421;	529	937.83575993;	579	13.42061.205537;
480		530	193.408831757;	580	421.268799581;
481	601.2617.34033;	531	13.10453.585049;	581	61.73.25588837;
482		532	37.2164927069;	582	
483	13.4186424941;	533		583	13.
484		534		584	37.457.1549.4441;
485		535	13.73.86327149;	585	61837.1893973;
486	241.877:263953;	536	181.10069.45289;	586	109.22777.47497;
487	13.73.2293.25849;	537	109.7537.101221;	587	13.13.73.9623689;
488	13.13.335575897;	538	349.240050257;	588	97.1232356369;
489	37.1545367933;	539	13.4801.1352317;	589	61.277.7122793;
490	9241.6238261;	540	13.6540789877;	590	42961.2820541;
491	97.599173273;	541	37.2315185813;	591	13.9384374437;
492	13.4507287541;	542	241.1213.295201;	592	13.757.12480913;
493	157.733.513313;	543		593	
494		544	13.54877.122761;	594	56809.2191429;
495	37.10729.151237;	545		595	
496	13.577.1549.5209;	546	373.238265017;	596	13.13.601.1242289;
497	433.140908081;	547	37.229.10566001;	597	313.405837121;
498	2833.21710461;	548	13.433.16020997;	598	97.229.5757001;
499	93337.664273;	549		599	
500	13.4807673077;	550	181.505557721;	600	13.25057:37.10753;

Simple Sextans, $N = (y^6 + 1^6) \div (y^2 + 1^2)$
[All divisors < 100,000 cast out.]

y	N	y	N	y	N
601	409.318987289;	651	24793.7244257;	701	
602	337.12037.32377;	652	13.37.375702673;	702	
603	5449.24263377;	653		703	61.11437.350089;
604	13.937.2557.4273;	654	73.421.5952577;	704	13.1693.2473.4513;
605	13.32917.313081;	655	313.588057577;	705	229.1078748269;
606	37.157.601.38629;	656	13.19249.740053;	706	73.11317.300721;
607	21817.6222409;	657	13.13.13.673.126013;	707	
608	73.1871932921;	658	37.577.8780617;	708	13.13729.1407829;
609	13.421.4513.5569;	659		709	13.193.100712509;
610		660	73.2599272937;	710	
611	4261.5113.6397;	661	13.	711	37.61.113225953;
612		662	109.1761994177;	712	4993.51470401;
613	13.109.5521.18049;	663	97.241.1129.7321;	713	13.16417.1210933;
614		664	1033.188178937;	714	13477.19284073;
615	37.29221.132313;	665	13.1609.9349453;	715	673.388336537;
616	13537.10636513;	666	2341.84041641;	716	
617	13.397.28080553;	667	60637.3264109;	717	13.37.181.3035653;
618	13.709.853.18553;	668	76261.2610973;	718	
619		669	13.229.67285993;	719	241.1801.615721;
620	733.201586597;	670	13.2857.5425561;	720	6481.41465521;
621	37.6673.602341;	671	1213.2749.60793;	721	13.601.1237.27961;
622	13.26713.431017;	672	7333.27809581;	722	13.109.349.549481;
623		673	97.25717.82237;	723	193.4813.294157;
624	9649.15712849;	674	13.37.349.1229329;	724	
625		675		725	
626	13.409.2677.10789;	676		726	13.37021.37.15601;
627	5581.27692173;	677	373.563176981;	727	73.241.2281.6961;
628	193.805898161;	678	13.157.103532053;	728	
629	27109.5774149;	679	757.280790413;	729	
630	13.	680	37.313.18462421;	730	13.10909.2002453;
631	13.61.673.297049;	681	73.1429.2061733;	731	181.2161.730021;
632		682	13.	732	37.7759643869;
633	73.2029.1083949;	683	13.421.39760921;	733	73.39301.100621;
634	577.6793.41221;	684		734	13.
635	13.	685	97.2269810633;	735	13.157.2089.68449;
636		686		736	277.1059328573;
637	37.397.877.12781;	687	13.	737	79777.3698209;
638		688	769.291357697;	738	
639	13.61.210246697;	689	37.673.9050221;	739	13.
640		690		740	
641		691	13.6397.2741521;	741	15493.19459717;
642	61.193.14429521;	692	61.3759184453;	742	
643	13.37.355383913;	693		743	13.
644	13.	694	3541.65510521;	744	
645		695	13.13.37.97.109.3529;	745	997.1657.186469;
646	109.26209.60961;	696	13.2713.6653389;	746	337.919019293;
647		697		747	13.
648	13.30553.443917;	698	181.1249.1049977;	748	13.37.1429.455437;
649	277.640468813;	699		749	
650	61.157.18639013;	700	13.61.1009.300073;	750	181.769.2273209;

Simple Sextans, $N = (y^6 + 1^6) \div (y^2 + 1^2)$.

[All divisors < 100,000 cast out.]

y	N	y	N	y	N
751	433.734634097;	801		851	13.
752	13.829.2389.12421;	802	433.955452061;	852	13.73.1093.508009;
753	61.5270469493;	803	51817.8023969;	853	
754	37.73.5869.20389;	804	13.109.294885673;	854	
755	109.613.853.5701;	805		855	
756	13.13.1932856969;	806	37.73.156248161;	856	13.
757	541.1753.346261;	807	157.19801.136429;	857	97.5560975249;
758		808	13.	858	349.38917.39901;
759	4129.80375089;	809	541.791764741;	859	37.2833.5194261;
760	13.97.264568741;	810	5581.6121.12601;	860	13.
761	13.61.7489.56473;	811		861	
762	54133.6228121;	812	13.6229.5368609;	862	
763	37.157.58343977;	813	13.82657.406573;	863	3853.143960581;
764	61.5585253061;	814	61.541.13303621;	864	109.6301:13.13.4801;
765	13.13.2017.1004737;	815	373.1182826237;	865	13.37.1163908321;
766		816		866	277.541.3753133;
767		817	13.3229.10613929;	867	97.409.14242321;
768		818	1021.438517393;	868	
769	13.37.409.1777609;	819	2341.192191221;	869	13.
770	97.4021.901273;	820		870	1789.320233009;
771	109.1093.2965993;	821	13.193.181080349;	871	
772	61.47521.122533;	822	37.61.7873.25693;	872	
773	13.80317.341953;	823		873	13.73.612054637;
774	13.	824	23857.19323793;	874	37.
775	34513.10452577;	825	13.61.7213.80989;	875	61.1993.4821637;
776	229.25657.61717;	826	13.13.2754437029;	876	
777		827	73.229.27980989;	877	13.16633.2735797;
778	13.57781.487741;	828	37.	878	13.2161.21153361;
779	73.5044594417;	829	5557.84991813;	879	73.97.181.465781;
780		830	13.	880	37.109.148696897;
781	75289.4941649;	831	109.1129.3875101;	881	36709.16410829;
782	13.97.12589.23557;	832		882	741721:13.62761;
783	241.8389.185917;	833	61.7893135253;	883	61.9965805853;
784	7321.51605161;	834	13.3697.10066321;	884	18541.32936341;
785	37.	835	157.193.757.21193;	885	313.1959874177;
786	13.1069.27464293;	836		886	13.61.16477.47161;
787	13.457.64571173;	837	37.	887	349.1773652357;
788	769.18301.27397;	838	13.	888	3673.169289641;
789	5689.68119489;	839	13.	889	97.6439242193;
790	613.635399977;	840		890	13.
791	13.37.337.2415073;	841	9001.55576681;	891	13.3109.15593593;
792	97.4056283489;	842		892	157.157.25683817;
793		843	13.37.1049940313;	893	
794		844	2281.222455881;	894	61.337.1201.25873;
795	13.	845	1201.424505401;	895	13.
796	277.1693.856081;	846	733.698838577;	896	37.37.457.709.1453;
797		847	13.8461.4679161;	897	1693.382395061;
798		848	3373.153308581;	898	181.229.15688837;
799	13.5437.5766121;	849	1009.2053.250813;	899	13.46573.1078849;
800	37.16453:13.73.709;	850	769.1021.664849;	900	73.8987660137;

Simple Sextans, $N = (y^6 + 1^6) \div (y^2 + 1^2)$.

[All divisors < 100,000 cast out.]

y	N	y	N
901	13537.48682873;	951	13.397.853.185797;
902	37.193.92697193;	952	73.
903	13.66841.765181;	953	
904	13.241.213163477;	954	37.97.13309.17341;
905		955	13.61.3889.269713;
906	829.997.815197;	956	13.
907	661.12457.82189;	957	34429.24362557;
908	13.	958	
909		959	
910	313.2190890677;	960	13.241.2053.132049;
911	37.	961	
912	13.181.1657.177433;	962	
913	109.3637.1752721;	963	
914	1117.15313.40801;	964	13.97.157.4362073;
915		965	1549.559831549;
916	13.193.10369.27061;	966	6529.133370989;
917	13.37.11161.131713;	967	
918	10357.68570329;	968	895357:13.241.313;
919	373.1789.1068913;	969	13.
920	157.4563007093;	970	37.661.36197893;
921	13.	971	397.2239164253;
922	757.3301.289189;	972	
923		973	13.73.109.733.11821;
924		974	
925	13.13.73.59341273;	975	55933.16156597;
926	277.20173.131581;	976	37.97.252828109;
927		977	13.
928	31489.23552257;	978	
929	13.	979	54469.16864789;
930	13.433.132892369;	980	3517.262259653;
931	181.23833.174157;	981	13.433.164529709;
932	457.1650999529;	982	13.
933	37.	983	
934	13.13.229.19663681;	984	8161.148 77921;
935	40357.18937693;	985	37.
936	61.61.206273401;	986	13.97.749535361;
937	6133.125685421;	987	
938	13.39877.1493293;	988	
939	37.337.7369.8461;	989	109.1693.5184433;
940	109.12073.593293;	990	13.9901.7463077;
941	8629.90865189;	991	37.
942	13.	992	157.6168031669;
943	13.	993	313.36901.84181;
944	61.1609.8090989 ;	994	13.2017.37230241;
945	36433.21889297;	995	13.13.74377.77977;
946	73.373.29412529;	996	1753.561377497;
947	13.61.1014206161;	997	61.
948	37.	998	73.10141.1340041;
949		999	13.6553.11691709;
950	40213.20254777;	1000	999999000001; Lo
		1001	421.2113.1128637;

Sextans, $N = (x^6 + y^6) \div (x^2 + y^2) \not> 9.10^6$; [$x$ and $y > 1$, y *even*].

x, y	N	x, y	N	x, y	N
3, 2	1:61;	51, 4	457.14713;	17, 10	64621;
5	541;	53, 4	7845793;	19	13.8017;
7	2221;	5, 6	1021;	21	13.13.13.73;
9	37:13.13;	7	1933;	23	313.757;
11	14173;	11	37.313;	27	61.7681;
13	27901;	13	23773;	29	37.109.157;
15	49741;	17	74413;	31	13.37.1741;
17	13.6337;	19	118621;	33	13.83617;
19	61.2113;	23	13.20161;	37	97.18013;
21	37.5209;	25	181:13.157;	39	2171341;
23	277741;	29	13.52177;	41	769.3469;
25	409:13.73;	31	890221;	43	37.73.1201;
27	373:13.109;	35	1457821;	47	13.359137;
29	349.2017;	37	1826173;	49	229.24169;
31	919693;	41	373.7417;	51	193.33757;
33	1181581;	43	3353533;	53, 10	7619581;
35	13.115057;	47	37.129769;	5, 12	17761;
37	1868701;	49	1009:13.433;	7	13.1237;
39	2307373;	53, 6	13.599281;	11	13.1381;
41	2819053;	3, 8	13:277;	13	109.229;
43	13.397.661;	5	3121;	17	37.1693;
45	421.9721;	7	3361;	19	13.7621;
47	4870861;	9	13:421;	23	224401;
49	1789:3217;	11	10993;	25	97.3313;
51	13.229.2269;	13	21841;	29	606913;
53, 2	13.37.16381;	15	61.661;	31	805873;
3, 4	193;	17	13.13.409;	35	313.4297;
5	13.37;	19	157.709;	37	13.73.1789;
7	1873;	21	170353;	41	13.200341;
9	5521;	23	13.19237;	43	61.52021;
11	13.997;	25	229:1549;	47	4582321;
13	26113;	27	181:37.73;	49	5439793;
15	13.3637;	29	13.50581;	53	61.109.1129;
17	79153;	31	97.8929;	55, 12	8735761;
19	37.3373;	33	1120321;	3, 14	109.337;
21	13.14437;	35	13.109717;	5	34141;
23	61.61.73;	37	1790641;	9	29101;
25	380881;	39	2220193;	11	13.37.61;
27	433.1201;	41	2722273;	13	97.349;
29	694081;	43	13.254197;	15	13.3457;
31	13.69877;	45	109.36469;	17	65293;
33	97.12049;	47	37.128173;	19	13.7537;
35	1481281;	49	13.97:61.73;	23	229.937;
37	13.142501;	51	6602833;	25	306541;
39	709.3229;	53, 8	7714801;	27	426973;
41	13.215317;	3, 10	9181;	29	73.73.109;
43	157.21589;	7	13.577;	31	73.10597;
45	613.6637;	9	8461;	33	13.77761;
47	13.372661;	11	12541;	37	13.126481;
49, 4	337.16993;	13, 10	21661;	39, 14	757.2713;

Sextans, $N = (x^6 + y^6) \div (x^2 + y^2) \ngtr 9.10^6$; [$x$ and $y > 1$, y *even*].

x, y	N	x, y	N	x, y	N
41, 14	13.194977;	53, 18	7085341;	13, 24	181.1453;
43	3094813;	55, 18	13.181.3517;	17	13.19141;
45	13.287857;	3, 20	13.12037;	19	254161;
47	337.13309;	7	61.2341;	23	306913;
51	6293821;	9	134161;	25	61:13.457;
53	7378333;	11	126241;	29	554641;
55, 14	277.31033;	13	73.1657;	31	701761;
3, 16	63313;	17	127921;	35	13.86677;
5	13.4597;	19	337.433;	37	1417393;
7	13.4261;	21	37.4813;	41	2189281;
9	51361;	23	13.97.181;	43	13.337.613;
11	49201;	27	13.30757;	47	97.40609;
13	50833;	29	13.97.421;	49	373:12637;
15	157.373;	31	61.73.157;	53	13.508021;
17	37.2029;	33	397.2293;	55, 24	7740001;
19	13.73.109;	37	1486561;	3, 26	61.7393;
21	13.11317;	39	277.6733;	5	73.6037;
23	209953;	41	181.12781;	7	426253;
25	73.4057;	43	2839201;	9	337.1213;
27	410353;	47	4156081;	11	457.853;
29	557521;	49	13.37.10321;	15	355501;
31	13.61.937;	51	13.109.4153;	17	345133;
33	13.74821;	53, 20	13.37.14401;	19	343261;
35	37.97.349;	3, 22	229981;	21	353341;
37	61.26053;	5	13.17137;	23	37.37.277;
39	37.53773;	7	73.2917;	25	425101;
41	2460961;	9	37.5449;	27	495613;
43	193.15601;	13	157.1153;	29	595741;
45	13.280597;	15	13.13537;	31	37.19753;
47	13.336901;	17	73.2437;	33	73.12421;
49	1993.2617;	19	189853;	35	1129501;
51	1741.3541;	21	13.16561;	37	1405693;
53	397.18229;	23	258061;	41	433.4957;
55, 16	8441761;	25	37.8713;	43	37.70969;
5, 18	97501;	27	181.2281;	45	3188701;
7	37.2473;	29	181.2953;	47	409.9397;
11	97.829;	31	13.53281;	49	4598701;
13	78781;	35	61.97.193;	51	37.147673;
17	13.7297;	37	13.111217;	53	6448573;
19	73.1621;	39	1811533;	55	7562701;
23	13.16417;	41	13.172801;	57, 26	8816653;
25	109:2689;	43	2758141;	3, 28	607681;
29	13.41521;	45	3354781;	5	373.1597;
31	717133;	47	13.311137;	9	13.42901;
35	13.109.853;	49	37.130729;	11	73.7321;
37	1535581;	51	577.9949;	13	157.3253;
41	37.64489;	53, 22	6765181;	15	37.73.181;
43	13.224977;	5, 24	318001;	17	13.36277;
47	397.10753;	7	37.8269;	19	61.7573;
49, 18	13.61:6421;	11, 24	276721;	23, 28	479761;

Sextans, $N = (x^6 + y^6) \div (x^2 + y^2) \ngtr 9.10^6$; [$x$ and $y > 1$, y *even*]

x, y	N	x, y	N	x, y	N
25, 28	13.13.3049;	45, 32	3075601;	55, 36	6909841;
27	13.193.229;	47	3666241;	3, 38	229.9049;
29	397.1669;	49	457.13.733;	5	61.33601;
31	784753;	51	13.396181;	7	13.155137;
33	946801;	53	13.466357;	9	277.7129;
37	97.14593;	55	13.337.1621;	11	13.373.397;
39	37.61.769;	57, 32	8277601;	13	421.4441;
41	2122513;	3, 34	13.102001;	15	13.139297;
43	13.13.15289;	5	37.35353;	17	613.2857;
45	3127681;	7	1282093;	21	1642813;
47	457.8233;	9	13.96097;	23	37.109.397;
51	13.109.3769;	11	433.2797;	25	1213.1297;
53	13.313.1549;	13	37.73.433;	27	1563901;
55	7393681;	15	1126861;	29	37.42649;
57, 28	349.24709;	19	193.5437;	31	1620973;
7, 30	768301;	21	181.5641;	33	13.130657;
11	13.55057;	23	13.109.709;	35	1816861;
13	37.18553;	25	1004461;	37	13.73.2089;
17	73.8677;	27	157.6529;	39	2202253;
19	37.16633;	29	13.73.1129;	41	13.73.2617;
23	613741;	31	1148941;	43	1249.2269;
29	181.4201;	33	1263373;	45	13.37.6781;
31	13.109.613;	35	13.109297;	47	769.4909;
37	13.111697;	37	157.10369;	49	4382893;
41	13.61.2677;	39	1891501;	51	37.157.877;
43	2564701;	41	2218861;	53	229.25849;
47	13.284737;	43	13.61.3301;	55, 38	277.24793;
49	1069.4129;	45	3096061;	3, 40	2545681;
53, 30	6172381;	47	313.11701;	7	13.109.1753;
3, 32	13.37.2161;	49	13.332737;	9	769.3169;
5	1023601;	53	5979613;	11	13.13.73.193;
7	277.3613;	55	13.109.4933;	13	37.62653;
9	409.2377;	57, 34	181.44953;	17	853.2557;
11	313.3001;	5, 36	13.126757;	19	13.162517;
13	61.14821;	7	13.13.61.157;	21	1153.1777;
15	868801;	11	1537441;	23	1993441;
17	836161;	13	1489153;	27	1925041;
19	757.1069;	17	1388593;	29	1921681;
21	791473;	19	13.103237;	31	313.6217;
23	13.73.829;	23	37.34429;	33	13.229.673;
25	13.13.4729;	25	1009.1249;	37	13.172597;
27	97.13.661;	29	37.35053;	39	97.25153;
29	13.68821;	31	13.181.577;	41	13.157.1321;
31	988033;	35	73.21817;	43	3020401;
33	37.30253;	37	61.29173;	47	61.73.877;
35	73.17737;	41	409.5689;	49	4483201;
37	337.4513;	43	2702113;	51	97.53233;
39	1804513;	47	13.284341;	53	457.13033;
41	37.58189;	49	4332721;	57, 40	7917601;
43, 32	181.14221;	53, 36	5929633;	5, 42	13.236017;

Sextans, $N = (x^6 + y^6) \div (x^2 + y^2) \not> 9.10^6$; $[x$ and $y > 1$, y *even*$]$.

x, y	N	x, y	N	x, y	N
11, 42	2912893;	27, 46	13.266641;	43, 50	13.388177;
13	409.6949;	29	13.13.20149;	47	61.91921;
17	1609.1669;	31	349.9649;	49	421:14281;
19	13.200401;	33	109.30817;	51	13.500977;
23	1321.1861;	35	37.91513;	53	13.547537;
25	2399821;	37	3454813;	57, 50	1993.4357;
29	769.3037;	39	37.96553;	3, 52	7287361;
31	13.180001;	41	109.34369;	5	193.37537;
37	2570941;	43	3983773;	7	73.98377;
41	37.80329;	45	733.5857;	9	37.313.613;
43	3268861;	47	2053.2281;	11	6999073;
47	13.13.24229;	49	13.397057;	15	6753841;
53	73.82837;	51	13.61.7237;	17	61.108421;
55, 42	709.9769;	53	13.61.8101;	19	733.8821;
3, 44	13.286981;	55	13.157.3541;	21	97.65089;
5	61.60661;	57, 46	97.241.349;	23	6161041;
7	3655633;	5, 48	13.403957;	25	37.162493;
9	13.13.61.349;	7	397.13093;	27	5871841;
13	37.93229;	11	13.61.6361;	29	5744833;
15	1297.2593;	13	4947601;	31	5636593;
17	13.251701;	17	4726081;	33	241.23041;
19	37.85933;	19	181.25453;	35	5499841;
21	3088801;	23	37.118093;	37	337.16273;
23	13.229.1009;	25	1993.2137;	39	13.13.13.13.193;
25	97.109.277;	29	61.66853;	41	97.57649;
27	97.29569;	31	13.37.8353;	43	5730721;
29	13.13.16729;	35	3986641;	45	157.37813;
31	673.4177;	37	13.309877;	47	6218161;
35	13.221317;	41	13.433.757;	49	37.177949;
37	2971873;	43	37.157.769;	51	7043713;
39	73.42697;	47	13.13.30169;	53	7606561;
41	1093.3037;	49	241.22993;	55, 52	61.135781;
43	13.275941;	53	6726961;	5, 54	2389.3529;
45	1429.2749;	55, 48	937.7993;	7	8362573;
47	853.5101;	3, 50	37.37:4549;	11	8164861;
49	13.109.3433;	7	37.165673;	13	829.9697;
51	1201.4561;	9	13.97:4801;	17	13.373.1597;
53	6200353;	11	577.10333;	19	7580701;
57, 44	8014033;	13	61.96001;	23	7240333;
3, 46	13.193.1777;	17	13.431617;	25	13.37:61.241;
5	4425181;	19	5477821;	29	6757981;
7	4376173;	21	109.49009;	31	157.42193;
9	73.59077;	23	5207341;	35	13.494737;
11	4236061;	27	349:13.1093;	37	6385213;
13	4148413;	29	4854781;	41	61.105361;
15	37.97.1129;	31	4771021;	43	13.502321;
17	3949453;	33	181.26041;	47	6941293;
19	1609.2389;	37	4701661;	49	277:37.709;
21	3738781;	39	4760941;	53	13.37.17053;
25, 46	13.272737;	41, 50	73.241.277;	55, 54	73.120997;

☞ *Continued on top of page* 170.

Sextans, $N = (x^6+y^6) \div (x^2+y^2) \not> 9.10^6$; [$x$ and $y > 1$, xy *odd*].

x, y	N	x, y	N	x, y	N
5, 3	13.37;	39, 7	2241313;	29, 13	61.9733;
7	13.157;	41	61.45013;	31	789673;
11	13633;	43	193.17257;	33	1030441;
13	37.733;	45	937.4273;	35	1322161;
17	81001;	47	4773841;	37	349.4789;
19	13.9781;	51	13.73.6997;	41	2570233;
23	275161;	53, 7	13.13.109.421;	43	3134881;
25	385081;	11, 9	13.877;	45	1669.2269;
29	699793;	13	21433;	47	193.23497;
31	13.70381;	17	61.1093;	49	5387593;
35	61.24421;	19	107641;	51	661.9613;
37	1861921;	23	243553;	53	73.101977;
41	2810713;	25	346561;	55, 13	8667961;
43	3402241;	29	757.853;	17, 15	13.13.409;
47	13.373837;	31	13.65557;	19	99721;
49	97.59209;	35	37.38053;	23	211441;
53, 3	7865281;	37	13.109.1249;	29	277.2053;
7, 5	1801;	41	13.157.1321;	31	709.1069;
9	13.397;	43	97.33769;	37	193.8377;
11	12241;	47	13.97.3733;	41	229.10909;
13	109.229;	49	5576881;	43	13.349.673;
17	13.61.97;	53	181.42373;	47	4433281;
19	121921;	55, 9	61.193.757;	49	37.142573;
21	184081;	13, 11	61.373;	53, 15	13.13.61.709;
23	13.61.337;	15	109.349;	19, 17	97.1129;
27	513841;	17	13.4861;	21	13.37.313;
29	13.52837;	19	101281;	23	210481;
31	900121;	21	109.1429;	25	37.7933;
33	37.31333;	23	37.6229;	27	109.3709;
37	1840561;	25	13.25357;	29	547753;
39	2276041;	27	13.35221;	31	13.56101;
41	37.75253;	29	620161;	33	337.2833;
43	13.313.829;	31	37.97.229;	35	661.1861;
47	73.157.421;	35	13.13.8089;	37	13.120157;
49	13.438877;	37	421.4093;	39	937.2089;
51	181.37021;	39	2144041;	41	13.277.673;
53, 5	7820881;	41	2637001;	43	73.109.373;
9, 7	4993;	43	13.37.6673;	45	3598921;
11	11113;	45	73.53017;	47	13.277.1201;
13	37.613;	47	61.75853;	49	37.139309;
15	97.433;	49	5488921;	53	37.193573;
17	71761;	51	13.37.13441;	55, 17	8359921;
19	37.3109;	53, 11	13.581941;	21, 19	165601;
23	13.19717;	15, 13	41161;	23	13.13.1297;
25	13.61.457;	17	63241;	25	13.22717;
27	13.38317;	19	97.1009;	27	13.30661;
29	13.51421;	21	148513;	29	13.41077;
31	878833;	23	219001;	31	706921;
33	1134961;	25	313561;	33	37.61.409;
37, 7	1809481;	27, 13	436801;	35, 19	1188721;

Sextans, $N = (x^6 + y^6) \div (x^2 + y^2) \not> 9.10^6$; $[x \text{ and } y > 1,\ xy \text{ odd}]$.

x, y	N	x, y	N	x, y	N
37, 19	1510273 ;	31, 27	37.20389 ;	57, 35	13.229.2713 ;
39	193.9817 ;	35	1139041 ;	39, 37	1201.1753 ;
41	37.63493 ;	37	13.13.8329 ;	41	2398633 ;
43	109.26437 ;	41	13.163981 ;	43	13.212437 ;
45	3499921 ;	43	37.61.1153 ;	45	61.52501 ;
47	661.6373 ;	47	3800761 ;	47	3729721 ;
49	13.181.2137 ;	49	4545913 ;	49	757.5749 ;
51	13.458197 ;	53	97.65713 ;	51	13.241.1621 ;
53	13.157.3433 ;	55, 27	7476841 ;	53	13.455317 ;
55, 19	13.73.8629 ;	31, 29	13.63277 ;	55	6883561 ;
23, 21	13.18541 ;	33	13.75181 ;	57, 37	181.44101 ;
25	309481 ;	35	1177681 ;	41, 39	2582401 ;
29	13.97.421 ;	37	157.9109 ;	43	2919913 ;
31	694201 ;	39	73.23857 ;	47	3833233 ;
37	229.6397 ;	41	577.3673 ;	49	673.6577 ;
41	829.2749 ;	43	2571073 ;	53	37.160309 ;
43	13.373.577 ;	45	13.238837 ;	55, 39	97.70753 ;
47	61.67213 ;	47	13.37.7753 ;	43, 41	13.241261 ;
53	6846193 ;	49	4452841 ;	45	97.36313 ;
55, 21	13.241.2557 ;	51	1213.4357 ;	47	1741.2293 ;
25, 23	339841 ;	53	241.25873 ;	49	4554481 ;
27	425641 ;	55	7313881 ;	51	13.229.1753 ;
29	337.1609 ;	57, 29	13.656221 ;	53	13.461101 ;
31	13.193.277 ;	33, 31	1062913 ;	55	37.186253 ;
33	13.68437 ;	35	13.95917 ;	57, 41	7920193 ;
35	157.7213 ;	37	109.13597 ;	45, 43	3775201 ;
37	1429801 ;	39	1775281 ;	47	13.37.8761 ;
39	1788673 ;	41	61.34981 ;	49	61.77773 ;
41	73.97.313 ;	43	13.197341 ;	51	853.6301 ;
43	457.5953 ;	45	61.50461 ;	53	6115441 ;
45	13.254557 ;	47	37.99469 ;	55	1321.5281 ;
47	13.433.709 ;	49	13.336997 ;	57, 43	13.612877 ;
49	4774513 ;	51	5189161 ;	47, 45	37.181.673 ;
51	157.36109 ;	53	73.83761 ;	49	13.241.1597 ;
53	6684361 ;	55	13.13.42409 ;	53, 45	13.313.1549 ;
55, 23	181.43261 ;	57, 31	8357233 ;	49, 47	13.109.3769 ;
27, 25	466441 ;	35, 33	1352521 ;	51	5899273 ;
29	572281 ;	37	1569241 ;	53	241.27241 ;
31	193.3697 ;	41	2181073 ;	55	13.565237 ;
33	13.68917 ;	43	97.26713 ;	57, 47	8258641 ;
37	13.61.1777 ;	47	73.181.277 ;	51, 49	1657.3793 ;
39	1753441 ;	49	13.13.25657 ;	53	61.277.409 ;
41	13.166597 ;	53, 33	13.433.1069 ;	55	7652401 ;
43	2653801 ;	37, 35	13.73.1789 ;	57, 49	13.37.17713 ;
47	1453.2677 ;	39	61.31981 ;	53, 51	7349473 ;
49	4654801 ;	41	13.73.2389 ;	55, 51	8047801 ;
51	5530201 ;	43	2654401 ;	55, 53	8543881 ;
53	97.67273 ;	47	13.109.2593 ;		
57, 25	37.73.3301 ;	51	337.15073 ;		
29, 27	37.37.457 ;	53, 35	37.160813 ;		

Sextans, $N = (x^6+y^6) \div (x^2+y^2) \not> 9.10^6$; [$x$ and $y > 1$, y *even*].

(*Continued from page* 167.)

x, y	N	x, y	N	x, y	N
19, 56	8832721 ;	39, 56	7378081 ;	35, 58	37.61.3853 ;
23	61.97.1429 ;	41	13.37.15361 ;	37	373.23017 ;
25	8265121 ;	43	73.102121 ;	39	37.230089 ;
27	1213.6661 ;	45	97.78193 ;	41	8487373 ;
29	877.9013 ;	47	13.598981 ;	43	733.11617 ;
31	13.595717 ;	51	73.115657 ;	45	73.117877 ;
33	37.205549 ;	53, 56	8915953 ;	47, 58	8765101 ;
37, 56	13.570421 ;	33, 58	8839021 ;		

☞ *These Tables, pages* 164–170, *show all Sextans* $\not>$ 9.10⁶ (*with* x *and* $y > 1$).

High Irreducible Sextans.

$\mathbf{N} = (x^6+y^6) = N_{ii}.N_{vi}$; $N_{ii} = (x^2+y^2)$; $N_{vi} = (x^6+y^6) \div (x^2+y^2) > 9.10^6$.

[9-*ans*, 12-*mans*, *and Aurifeuillians excluded.*]

x, y	x^2+y^2	N_{vi}	Fig.	x, y	x^2+y^2	N_{vi}	Fig.
2^6, 3	5.821 ;	73.229321 ;	11	2^6, 7	5.829 ;	13.229.5569 ;	11
2^8, 3	5.13109 ;	2833.1515841 ;	15	2^7, 7	16433 ;	267635041 ? ‡	13
				2^8, 7	5.13.1009 ;	7753.553561 ;	15
2^6, 5	13.317 ;	157.106213 ;	11	2^6, 11	4217 ;	13.1253557 ;	11
2^7, 5	61.269 ;	9349.28669 ;	13	2^7, 11	5.3301 ;	97.2747089 ;	13
2^8, 5	53.1237 ;	13.37.8925841 ;	15	2^8, 11	65657 ;	13.329773237 ? ‡	15
10^2, 3	10009 ;	2281.43801 ;	13	11^2, 3	2.25.293 ;	337.635689 ;	13
10^2, 7	13.773 ;	5881.16921 ;	13	11^2, 5	2.7333 ;	13.1453.11329 ;	13
10^2, 11	29.349 ;	13.7600357 ;	13	11^2, 7	2.5.13.113 ;	157.1360789 ;	13

High Aurifeuillian Sextans. $\mathbf{N} = N_{ii}.L.M$; N_{ii}, L, M $> 9.10^6$.

$\mathbf{N} = (x^6+y^6) = N_{ii}.N_{vi}$; $N_{ii} = (x^2+y^2)$; $N_{vi} = (x^6+y^6) \div (x^2+y^2) = L.M$;

$xy = 2\xi^2\eta^2$ or $6\xi^2\eta^2$.

x, y	x^2+y^2	L	M	Fig.	
2^{13}, 3^2	7817 : 5.17.101 ;	13.4925653 ;	37.1900861 ;	24	Bin-Aurifn.
5^2	7577 : 17.521 ;	13.1033.4621 ;	37.73.97.277 ;	24	
7^2	5.13.113 : 9137 ;	37.61.26641 ;	74896609 ;	24	
11^2	5.1381 : 9721 ;	56410033 ;	13.73.241.349 ;	24	
2^{15}, 3^2	32009 : 5.6709 ;	1048864081 ? ‡	13.84554581 ;	28	
5^2	31513 : 13.2621 ;	3373.306133 ;	1116536689 ? ‡	28	
7^2	25.17.73 : 53.653 ;	181.5616253 ;	13.87242917 ;	28	
11^2	17.29.61 : 5.37.193 ;	985105969 ? ‡	13.900259909 ? ‡	28	
2^{13}, 3	13.5162221 ;	1741.36781 ;	7069.9949 ;	24	Sext-Aurn.
2^{15}, 3	‡	1009.1039513 ;	13.73.97.11941 ;	28	

High Numbers, $\mathbf{N} = (X^6 + Y^6)$; $[X = 3^m,\ Y = 2^n$; m, n *odd*$]$.
$\mathbf{N} = N_{ii} . N_{vi}$; $N_{ii} = (X^2 + Y^2)$; $N_{vi} = (X^6 + Y^6) \div (X^2 + Y^2) = L . M$.
$[6XY = \square$; N_{vi} is a Sext-Aurifeuillian.$]$

m, n	N_{ii}	L	M	Fig.
1, 1	13;	1;	61;	3
3	73;	13;	277;	6
5	1033;	13·37;	2161;	10
7	13·13·97;	11257;	23833;	13
9	262153;	13·73·229;	316201;	17
11	181.23173;	3818953;	13·37·61·157;	20
3, 1	733;	373;	13·109;	9
3	13; 61;	181;	37·73;	9
5	1753;	97;	13·661;	10
7	109·157;	13·397;	49801;	13
9	73; 13·277;	149113;	37·12421;	17
11	4195033;	13·243517;	5556121;	20
5, 1	59053;	13·3637;	37·1993;	15
3	59113;	157·241;	13·73·97;	15
5	13; 4621;	24001;	61; 2341;	15
7	241·313;	37·229;	13·25309;	15
9	321193;	73·577;	13·13·13·613;	17
11	13·97·3373;	241·7417;	9705193 ? †	20
7, 1	13·97·3793;	4441477;	5150713;	21
3	73·65521;	13·317257;	577·9613;	21
5	4783993;	13·37·7393;	6431857;	21
7	13; 369181;	61; 43261;	8639041;	21
9	5045113;	13·97·1117;	73·210961;	21
11	37·242629;	459961;	13·3412957;	21
9, 1	†	†	13· †	25
3	733; 13·109·373;	829·444817;	37·73·150697;	25
5	229·1021·1657;	†	13· †	25
7	†	13·97·252157;	†	25
9	13; 61; 37·73·181;	109·2393389;	13177·43633;	25
11	†	13·13·1033057;	†	

Trinomial Dimorph Sextans.

$N_{vi} = t^8 + 14t^4u^4 + u^8 = N(y) = N(z) = L . M$; $N(y)$ means $(x^6 + y^6) \div (x^2 + y^2)$.
$x = t^2 + u^2,\ y = t^2 \sim u^2,\ z = 2tu$; $L = x^2 - yz$, $M = x^2 + yz$.
Ex. $N_1 = (7^{8n} + 2.7^{4n+1} + 1)$, $N_2 = (7^{8n} + 14^{4n+1} + 2^{8n})$, $N_3 = 14^{8n} + 14^{4n+1} + 1$.

N	t, u	x, y, z	L	M	Fig.
N_1	1, 7	50, 48, 14	4·457;	4·13·61;	7
N_1	1, 49	2402, 2400, 98	4·829·1669;	4·13·37·3121;	14
N_1	1, 343	117650, 117648, 686	4·277·12419509 ? †	4·3480557257 ? †	21
N_2	2, 7	53, 45, 28	1549;	13·313;	7
N_2	2, 49	2405, 2397, 196	5314213;	73·85669;	14
N_2	2, 343	117653, 117645, 1372	61·224275729 ? †	13·733·1469581;	21
N_3	1, 14	197, 195, 28	33349;	44269;	10
N_3	1, 196	38417, 38415, 392	109·13401901 ? †	241·457·13537;	19

Simple Bin-Aurifeuillian Sextans.

$$N = (1^6 + y^6) \div (1^2 + y^2) = L.M; \quad [y = 2\eta^2].$$

$$P = (1 + y + y^2), \quad Q = 2\eta\,(1 + y); \qquad L = (P - Q), \quad M = (P + Q).$$

[All divisors < 32,900 cast out.]

η	y	L	M	η	y	L	M
1	2	1 ;	13 ;	34	2 312	73·97·733 ;	13·423457 ;
2	8	37 ;	109 ;	35	2 450	313·18637 ;	13·37·12841 ;
3	18	229 ;	457 ;	36	2 592	6534361 ;	6907753 ;
4	32	13·61 ;	1321 ;	37	2 738	7296769 ;	7702069 ;
5	50	13·157 ;	3061 ;	38	2 888	13·241·2593 ;	61·229·613 ;
6	72	13·337 ;	6133 ;	39	3 042	37·243697 ;	73·130057 ;
7	98	8317 ;	13·853 ;	40	3 200	9987121 ; B	13·807637 ;
8	128	14449 ;	13·1429 ;	41	3 362	11030641 ;	1117·10369 ;
9	162	23473 ;	13·37·61 ;	42	3 528	433·28069 ;	73·174613 ;
10	200	97·373 ;	44221 ;	43	3 698	13·1027753 ;	109·128413 ;
11	242	193·277 ;	64153 ;	44	3 872	13·61·18481 ;	73·210097 ;
12	288	13·5869 ;	37·2437 ;	45	4 050	13·1093·1129 ;	16771141 ;
13	338	105769 ;	123397 ;	46	4 232	2089·8389 ;	13·37·38053 ;
14	392	143053 ;	13·12697 ;	47	4 418	19107757 ;	13·157·9769 ;
15	450	189421 ;	216481 ;	48	4 608	1621·12829 ;	13·1667749 ;
16	512	246241 ;	279073 ;	49	4 802	3229·6997 ;	37·636073 ;
17	578	13·24229 ;	37·61·157 ;	50	5 000	24504901 ;	25505101 ;
18	648	13·30553 ;	443917 ;	51	5 202	13·2041177 ;	27596713 ;
19	722	13·109·349 ;	549481 ;	52	5 408	61·470317 ;	1381·21589 ;
20	800	37·16453 ;	13·73·709 ;	53	5 618	73·424273 ;	13·181·13669 ;
21	882	741721 ;	13·62761 ;	54	5 832	33388093 ;	37·37·25309 ;
22	968	895357 ;	13·241·313 ;	55	6 050	35942941 ;	37274161 ;
23	1 058	61·17569 ;	1169137 ;	56	6 272	13·13·373·613 ;	229·174877 ;
24	1 152	1272913 ;	829·1669 ;	57	6 498	13·37·86257 ;	61·704449 ;
25	1 250	13·37·3121 ;	1626301 ;	58	6 728	13·457·7489 ;	46053277 ;
26	1 352	97·18133 ;	1069·1777 ;	59	6 962	47654773 ;	13·661·5737 ;
27	1 458	109·18793 ;	13·169693 ;	60	7 200	157·324733 ;	13·1069·3793 ;
28	1 568	37·97·661 ;	337·7561 ;	61	7 442	54482761 ;	13·853·5077 ;
29	1 682	73·37441 ;	2928421 ;	62	7 688	37·1571881 ;	6961·8629 ;
30	1 800	13·193·1249 ;	3349861 ;	63	7 938	181·277·1237 ;	97·660001 ;
31	1 922	13·73·3769 ;	1597·2389 ;	64	8 192	13·13·313·1249 ;	3121·21841 ;
32	2 048	13·312709 ;	4327489 ;	65	8 450	37·61·31153 ;	72509581 ;
33	2 178	1009·4561 ;	13·13·28933 ;	66	8 712	109·685849 ;	13·577·10273 ;

Simple Bin-Aurifeuillian Sextans,

$$N = (1^6 + y^6) \div (1^2 + y^2) = L.M; \quad [y = 2\eta^2].$$

$$P = (1 + y + y^2), \quad Q = 2\eta(1 + y); \qquad L = (P - Q), \quad M = (P + Q).$$

[All divisors < 32,900 cast out.]

η	y	L	M	η	y	L	M
67	8 978	79410277 ;	433·188953 ;	100	20 000	396019801 ;	13·337·92221 ;
68	9 248	6361.13249 ;	86792617 ;	101	20 402	412140601 ;	349·829.1453 ;
69	9 522	13.2161.3181 ;	97·541·1753 ;	102	20 808	37·73·181.877 ;	437238709 ;
70	9 800	13.181.40237 ;	61.1597081 ;	103	21 218	13·397·86389 ;	937·485161 ;
71	10 082	13·7709617 ;	97·397·2677 ;	104	21 632	73·613.10357 ;	472464721 ;
72	10 368	106012657 ;	13·37·226609 ;	105	22 050	61·7894981 ;	13·13·2904469 ;
73	10 658	112047409 ;	13·8858449 ;	106	22 472	277·397·4549 ;	421.1210873 ;
74	10 952	193·613141 ;	13·9352177 ;	107	22 898	73·97·109·673 ;	1009·524521 ;
75	11 250	124886101 ;	128261401 ;	108	23 328	13·41475373 ;	21661·25357 ;
76	11 552	37·457·7789 ;	313·432001 ;	109	23 762	13·3637·11833 ;	37·15400993 ;
77	11 858	13.10676749 ;	8677·16417 ;	110	24 200	13·157·284341 ;	1549·381529 ;
78	12 168	146174029 ;	61.2458537 ;	111	24 642	241.2497021 ;	13.109.181.2389 ;
79	12 482	13.181.67057 ;	193·373·2137 ;	112	25 088	623812897 ;	13·73·669181 ;
80	12 800	1297.124753 ;	165900961 ;	113	25 538	37·37·61·7741 ;	13·13·1753·2221 ;
81	13 122	170074081 ;	11677.14929 ;	114	25 992	669683653 ;	193·3531277 ;
82	13 448	13.409.33601 ;	109.1679521 ;	115	26 450	241·241.11941 ;	73·9667297 ;
83	13 778	187559749 ;	37·5192821 ;	116	26 912	13·2557·21601 ;	18397·39709 ;
84	14 112	13.61.248161 ;	201533641 ;	117	27 378	24097·30841 ;	73·10356013 ;
85	14 450	7177·28753 ;	13.1093.14869 ;	118	27 848	8017·95917 ;	13·61.181·5449 ;
86	14 792	216273661 ;	13·13·37·35401 ;	119	28 322	795423133 ;	193·4191217 ;
87	15 138	226539997 ;	13·97·183829 ;	120	28 800	822556561 ;	37·22604893 ;
88	15 488	11701.20269 ;	242619697 ;	121	29 282	13·421.155377 ;	613.1410361 ;
89	15 842	248164753 ;	5557·45673 ;	122	29 768	13·67607689 ;	1741.513169 ;
90	16 200	13.19964617 ;	109.2434609 ;	123	30 258	13·97·337·2137 ;	37·3529·7069 ;
91	16 562	271301941 ;	37·7495429 ;	124	30 752	938089513 ;	13·73334077 ;
92	16 928	229.1237813 ;	13.1801.12373 ;	125	31 250	97·9987433 ;	13·349·216973 ;
93	17 298	1741.170029 ;	73·4143229 ;	126	31 752	61·73·224617 ;	13·241·324361 ;
94	17 672	37·8351209 ;	315639781 ;	127	32 258	421·2452297 ;	13033·80473 ;
95	18 050	13·24799297 ;	329250241 ;	128	32 768	109·181·54001 ;	37·29247661 ; L
96	18 432	13·25862917 ;	343296193 ;	160	51 200	37·61·97·12049 ;	13·2857·70141 ;
97	18 818	13·3373·7993 ;	157·733·3109 ;				
98	19 208	277·541·2437 ;	1069·348673 ;	200	80 000	13·489852277 ;	61.105443941 ;
99	19 602	37.1021.10069 ;	13·61·489457 ;	201	80 802	13·673·742549 ;	20389·321817 ;

Bin-Aurifeuillian Sextans.

$$N = (x^6 + y^6) \div (x^2 + y^2) = L.M\,; \quad [x = \xi^2 > 1,\ y = 2\eta^2\,;\ \ L, M < 9.10^6].$$
$$P = (x^2 + xy + y^2),\ \ Q = 2\xi\eta\,(x + y)\,; \quad L = (P - Q),\ \ M = (P + Q).$$

ξ, η	x, y	L M	ξ, η	x, y	L M
3, 1	9, 2	37:13.13;	51, 2	2601, 8	73.85669:7318309;
5	25	409:13.73;	53, 2	2809, 8	7315813:1453.5857;
7	49	1789:3217;	5, 3	25, 18	109:2689;
9	81	5233:8221;	7	49	13.61:6421;
11	121	13.937:73.241;	11	121	13.613:26317;
13	169	37.661:33349;	13	169	17341:193.241;
15	225	44269:13.61.73;	17	289	13.4441:37.3253;
17	289	74209:93997;	19	361	93937:13.13873;
19	361	37.3169:241.601;	23	529	13.16477:365173;
21	441	13.13597:213973;	25	625	305749:37.13477;
23	529	13.109.181:305329;	29	841	573277:13.13.13.397;
25	625	13.27733:423229;	31	961	433.1753:673.1669;
27	729	61.8089:13.44029;	35	1225	1261969:13.37.3709;
29	841	660073:13.97.601;	37	1369	13.122401:61.97.373;
31	961	865741:13.75781;	41	1681	1069.2281:37.61.4129;
33	1089	73.15289:37.34057;	43	1849	13.228517:3934093;
35	1225	1417189:193.8233;	47	2209	541.7933:73.75997;
37	1369	13.136573:157.12601;	49	2401	13.37.10597:6519529;
39	1521	2197693:2435281;	53, 3	2809, 18	109.64609:2029.4357;
41	1681	37.72733:13.13.97.181;	3, 4	9, 32	409:2377;
43	1849	61.61.877:3581689;	5	25	13.13:4729;
45	2025	3922249:97.193.229;	7	49	457:13.733;
47	2209	13.359713:5091937;	9	81	13.157:18313;
49	2401	13.425701:6005101;	11	121	6073:61.541;
51	2601	13.661.757:7035913;	13	169	73.193:55897;
53, 1	2809, 2	997.7621:13.61.10333;	15	225	37.757:89689;
3, 2	9, 8	13:421;	17	289	181.277:13.97.109;
5	25	229:1549;	19	361	13.6397:97.2089;
7	49	13.97:61.73;	21	441	157.829:13.37.601;
9	81	13.313:10477;	23	529	194569:401017;
11	121	13.769:37.577;	25	625	280249:13.37.1129;
13	169	20773:39181;	27	729	13.30109:337.2137;
15	225	97.397:13.5113;	29	841	73.7297:61.15373;
17	289	65701:13.8161;	31	961	13.54541:541.2221;
19	361	105229:13.12409;	33	1089	925849:13.313.373;
21	441	160357:235789;	35	1225	13.91453:37.51157;
23	529	234733:13.25657;	37	1369	1504297:2333689;
25	625	61.5449:277.1657;	39	1521	37.50773:181.15733;
27	729	397.1153:616933;	41	1681	829.2797:3442441;
29	841	13.47353:37.21961;	43	1849	109.25981:13.433.733;
31	961	61.13297:37.157.181;	45	2025	13.263533:4906969;
33	1089	13.80761:109.12289;	47	2209	61.193.349:13.277.1609;
35	1225	13.102913:577.2917;	49	2401	37.229.577:6796393;
37	1369	13.13.9949:2088973;	51, 4	2601, 32	577.1009:13.609517;
39	1521	97.21517:1249.2053;	3, 5	9, 50	13.97:4801;
41	1681	2562277:13.239713;	7	49	421:14281;
43	1849	37.73.1153:13.288697;	9	81	1321:37.673;
45	2025	37.101377:13.61.5653;	11	121	13.337:97.433;
47	2209	4480621:5314213;	13	169	61.181:157.433;
49, 2	2401, 8	5311909:13.481249;	17, 5	289, 50	42841:37.4273;

Bin-Aurifeuillian Sextans.

ξ, η	x, y	L M	ξ, η	x, y	L M
19, 5	361, 50	73.997 : 228961 ;	51, 7	2601, 98	13.13.109.277 : 8956789 ;
21	441	13.37.241 : 61.5281 ;	3, 8	9, 128	61.181 : 13.1861 ;
23	529	175621 : 13.33997 ;	5	25	13.613 : 37.877 ;
27	729	13.27697 : 780721 ;	7	49	5233 : 37.1213 ;
29	841	13.37957 : 61.16561 ;	9	81	3217 : 63409 ;
31	961	660661 : 13.97.1021 ;	11	121	2689 : 13.6949 ;
33	1089	867001 : 1618741 ;	13	169	4801 : 37.3469 ;
37	1369	13.313.349 : 2470141 ;	15	225	13.853 : 73.2473 ;
39	1521	1779301 : 3004681 ;	17	289	23473 : 193.1297 ;
41	1681	2202601 : 13.278617 ;	19	361	44257 : 341569 ;
43	1849	1129.2389 : 4330321 ;	21	441	76129 : 13.13.2713 ;
47	2209	13.61.4957 : 6054361 ;	23	529	13.9397 : 109.5557 ;
49	2401	97.48313 : 13.545257 ;	25	625	13.14293 : 788209 ;
51, 5	2601, 50	5545741 : 13.634597 ;	27	729	270913 : 13.77797 ;
5, 6	25, 72	1789 : 13.1033 ;	29	841	381697 : 13.98533 ;
7	49	13.73 : 21277 ;	31	961	13.40213 : 1603057 ;
11	121	3061 : 54013 ;	33	1089	61.73.157 : 37.53629 ;
13	169	8317 : 37.37.61 ;	35	1225	916129 : 2431489 ;
17	289	35869 : 13.73.193 ;	37	1369	1179553 : 13.97.2341 ;
19	361	62773 : 13.37.541 ;	39	1521	61.24517 : 313.11353 ;
23	529	97.1621 : 241.2029 ;	41	1681	13.37.3889 : 4244017 ;
25	625	231709 : 13.49993 ;	43	1849	2311681 : 5032033 ;
29	841	109.4177 : 61.17881 ;	45	2025	73.38713 : 349.16981 ;
31	961	601.1021 : 13.106321 ;	47	2209	3421393 : 13.73.7309 ;
35	1225	13.80713 : 2138749 ;	49, 8	2401, 128	13.315829 : 8071249 ;
37	1369	1338109 : 2617717 ;	5, 9	25, 162	73.193 : 13.3673 ;
41	1681	37.56473 : 229.16657 ;	7	49	13.769 : 181.349 ;
43	1849	73.35149 : 13.181.1933 ;	11	121	61.73 : 109.1069 ;
47	2209	13.289033 : 61.157.661 ;	13	169	4729 : 61.2617 ;
49, 6	2401, 72	313.14341 : 7396981 ;	17	289	13.1429 : 97.3037 ;
3, 7	9, 98	6073 : 15061 ;	19	361	97.373 : 13.157.193 ;
5	25	13.313 : 61.349 ;	23	529	37.2857 : 677857 ;
9	81	1549 : 13.37.97 ;	25	625	13.12613 : 872269 ;
11	121	2377 : 69829 ;	29	841	346201 : 1393333 ;
13	169	6133 : 277.373 ;	31	961	478813 : 13.13.37.277 ;
15	225	14449 : 37.4057 ;	35	1225	709.1201 : 13.199933 ;
17	289	13.37.61 : 213553 ;	37	1369	1102537 : 3141829 ;
19	361	13.4093 : 297397 ;	41	1681	1764193 : 1213.3697 ;
23	529	139393 : 181.3001 ;	43	1849	13.37.4549 : 5301097 ;
25	625	13.16033 : 714529 ;	47	2209	13.73.3433 : 37.196477 ;
27	729	209881 : 13.71161 ;	49, 9	2401, 162	3919441 : 181.46633 ;
29	841	418069 : 1180537 ;	3, 10	9, 200	13.37.61 : 54421 ;
31	961	13.43669 : 1486909 ;	7	49	17341 : 13.37.181 ;
33	1089	433.1741 : 13.142.357 ;	9	81	13.937 : 113341 ;
37	1369	337.3733 : 2777833 ;	11	121	8221 : 13.11497 ;
39	1521	1093.1453 : 193.17389 ;	13	169	6421 : 198301 ;
41	1681	73.27109 : 4021249 ;	17	289	15061 : 13.26737 ;
43	1849	13.97.1933 : 313.15277 ;	19	361	13.37.61 : 455701 ;
45	2025	13.13.17581 : 1993.2833 ;	21	441	193.277 : 591901 ;
47, 7	2209, 98	3587761 : 13.509521 ;	23, 10	529, 200	73.1237 : 13.58537 ;

Bin-Aurifeuillian Sextans.

ξ, η	x, y	L M
27, 10	729, 200	193·1117:1218901 ;
29	841	13·23977:1519261 ;
31	961	349·1249:1875541 ;
33	1089	61.9721:13.176497 ;
37	1369	241.4261:13·73·3529 ;
39	1521	397·3313:1801.2221 ;
41	1681	13·127657:433·10957 ;
43	1849	2066461:13·430057 ;
47	2209	3097021:7625941 ;
49, 10	2401, 200	181.20641:13·679537 ;
3, 11	9, 242	44257:13·5953 ;
5	25	35869:37·2557 ;
7	49	37·757:13·9049 ;
9	81	20773:13·11437 ;
13	169	10477:37·6637 ;
15	225	13·733:61.5209 ;
17	289	13·1033:410617 ;
19	361	13·1861:73·7237 ;
21	441	44221:675313 ;
23	529	13·5869:856549 ;
25	625	73·1693:1077289 ;
27	729	61.3109:1343197 ;
29	841	278413:13·127717 ;
31	961	394201:2035093 ;
35	1225	109.6661:13·277·829 ;
37	1369	952669:13·229.1201 ;
39	1521	349·3517:1201·3541 ;
41	1681	13·119737:37·135829 ;
43	1849	13·149749:61.96769 ;
47, 11	2209, 242	2938489:157·51001 ;
5, 12	25, 288	13·4093:181.709 ;
7	49	42841:97·1609 ;
11	121	37·661:13·18493 ;
13	169	73·241:277.1093 ;
17	289	14281:485113 ;
19	361	61.349:613177 ;
23	529	64153:13·74317 ;
25	625	105769:13·92413 ;
29	841	13·18973:37·157·313 ;
31	961	13·73·373:37·59797 ;
35	1225	61.10909:3207289 ;
37	1369	879961:13·157·1873 ;
41	1681	13·111949:337·15817 ;
43	1849	37·49429:421.14821 ;
47, 12	2209, 288	2782201:13.647341 ;
3, 13	9, 338	73·1237:97·1489 ;
5	25	76129:170509 ;
7	49	62773:203641 ;
9	81	181.277:73·3373 ;
11, 13	121, 338	97·397:301057 ;
15, 13	225, 338	37·577:61.7549 ;
17	289	18313:572581 ;
19	361	21277:711889 ;
21	441	37·877:883117 ;
23	529	54421:397·2749 ;
25	625	37·2437:1342069 ;
27	729	143053:157·10453 ;
29	841	157·1381:313.6373 ;
31	961	315589:97·24841 ;
33	1089	443881:601.4813 ;
35	1225	606589:37·37·2521 ;
37	1369	808993:1489.2749 ;
41	1681	1129.1201:1753·3229 ;
43	1849	61.28081:853·7741 ;
45	2025	2134609:7664029 ;
47, 13	2209, 338	2628133:8853001 ;
3, 14	9, 392	73·1693:13·37·397 ;
5	25	37·2857:13.109.157 ;
9	81	73·997:311173 ;
11	121	13·4441:97·3853 ;
13	169	44269:109.4153 ;
15	225	33349:13·42433 ;
17	289	26317:157·4297 ;
19	361	37.673:826093 ;
23	529	13·3673:1093.1129 ;
25	625	13·5953:241.6229 ;
27	729	123397:13.61.2293 ;
29	841	189421:13.168601 ;
31	961	277.1009:13.181.1117 ;
33	1089	398029:3134917 ;
37	1369	13·56929:4388869 ;
39	1521	541.1801:5152333 ;
41	1681	313·4021:13·462937 ;
43	1849	1599109:757·9241 ;
45, 14	2025, 392	2002669:8093509 ;
7, 15	49, 450	13·9397:157·2113 ;
11	121	13·6397:37·12433 ;
13	169	65701:548521 ;
17	289	39181:13.181.337 ;
19	361	61.541:13·73·1009 ;
23	529	37·1213:13·107377 ;
29	841	13·12697:37·65173 ;
31	961	246241:2870701 ;
37	1369	13·51817:4711801 ;
41	1681	1163581:13·492757 ;
43, 15	1849, 450	1487641:13·570697 ;
3, 16	9, 512	157·1381:13·24373 ;
5	25	61.3109:193·1873 ;
7	49	13·12613:73·5689 ;
9, 16	81, 512	139393:13·36997 ;

Bin-Aurifeuillian Sextans.

ξ, η	x, y	L	M
11, 16	121, 512	13·37·241	561553 ;
13,	169	93937	660529 ;
15	225	74209	13·60133 ;
17	289	13·61·73	397·2341 ;
19	361	193·241	13·85237 ;
21	441	97·433	73·18121 ;
23	529	13·37·97	1579009 ;
25	625	181·349	1882369 ;
27	729	37·2557	2239057 ;
29	841	97·1489	13·37·5521 ;
31	961	216481	3138913 ;
33	1089	13·24229	13·73·1453 ;
35	1225	444529	13·333493 ;
37	1369	13·46933	5064337 ;
39	1521	817153	5891521 ;
41	1681	109·9829	13·97·5413 ;
43, 16	1849, 512	13·73·1453	7876369 ;
3, 17	9, 578	277·1009	399241 ;
5	25	13·18973	451669 ;
7	49	193·1117	13·39541 ;
9	81	13·14293	577·1021 ;
11	121	97·1621	61·11149 ;
13	169	157·829	790501 ;
15	225	105229	313·2953 ;
19	361	13·5113	1279657 ;
21	441	55897	13·13·8941 ;
23	529	54013	37·73·661 ;
25	625	63409	13·241·673 ;
27	729	13·37·181	2486713 ;
29	841	181·709	61·47977 ;
31	961	13·37·397	3435169 ;
33	1089	279073	13·373·829 ;
35	1225	13·30553	4688329 ;
37	1369	73·7537	5448853 ;
39	1521	373·1993	1237·5101 ;
41	1681	109·9013	73·99733 ;
43, 17	1849, 578	1273333	13·37·17401 ;
5, 18	25, 648	315589	13·13·3301 ;
7	49	278413	629701 ;
11	121	13·16033	709·1153 ;
13	169	175621	373·2521 ;
17	289	37·3169	73·17317 ;
19	361	93997	61·24169 ;
23	529	157·433	13·155161 ;
25	625	69829	13·37·4909 ;
29	841	13·9049	3226669 ;
31	961	170509	13·289369 ;
35	1225	37·61·157	829·6121 ;
37	1369	13·109·349	97·60493 ;
41, 18	1681, 648	897349	13·597889 ;
43, 18	1849, 648	241·4861	997·8929 ;
3, 19	9, 722	444529	97·6301 ;
5	25	398029	13·52453 ;
7	49	13·73·373	764149 ;
9	81	13·23977	37·23269 ;
11	121	270913	975661 ;
13	169	231709	1112017 ;
15	225	194569	1274149 ;
17	289	160357	13·37·3049 ;
21	441	13·8161	733·2677 ;
23	529	89689	181·12577 ;
25	625	37·37·61	13·203293 ;
27	729	13·6949	3067789 ;
29	841	113341	3558193 ;
31	961	97·1609	13·61·5197 ;
33	1089	13·109·157	421·11317 ;
35	1225	13·24373	37·148537 ;
37	1369	443917	61·103669 ;
39	1521	37·16453	7257013 ;
41, 19	1681, 722	181·4513	61·109·1249 ;
3, 20	9, 800	73·7537	109·6829 ;
7	49	443881	13·70717 ;
9	81	394201	13·61·1297 ;
11	121	346201	1156681 ;
13	169	299881	1307641 ;
17	289	13·16477	877·1933 ;
19	361	13·13597	1941481 ;
21	441	241·601	13·171517 ;
23	529	37·3253	61·42061 ;
27	729	277·373	13·37·73·97 ;
29	841	109·1069	3923641 ;
31	961	13·11437	37·122053 ;
33	1089	203641	13·399277 ;
37	1369	399241	6819481 ;
39	1521	549481	7791001 ;
41, 20	1681, 800	741721	8879401 ;
5, 21	25, 882	13·46933	991069 ;
11	121	349·1249	13·37·2833 ;
13	169	381697	1529389 ;
17	289	280249	1952437 ;
19	361	234733	2218561 ;
23	529	13·12409	37·73·1069 ;
25	625	13·97·109	3302149 ;
29	841	37·3469	13·229·1453 ;
31	961	13·11497	4948633 ;
37, 21	1369, 882	193·1873	13·73·7753 ;
3, 22	9, 968	181·4513	673·1597 ;
5	25	373·1993	13·90793 ;
7	49	13·51817	61·21313 ;
9, 22	81, 968	606589	13·110569 ;

Bin-Aurifeuillian Sextans.

ξ, η	x, y	L M	ξ, η	x, y	L M
13, 22	169, 968	478813:1779541 ;	27, 25	729, 1250	13·25657:5676841 ;
15	225	418069:241·8269 ;	29	841	13·37·601:193·32917 ;
17	289	13·27697:2240533 ;	31	961	13·37·541:109·65269 ;
19	361	305749:13·337·577 ;	33, 25	1089, 1250	193·1297:997·7993 ;
21	441	13·109·181:2860309 ;	3, 26	9, 1352	1627837:349·5881 ;
23	529	213973:37·73·1201 ;	5	25	37·109·373:2220349 ;
25	625	13·13873:61·193·313 ;	7	49	109·12721:37·193·337 ;
27	729	37·4273:13·157·2053 ;	9	81	1273333:2614621 ;
29	841	37·4057:97·157·313 ;	11	121	1163581:2848693 ;
31	961	61·2617:13·193·2161 ;	15	225	952669:1597·2137 ;
35	1225	73·3373:13·538513 ;	17	289	709·1201:349·10753 ;
37	1369	157·2113:73·181·601 ;	19	361	433·1741:4138741 ;
39, 22	1521, 968	451669:109·109·757 ;	21	441	660661:2017·2269 ;
3, 23	9, 1058	13·13·37·157:1276213 ;	23	529	573277:421·12049 ;
5	25	897349:37·37717 ;	25	625	61·8089:73·229·337 ;
7	49	817153:13·37·3181 ;	27	729	423229:6266677 ;
9	81	13·56929:1683169 ;	29	841	365173:6979261 ;
11	121	61·10909:13·142969 ;	31	961	61·5281:7779253 ;
13	169	61·9721:2060473 ;	33, 26	1089, 1352	297397:37·234457 ;
15	225	13·40213:2293309 ;	5, 27	25, 1458	1762429:13·37·73·73 ;
17	289	109·4177:13·197077 ;	7	49	37·44053:97·28549 ;
19	361	13·30109:37·77617 ;	11	121	13·73·1453:3254749 ;
21	441	61·5449:3228457 ;	13	169	313·4021:337·10513 ;
25	625	235789:97·42337 ;	17	289	241·4261:4234393 ;
27	729	97·2089:1381·3361 ;	19	361	916129:37·125641 ;
29	841	13·73·193:1453·3613 ;	23	529	13·54541:5644741 ;
31	961	73·2473:1033·5749 ;	25	625	13·47353:2113·2953 ;
33	1089	198301:13·193·2677 ;	29	841	277·1657:13·589189 ;
35	1225	13·18493:373·20353 ;	31, 27	961, 1458	401017:13·241·2713 ;
37, 23	1369, 1058	311173:13·659437 ;	3, 28	9, 1568	13·169837:109·25117 ;
5, 24	25, 1152	73·14713:61·97·277 ;	5	25	2052409:1657·1777 ;
7	49	109·9013:13·137653 ;	9	81	13·135469:3423289 ;
11	121	808993:13·73·2269 ;	11	121	13·13·9601:3703417 ;
13	169	109·6661:97·24481 ;	13	169	1487641:4016713 ;
17	289	13·43669:97·30097 ;	15	225	1129·1201:13·193·1741 ;
19	361	13·37957:3253153 ;	17	289	349·3517:13·366397 ;
23	529	13·27733:61·66757 ;	19	361	1102537:13·397·1009 ;
25	625	305329:37·123517 ;	23	529	867001:13·37·13033 ;
29	841	228961:13·444421 ;	25	625	433·1753:73·94513 ;
31	961	213553:157·41413 ;	27	729	660073:37·241·853 ;
35, 24	1225, 1152	37·6637:13·633253 ;	29, 28	841, 1568	13·44029:8396809 ;
3, 25	9, 1250	13·106537:1762681 ;	3, 29	9, 1682	73·181·193:13·241429 ;
7	49	241·4861:2080801 ;	5	25	37·64237:3366829 ;
9	81	109·9829:61·37201 ;	7	49	13·37·4597:109·33181 ;
11	121	541·1801:13·13·37·397 ;	9	81	2051641:13·299401 ;
13	169	879961:2724661 ;	11	121	13·337·433:4197601 ;
17	289	61·73·157:3315421 ;	13	169	37·109·433:4537597 ;
19	361	601·1021:3674521 ;	15	225	1599109:13·378253 ;
21	441	73·7297:13·314137 ;	17	289	13·111949:757·7057 ;
23, 25	529, 1250	397·1153:13·61·5737 ;	19, 29	361, 1682	397·3313:13·447541 ;

Bin-Aurifeuillian Sextans.

ξ, η	x, y	L	M	ξ, η	x, y	L	M
21, 29	441, 1682	1179553:	6351181 ;	13, 32	169, 2048	73.37321:	2293.2797 ;
23	529	13.80713:	6948217 ;	15	225	2523649:	37.186157 ;
25	625	925849:	97.78517 ;	17	289	37.109.577:	13.570181 ;
27, 29	729, 1682	61.13297:	73.114553 ;	19	361	2134609:	13.109.5641 ;
7, 30	49, 1800	2554021:	13.315937 ;	21, 32	441, 2048	13.149749:	37.233437 ;
11	121	1093.2017:	109.157.277 ;	5, 33	25, 2178	13.97.3229:	181.30529 ;
13	169	2036941:	5108581 ;	7	49	3823933:	13.61.7417 ;
17	289	61.28081:	13.37.12421 ;	13	169	97.32233:	421.16993 ;
19	361	13.119737:	877.7393 ;	17	289	13.206821:	613.13417 ;
23, 30	529, 1800	337.3733:	61.126001 ;	19, 33	361, 2178	13.13.14653:	73.97.1249 ;
3, 31	9, 1922	13.257869:	37.110017 ;	3, 34	9, 2312	61.80209:	13.449209 ;
5	25	3139189:	13.334333 ;	5	25	13.354553:	997.6217 ;
7	49	2935249:	709.6553 ;	7	49	541.8017:	1297.5077 ;
9	81	2738653:	61.73.1117 ;	9	81	1777.2293:	37.73.2593 ;
11	121	13.195997:	181.29473 ;	11	121	3819853:	13.573817 ;
13	169	37.63841:	5732809 ;	13	169	3571429:	7957837 ;
15	225	1009.2161:	13.109.4357 ;	15, 34	225, 2312	13.256033:	1021.8329 ;
17	289	2002669:	61.313.349 ;	3, 35	9, 2450	5508241:	6541021 ;
19	361	37.49429:	7207621 ;	9	81	241.19141:	13.373.1609 ;
21	441	13.127657:	109.229.313 ;	11	121	37.117133:	1669.4969 ;
23, 31	529, 1922	61.24517:	13.652753 ;	13, 35	169, 2450	181.22441:	73.120937 ;
3, 32	9, 2048	337.11329:	1777.2593 ;	5, 36	25, 2592	757.7717:	7726009 ;
5	25	3582769:	13.377653 ;	7, 36	49, 2592	5516809:	13.433.1453 ;
7	49	13.258277:	601.8713 ;	3, 37	9, 2738	337.20509:	13.625477 ;
9	81	13.37.6529:	1201.4657 ;	5, 37	25, 2738	61.107269:	8588029 ;
11, 32	121, 2048	157.18661:	1213.4933 ;				

☞ *The Tables above, pages* 174–179, *show all Bin-Aurifeuillian Sextans with* $x > 1$ *and* $L, M < 9.10^6$.

Simple Sext-Aurifeuillian Sextans, S′, [*Species* i].

$$S' = (1^6 + y^6) \div (1^2 + y^2) = L.M\,; \quad [y = 6\eta^2].$$

$$P = (1^2 + 3y + y^2), \quad Q = 6\eta\,(1+y)\,; \qquad L = (P - Q), \quad M = (P + Q).$$

[All factors < 32,900 cast out.]

η	y	L	M	η	y	L	M
1	6	13 ;	97 ;	11	726	13.37021 ;	37.15601 ;
2	24	349 ;	13.73 ;	12	864	109.6301 ;	13.13.4801 ;
3	54	2089 ;	13.313 ;	13	1 014	193.4933 ;	397.2797 ;
4	96	7177 ;	11833 ;	14	1 176	13.37.2677 ;	1485373 ;
5	150	13.13.109 ;	27481 ;	15	1 350	1704961 ;	13.277.541 ;
6	216	73.541 ;	55117 ;	16	1 536	73.97.313 ;	13.193189 ;
7	294	74929 ;	99709 ;	17	1 734	2834989 ;	3188929 ;
8	384	37.3517 ;	13.12853 ;	18	1 944	13.274993 ;	3995029 ;
9	486	241.877 ;	263953 ;	19	2 166	4451017 ;	73.67741 ;
10	600	13.25057 ;	37.10753 ;	20	2 400	61.89821 ;	6055321 ;

Simple Sext-Aurifeuillian Sextans, S′, [*Species* i].

η	y	L	M	η	y	L	M
21	2 646	6675733 ;	13·409·1381 ;	51	15 606	37·6454549 ;	13·19105369 ;
22	2 904	97·83077 ;	2713·3253 ;	52	16 224	258204649 ;	268329049 ;
23	3 174	13·741973 ;	37·284377 ;	53	16 854	13·109·196717 ;	193·373·4021 ;
24	3 456	13·881269 ;	61·204133 ;	54	17 496	73·1453·2833 ;	13·13·1845157 ;
25	3 750	397·34033 ;	13·61·18457 ;	55	18 150	323487121 ;	13·601·42937 ;
26	4 056	37·427849 ;	17096197 ;	56	18 816	347775793 ;	109·397·8329 ;
27	4 374	13·37·38329 ;	157·126457 ;	57	19 494	13·28723633 ;	73·5297833 ;
28	4 704	21351289 ;	13·1764013 ;	58	20 184	13009·30781 ;	241·1719829 ;
29	5 046	613·40129 ;	13·37·157·349 ;	59	20 886	3217·133321 ;	443681653 ;
30	5 400	61·462361 ;	673·44797 ;	60	21 600	458848441 ;	13·37·986281 ;
31	5 766	13·181·13681 ;	61·562897 ;	61	22 326	97·5055109 ;	506688937 ;
32	6 144	36587329 ;	38947009 ;	62	23 064	13·13·3097261 ;	433·1248493 ;
33	6 534	41418829 ;	44006689 ;	63	23 814	13·37·1160449 ;	157·3669937 ;
34	6 936	2749·16993 ;	13·3811081 ;	64	24 576	37·16070701 ;	13·47191621 ;
35	7 350	1021·51421 ;	55588261 ;	65	25 350	11677·54193 ;	7681·84961 ;
36	7 776	13·61·74161 ;	97·181·3541 ;	66	26 136	13·51755281 ;	37·3373·5557 ;
37	8 214	13·61·82813 ;	157·441517 ;	67	26 934	714693289 ;	13·73·769·1009 ;
38	8 664	73115269 ;	13·5928193 ;	68	27 744	758492809 ;	13·1741·34513 ;
39	9 126	709·114493 ;	85446973 ;	69	28 566	277·2903521 ;	109·7595677 ;
40	9 600	13·109·229·277 ;	181·522061 ;	70	29 400	13·65546137 ;	876796621 ;
41	10 086	99276253 ;	13·61·131449 ;	71	30 246	73·433·28537 ;	229·4051513 ;
42	10 584	109385389 ;	13·8824633 ;	72	31 104	954114769 ;	980989489 ;
43	11 094	120247609 ;	10957·11497 ;	73	31 974	661·1525609 ;	13·79725973 ;
44	11 616	13·2617·3877 ;	769·179497 ;	74	32 856	3313·321469 ;	1069·1023577 ;
45	12 150	37·397·9829 ;	150939721 ;	75	33 750	13·313·276229 ;	73·97·163021 ;
46	12 696	181·871393 ;	613·268729 ;				
47	13 254	171970369 ;	13·37·193·1933 ;	80	38 400	16729·87049 ;	13·9337·12301 ;
48	13 824	187162849 ;	37·5273677 ;				
49	14 406	13·15641569 ;	211811713 ;	100	60 000	37·96329173 ;	9157·397093 ;
50	15 000	13·168164977 ;	1069·214729 ;	101	61 206	13·37·7711573 ;	3853·981949 ;

Sext-Aurifeuillian Sextans, S′, [*Species* i].

$$S' = (x^6+y^6)\div(x^2+y^2) = L.M\,; \qquad x = \xi^2, (>1)\,; \quad y = 6\eta^2.$$

$$P = (x^2+3xy+y^2),\ Q = 6\xi\eta\,(x+y)\,; \qquad L = (P-Q),\ M = (P+Q)\,; \ [L \text{ and } M \not> 9.10^6].$$

ξ, η	x, y	L	M	ξ, η	x, y	L	M
5, 1	25, 6	181 :	13.157 ;	7, 4	49, 96	37.37 :	13.3853 ;
7	49	1009 :	13.433 ;	11	121	13.109 :	193.601 ;
11	121	37.229 :	25237 ;	13	169	3769 :	169129 ;
13	169	17989 :	45289 ;	17	289	13.1453 :	333049 ;
17	289	13.4513 :	157.757 ;	19	361	13.37.73 :	451897 ;
19	361	13.7309 :	178693 ;	23	529	229.421 :	13.60493 ;
23	529	73.2953 :	37.9817 ;	25	625	73.2017 :	241.4201 ;
25	625	307261 :	13.38197 ;	29	841	13.23581 :	661.2437 ;
29	841	575077 :	37.23509 ;	31	961	423097 :	1995913 ;
31	961	760993 :	13.86209 ;	35	1225	753001 :	13.228637 ;
35	1225	97.13033 :	13.181.757 ;	37	1369	13.75133 :	3578569 ;
37	1369	1593589 :	73.109.277 ;	41	1681	181.8677 :	13.457.853 ;
41	1681	2441053 :	73.44809 ;	43	1849	13.97.1549 :	1201.4969 ;
43	1849	13.228733 :	3930709 ;	47, 4	2209, 96	2925049 :	673.12073 ;
47	2209	13.37.8929 :	5544109 ;	7, 5	49, 150	13.397 :	88741 ;
49	2401	5100397 :	2137.3049 ;	11	121	2161 :	157.1153 ;
53, 1	2809, 6	13.541993 :	8836249 ;	13	169	37.73 :	97.2593 ;
5, 2	25, 24	61 :	13.457 ;	17	289	13.937 :	459961 ;
7	49	373 :	12637 ;	19	361	24001 :	13.13.37.97 ;
11	121	4789 :	13.3313 ;	23	529	71881 :	1008901 ;
13	169	11197 :	71413 ;	29	841	241.1021 :	1970401 ;
17	289	61.673 :	37.4561 ;	31	961	13.26557 :	73.33037 ;
19	361	69109 :	244669 ;	37	1369	826621 :	433.9697 ;
23	529	165877 :	13.36241 ;	41	1681	109.12409 :	37.158293 ;
25	625	13.13.1429 :	630901 ;	43, 5	1849, 150	13.61.2137 :	6852061 ;
29	841	13.157.229 :	1069429 ;	5, 6	25, 216	20101 :	106861 ;
31	961	421.1489 :	13.104593 ;	7	49	14029 :	13.11353 ;
35	1225	109.9769 :	769.2749 ;	11	121	13.13.37 :	273157 ;
37	1369	1354813 :	13.193.1033 ;	13	169	4549 :	364909 ;
41	1681	13.241.673 :	3786229 ;	17	289	8389 :	13.48193 ;
43	1849	2586037 :	541.8353 ;	19	361	13.1249 :	805573 ;
47	2209	13.290761 :	6298717 ;	23	529	13.37.109 :	1286149 ;
49, 2	2401, 24	73.61813 :	13.109.5197 ;	25	625	85381 :	1599181 ;
5, 3	25, 54	13.37 :	61.241 ;	29	841	37.5281 :	13.181.1021 ;
7	49	277 :	37.709 ;	31	961	337.829 :	397.7321 ;
11	121	13.193 :	71809 ;	35	1225	13.13.3109 :	97.42853 ;
13	169	6673 :	37.3001 ;	37	1369	13.53593 :	229.21481 ;
17	289	28297 :	238213 ;	41	1681	601.1933 :	13.520129 ;
19	361	49789 :	109.3061 ;	43, 6	1849, 216	61.24049 :	13.373.1621 ;
23	529	73.1741 :	13.61.769 ;	5, 7	25, 294	73.577 :	229.769 ;
25	625	13.14557 :	313.2557 ;	11	121	16069 :	13.73.421 ;
29	841	13.29173 :	1313629 ;	13	169	11257 :	37.61.229 ;
31	961	13.97.409 :	577.2857 ;	17	289	13.661 :	229.3673 ;
35	1225	73.12277 :	157.15973 ;	19	361	37.337 :	13.81373 ;
37	1369	13.73.1213 :	709.4297 ;	23	529	157.241 :	13.125221 ;
41	1681	193.9433 :	13.37.9109 ;	25	625	63361 :	1993261 ;
43	1849	2248333 :	109.47653 ;	29	841	13.61.193 :	2917909 ;
47	2209	3325957 :	13.13.42337 ;	31	961	223549 :	3491569 ;
49, 3	2401, 54	3991369 :	13.640153 ;	37	1369	583753 :	13.442489 ;
5, 4	25, 96	2521 :	37.853 ;	41, 7	1681, 294	13.13.5881 :	37.109.1933 ;

Sext-Aurifeuillian Sextans, S′, [*Species* i].

ξ, η	x, y	L M	ξ, η	x, y	L M
5, 8	25, 384	78721:13.21157;	17, 12	289, 864	157.1069:13.37.6217;
7	49	61.997:13.27061;	19	361	73.1873:13.13.20641;
11	121	34849:97.5857;	23	529	90697:1549.3037;
13	169	25633:715777;	25	625	13.61.97:709.7669;
17	289	14737:13.85621;	29	841	73609:193.37273;
19	361	13.1093:277.4957;	31, 12	961, 864	13.6733:61.134989;
23	529	28753:37.73.757;	5, 13	25, 1014	97.7213:37.40813;
25	625	13.3637:2468881;	7	49	73.8209:37.47569;
29	841	118369:1069.3301;	11	121	337.1297:2384749;
31	961	109.1621:13.37.8689;	17	289	263077:3718633;
35	1225	13.27397:73.193.409;	19	361	37.61.97:457.9397;
37	1369	61.73.109:6712033;	23	529	149113:5685397;
41, 8	1681, 384	845809:61.157.937;	25	625	124021:6516121;
5, 9	25, 486	135301:411241;	29, 13	841, 1014	97789:2473.3433;
7	49	13.8293:512269;	5, 14	25, 1176	97.9973:13.152017;
11	121	66697:13.60601;	11	121	37.16921:3022933;
13	169	51349:970969;	13	169	97.5557:193.18013;
17	289	29629:13.111733;	17	289	13.30313:97.109.433;
19	361	23833:1761877;	19	361	13.61.421:109.48073;
23	529	13.2053:2547949;	23	529	235069:6823189;
25	625	37.1033:3037921;	25, 14	625, 1176	196501:13.596977;
29	841	91573:13.193.1693;	7, 15	49, 1350	61.97.193:1657.1753;
31	961	138577:1549.3217;	11	121	870901:13.291037;
35	1225	13.37.601:73.92557;	13	169	613.1237:4312741;
37, 9	1369, 486	400069:13.600973;	17	289	828901:13.449557;
7, 10	49, 600	178021:723181;	19	361	489061:421.15061;
11	121	73.1597:13.37.2221;	23, 15	529, 1350	355261:13.181.3457;
13	169	92941:157.8233;	5, 16	25, 1536	13.132757:3224401;
17	289	56941:457.4093;	7	49	1522369:3652609;
19	361	109.409:2235661;	11	121	13.90901:4681297;
23	529	13.2617:1381.2281;	13	169	61.17029:877.6037;
29	841	37.1993:13.13.30109;	17	289	37.21517:181.37309;
31, 10	961, 600	61.1801:13.37.12301;	19, 16	361, 1536	577.1201:7613233;
5, 11	25, 726	13.25717:601.1381;	5, 17	25, 1734	2240341:97.41593;
7	49	278149:994249;	7	49	313.6361:13.421.829;
13	169	155809:73.23173;	11	121	13.157.769:5732149;
17	289	101209:2378869;	13, 17	169, 1734	97.14341:241.26713;
19	361	80557:1237.2269;	5, 18	25, 1944	13.220177:37.109.1237;
23	529	13.4153:37.181.577;	7	49	13.61.3229:37.150649;
25	625	49801:13.577.601;	11	121	1321.1549:97.229.313;
29	841	109.613:13.466561;	13, 18	169, 1944	37.49369:409.18973;
31, 11	961, 726	13.73.97:37.189061;	5, 19	25, 2166	37.97453:181.33721;
5, 12	25, 864	13.157.241:1131961;	7	49	37.87697:13.521533;
7	49	13.31981:1336057;	11, 19	121, 2166	13.201889:8360353;
11	121	294649:1854889;	7, 20	49, 2400	13.541.577:1093.7477;
13, 12	169, 864	246217:2179993;	5, 21	25, 2646	5517661:13.683317;

☞ *This Table, pages* 181, 182, *contains all Sext-Aurifeuillian Sextans of Species* i ($x = \xi^2$, $y = 6\eta^2$), *with* $x > 1$ *giving* L, M $\ngtr 9.10^6$.

Sext-Aurifeuillian Sextans, S″, [*Species* ii].

$$S'' = (x^6+y^6) \div (x^2+y^2) = L.M\,; \qquad x = 3\xi^2 \quad y = 2\eta^2.$$

$$P = (x^2+3xy+y^2),\ \ Q = 6\xi\eta\,(x+y)\,; \qquad L = (P-Q),\ \ M = (P+Q)\,; \quad [L \text{ and } M \ngtr 9.10^6].$$

ξ, η	x, y	L M	ξ, η	x, y	L M
1, 1	3, 2	1:61 ;	11, 5	363, 50	13.37.109:325009 ;
3	27	373:13.109 ;	13	507	118369:73.7573 ;
5	75	3769:8389 ;	17	867	61.6829:13.103993 ;
7	147	13.1249:28753 ;	19	1083	13.53233:1983649 ;
9	243	13.3637:37.1993 ;	21	1323	457.2377:2816269 ;
11	363	61.1801:13.12157 ;	23	1587	13.125353:229.16981 ;
13	507	61.3613:409.733 ;	27	2187	1201.2749:6925489 ;
15	675	13.37.829:520609 ;	29, 5	2523, 50	37.73.1669:13.691153 ;
17	867	37.18061:13.193.337 ;	1, 7	3, 98	13.13.37:14737 ;
19	1083	709.1489:13.100237 ;	3	27	2521:13.2617 ;
21	1323	229.6949:37.61.853 ;	5	75	13.73:73609 ;
23	1587	13.177601:1237.2221 ;	9	243	11197:268993 ;
25	1875	97.33457:3808429 ;	11	363	13.37.73:193.2389 ;
27	2187	4441477:5150713 ;	13	507	85381:746041 ;
29	2523	5941321:13.109.4813 ;	15	675	109.1621:13.88513 ;
31, 1	2883, 2	157.49633:8865601 ;	17	867	97.3373:181.9421 ;
1, 2	3, 8	13:277 ;	19	1083	558457:97.25189 ;
3	27	181:37.73 ;	23	1587	13.13.8089:4622461 ;
5	75	13.193:37.337 ;	25	1875	2004829:13.61.7753 ;
7	147	13.937:37.1033 ;	27, 7	2187, 98	13.218797:8026741 ;
9	243	157.241:13.73.97 ;	1, 8	3, 128	11257:23833 ;
11	363	91573:189517 ;	3	27	13.397:49801 ;
13	507	188941:313.1117 ;	5	75	2089:99529 ;
15	675	348949:594829 ;	7	147	13.157:186841 ;
17	867	13.45697:951061 ;	9	243	37.229:13.25309 ;
19	1083	37.61.421:13.193.577 ;	11	363	28297:13.42061 ;
21	1323	181.7993:13.162889 ;	13	507	71881:864361 ;
23	1587	2116501:337.8893 ;	15	675	13.61.193:1309369 ;
25	1875	2995789:13.317353 ;	17	867	13.37.601:1912921 ;
27	2187	13.317257:577.9613 ;	19	1083	500713:13.13.16033 ;
29, 2	2523, 8	5545357:7306933 ;	21	1323	812137:3737353 ;
1, 4	3, 32	13.37:2161 ;	23	1587	73.17137:13.13.13.2293 ;
3	27	97:13.661 ;	25	1875	37.49957:661.10069 ;
5	75	1009:13.2053 ;	27, 8	2187, 128	61.43261:8639401 ;
7	147	6673:109.613 ;	1, 10	3, 200	29629:13.4153 ;
9	243	24001:61.2341 ;	3	27	16069:97789 ;
11	363	63361:13.13.1609 ;	7	147	13.313:337.877 ;
13	507	138577:109.4357 ;	9	243	13.433:109.4441 ;
15	675	13.20533:673.1153 ;	11	363	17989:409.1861 ;
17	867	469153:61.19717 ;	13	507	49789:1021.1129 ;
19	1083	433.1777:733.2437 ;	17	867	223549:13.184633 ;
21	1323	13.91957:1117.2293 ;	19	1083	400069:13.229.1117 ;
23	1587	13.37.3697:61.58453 ;	21	1323	37.17977:13.346393 ;
25	1875	2552449:13.73.5101 ;	23, 10	1587, 200	193.5413:277.21577 ;
27	2187	13.37.7393:6431857 ;	1, 11	3, 242	109.409:13.61.97 ;
29, 4	2523, 32	4830481:13.241.2677 ;	3	27	25633:132157 ;
1, 5	3, 50	37.37:4549 ;	5	75	14029:13.13.1321 ;
3	27	349:13.1093 ;	7	147	7177:13.28201 ;
7	147	4789:13.6733 ;	9	243	13.457:37.15733 ;
9, 5	243, 50	13.1453:177109 ;	13, 11	507, 242	61.673:73.18169 ;

Sext-Aurifeuillian Sextans, S″, [*Species* ii].

ξ, η	x, y	L M	ξ, η	x, y	L M
15, 11	675, 242	229.421:1912069 ;	5, 19	75, 722	235069:13.87973 ;
17	867	37.5281:13.206461 ;	7	147	157.1069:1554757 ;
19	1083	13.27397:3679261 ;	9	243	73.1597:2096761 ;
21	1323	13.61.757:1693.2917 ;	11	363	78721:13.37.5821 ;
23	1587	952873:6505717 ;	13	507	55117:997.3709 ;
25, 11	1875, 242	13.181.613:37.227797 ;	15	675	13.3853:4827829 ;
1, 13	3, 338	90697:37.3889 ;	17, 19	867, 722	71413:1669.3733 ;
3	27	56941:421.541 ;	1, 20	3, 800	13.42373:613.1213 ;
5	75	34849:356989 ;	3	27	193.2113:13.229.337 ;
7	147	20101:241.2281 ;	7	147	37.61.97:73.24793 ;
9	243	11833:733.1129 ;	9	243	155809:2408689 ;
11	363	12637:1215553 ;	11	363	13.8293:421.7549 ;
15	675	69109:109.22381 ;	13	507	74929:37.112237 ;
17	867	73.2017:3342901 ;	17	867	71809:73.94153 ;
19	1083	337.829:4491217 ;	19, 20	1083, 800	157.757:829.10501 ;
21	1323	61.73.109:397.14929 ;	1, 22	3, 968	457.1789:13.82609 ;
23, 13	1587, 338	37.21313:7695481 ;	3	27	13.47857:37.38113 ;
1, 14	3, 392	124021:61.3121 ;	5	75	13.36313:397.4657 ;
3	27	80557:13.22441 ;	7	147	355261:193.12517 ;
5	75	51349:443629 ;	9	243	263077:3140413 ;
9	243	13.13.109:978541 ;	13	507	135301:37.140473 ;
11	363	61.241:13.108457 ;	15	675	99709:13.73.6961 ;
13	507	25237:1988653 ;	17, 22	867, 968	88741:109.76369 ;
15	675	13.4513:37.74257 ;	1, 23	3, 1058	433.2269:13.98101 ;
17	867	73.1741:13.286369 ;	3	27	756601:1654981 ;
19	1083	241.1021:13.381097 ;	5	75	13.61.733:193.11113 ;
23, 14	1587, 392	13.55009:1609.5197 ;	7	147	13.109.313:97.28573 ;
1, 16	3, 512	13.73.229:316201 ;	9	243	13.61.421:3565537 ;
3	27	149113:37.12421 ;	11	363	246217:60373 ;
5	75	101209:13.51133 ;	13	507	178021:109.53149 ;
7	147	66697:13.61.1201 ;	15, 23	675, 1058	37.3517:1304749 ;
9	243	73.577:13.13.13.613 ;	1, 25	3, 1250	1385809:73.24133 ;
11	363	27481:1875481 ;	3	27	13.83833:37.73.829 ;
13	507	37.709:181.14197 ;	7	147	668509:13.37.7489 ;
15	675	45289:3463849 ;	9	243	517249:13.181.1933 ;
17	867	13.7309:4596073 ;	11	363	13.30313:349.16381 ;
19	1083	13.14557:37.397.409 ;	13, 25	507, 1250	294649:7146949 ;
21, 16	1323, 512	13.26557:349.22189 ;	1, 26	3, 1352	1628701:2051461 ;
1, 17	3, 578	13.13.1657:398557 ;	3	27	61.21193:2583517 ;
3	27	196501:13.43597 ;	5	75	1024669:3250789 ;
5	75	73.1873:13.37.1669 ;	7	147	808837:1429.2857 ;
7	147	92941:37.30493 ;	9	243	37.109.157:5111941 ;
9	243	61.997:1568173 ;	11, 26	363, 1352	489061:6374941 ;
11	363	73.541:13.165469 ;	1, 28	3, 1568	13.169909:2736673 ;
13	507	37.853:2908981 ;	3	27	61.29221:601.5641 ;
15	675	13.3313:73.53113 ;	5	75	13.110533:37.113437 ;
19	1083	165877:6603913 ;	9	243	922513:13.277.1777 ;
21, 17	1323, 578	13.23581:1321.6397 ;	11, 28	363, 1568	729457:2029.3877 ;
1, 19	3, 722	445141:13.46957 ;	1, 29	3, 1682	13.61.3217:3137461 ;
3, 19	27, 722	13.61.409:73.73.157 ;	3, 29	27, 1682	2073997:3858193 ;

Continued centre of page 194.

Trin-Aurifeuillian Sextans (T).

$$T = (x^6+y^6)\div(x^2+y^2) = (y^6+3^3.z^6)\div(y^2+3z^2) = L.M\,; \quad [L, M \not> 9.10^6].$$
$$y^2-x^2 = 3z^2\,; \qquad L = (y^2-3yz+3z^2), \quad M = (y^2+3yz+3z^2).$$

y , x , z	L : M	y , x , z	L : M
		2, 1, 1	1:13;
7, 1, 4	13:181;	14, 13, 3	97:349;
13, 11, 4	61:373;	14, 11, 5	61:13.37;
19, 13, 8	97:1009;	26, 23, 7	277:37.37;
31, 23, 12	277:13.193;	26, 1, 15	181:2521;
37, 13, 20	349:4789;	38, 37, 5	13.73:2089;
43, 11, 24	13.37:6673;	38, 11, 21	373:13.397;
49, 47, 8	13.109:3769;	62, 59, 11	2161:13.13.37;
61, 37, 28	13.73:11197;	62, 13, 35	1009:14029;
67, 61, 16	13.157:37.229;	74, 73, 7	13.313:7177;
73, 23, 40	37.37:13.1453;	74, 47, 33	13.109:16069;
79, 71, 20	37.73:13.937;	86, 83, 13	4549:11257;
91, 59, 40	2161:24001;	86, 61, 35	13.157:20101;
91, 37, 48	2089:28297;	98, 71, 39	37.73:25633;
97, 1, 56	2521:13.37.73;	98, 23, 55	13.193:34849;
103, 97, 20	13.433:17989;	122, 121, 9	11833:13.13.109;
109, 107, 12	8389:13.1249;	122, 47, 65	3769:51349;
127, 73, 60	13.313:49789;	134, 109, 45	13.457:73.577;
133, 83, 60	4549:13.37.109;	134, 13, 77	4789:66697;
133, 109, 44	13.457:61.673;	146, 143, 17	14737:29629;
139, 11, 80	13.397:71881;	146, 97, 63	13.433:61.997;
151, 143, 28	37.337:157.241;	158, 131, 51	13.661:56941;
157, 59, 84	13.13.37:85381;	158, 11, 91	6673:92941;
163, 131, 56	13.661:63361;	182, 179, 19	23833:109.409;
169, 73, 88	7177:229.421;	182, 61, 99	37.229:73.1597;
181, 157, 52	12637:69109;	182, 181, 11	27481:73.541;
193, 191, 16	28753:13.3637;	182, 107, 85	8389:101209;
199, 193, 28	25237:13.4513;	194, 169, 55	61.241:78721;
211, 83, 112	11257:13.61.193;	194, 167, 57	13.1093:80557;
217, 167, 80	13.1093:118369;	206, 157, 77	12637:13.8293;
217, 121, 104	11833:73.2017;	206, 37, 117	11197:155809;
223, 169, 84	61.241:73.1741;	218, 143, 95	37.337:73.1873;
229, 13, 132	14029:37.5281;	218, 71, 119	13.937:157.1069;
241, 143, 112	14737:109.1621;	254, 253, 13	55117:74929;
247, 239, 36	37.1033:91573;	254, 107, 133	13.1249:37.61.97;
247, 47, 140	16069:223549;	266, 263, 23	13.4153:90697;
259, 227, 72	13.2053:138577;	266, 47, 143	17989:246217;
259, 253, 32	45289:13.7309;	266, 241, 65	37.853:135301;
271, 121, 140	13.13.109:241.1021;	266, 23, 153	13.1453:263077;
277, 61, 156	20101:337.829;	278, 251, 69	13.2617:149113;
283, 229, 96	37.709:13.14557;	278, 229, 91	37.709:178021;
301, 299, 20	37.1993:61.1801;	302, 227, 115	13.2053:235069;
301, 277, 68	13.3313:165877;	302, 59, 171	24001:13.61.421;
307, 179, 144	23833:13.37.601;	314, 311, 25	13.61.97:124021;
313, 71, 176	25633:13.27397;	314, 193, 143	25237:294649;
331, 181, 160	27481:13.26557;	326, 299, 75	49801:196501;
337, 241, 136	37.853:13.23581;	326, 37, 187	28297:13.30313;
343, 143, 180	29629:400069;	338, 337, 15	99709:37.3517;
349, 251, 140	13.2617:97.3373;	338, 191, 161	28753:355261;
361, 23, 208	34849:61.73.109;	362, 313, 105	13.3853:278149;
367, 359, 44	13.73.97:188941;	362, 1, 209	13.37.73:489061;

Trin-Aurifeuillian Sextans (T).

y, x, z	L	M	y, x, z	L	M
373, 349, 76	71413:	13.13.1429;	386, 239, 175	37.1033:	13.109.313;
379, 347, 88	109.613:	13.20533;	386, 143, 207	157.241:	517249;
397, 181, 204	73.541:	13.13.3109;	398, 277, 165	13.3313:	337.1297;
403, 109, 224	73.577:	583753;	398, 109, 221	61.673:	37.15373;
403, 397, 40	157.757:	73.2953;	422, 419, 29	37.3889:	13.73.229;
409, 313, 152	13.3853:	423097;	422, 253, 195	45289:	97.5557;
421, 179, 220	109.409:	13.61.757;	434, 407, 87	97789:	13.61.409;
427, 299, 176	49801:	500713;	434, 73, 247	49789:	577.1201;
427, 373, 120	71809:	13.29173;	434, 433, 17	13.12853:	241.877;
433, 431, 24	13.12157:	61.3613;	434, 191, 225	13.3637:	37.109.157;
439, 47, 252	51349:	13.55009;	446, 421, 85	106861:	13.25717;
457, 407, 120	13.6733:	61.6829;	446, 83, 253	13.37.109:	729457;
463, 263, 220	13.4153:	37.17977;	458, 409, 119	88741:	13.31981;
469, 131, 260	56941:	37.21313;	458, 383, 145	73609:	13.36313;
469, 253, 228	55117:	13.53593;	482, 479, 31	61.3121:	13.13.1657;
481, 383, 168	73609:	558457;	482, 193, 255	13.4513:	37.21517;
481, 97, 272	61.997:	845809;	494, 347, 203	109.613:	668509;
487, 481, 44	178693:	307261;	494, 131, 275	63361:	61.14401;
499, 13, 288	66697:	61.97.157;	494, 467, 93	132157:	193.2113;
511, 503, 52	189517:	348949;	494, 373, 187	71809:	37.16921;
511, 457, 132	37.3001:	13.97.409;	518, 443, 155	99529:	13.61.733;
523, 491, 104	61.2341:	469153;	518, 11, 299	71881:	1001173;
541, 517, 92	37.4561:	13.157.229;	518, 349, 221	71413:	613.1237;
547, 253, 280	74929:	13.13.5881;	518, 157, 285	69109:	13.73453;
553, 311, 264	13.61.97:	952873;	542, 541, 19	263953:	13.25057;
553, 169, 304	78721:	37.29389;	542, 299, 261	37.1993:	922513;
559, 167, 308	80557:	37.30097;	554, 529, 95	229.769:	13.157.241;
559, 409, 220	88741:	826621;	554, 407, 217	13.6733:	808837;
571, 443, 208	99529:	812137;	566, 517, 133	13.11353:	73.8209;
577, 481, 184	193.601:	753001;	566, 59, 325	85381:	109.10909;
589, 587, 28	409.733:	13.37.829;	602, 359, 279	13.73.97:	409.2689;
589, 11, 340	92941:	13.99577;	602, 239, 319	91573:	13.241.397;
601, 263, 312	90697:	1215769;	602, 481, 209	193.601:	870901;
607, 407, 260	97789:	193.5413;	602, 73, 345	229.421:	13.61.1693;
613, 563, 140	177109:	13.53233;	614, 611, 35	316201:	445141;
619, 107, 352	101209:	13.97.1117;	614, 253, 323	13.7309:	733.1753;
631, 337, 308	99709:	13.97369;	626, 599, 105	421.541:	13.47857;
637, 421, 276	106861:	601.1933;	626, 457, 247	37.3001:	61.17029;
637, 613, 100	244669:	421.1489;	662, 661, 21	37.10753:	13.37021;
643, 157, 360	13.8293:	181.8269;	662, 299, 341	61.1801:	1129.1297;
661, 61, 380	73.1597:	13.124897;	674, 649, 105	13.21157:	97.7213;
673, 577, 200	169129:	13.75133;	674, 167, 377	118369:	109.15073;
679, 671, 60	313.1117:	13.45697;	686, 683, 37	398557:	13.42373;
679, 673, 52	37.9817:	575077;	686, 397, 323	157.757:	13.111409;
691, 659, 120	13.13.1609:	433.1777;	698, 671, 111	13.22441:	756601;
703, 311, 364	124021:	1659373;	698, 169, 391	73.1741:	109.16189;
703, 649, 156	238213:	73.12277;	722, 647, 185	13.13.1321:	1024669;
709, 467, 308	132157:	13.181.613;	722, 601, 231	157.1153:	13.90901;
721, 143, 408	73.1873:	769.2473;	734, 491, 315	61.2341:	13.37.3181;
721, 337, 368	37.3517:	13.132469;	734, 227, 403	138577:	1913389;

Trin-Aurifeuillian Sextans (T).

y , x , z	L M	y , x , z	L M
727, 241, 396	135301:13.143281 ;	746, 577, 273	169129:97.14341 ;
733, 517, 300	13.11353:61.24049 ;	746, 121, 425	73.2017:13.37.4261 ;
739, 611, 240	186841:73.17137 ;	758, 611, 259	186841:1364773 ;
751, 601, 260	157.1153:109.12409 ;	758, 83, 435	13.61.193:2131429 ;
757, 419, 364	37.3889:1797181 ;	794, 793, 23	37.15601:109.6301 ;
763, 251, 416	149113:241.8521 ;	794, 431, 385	13.12157:1992181 ;
763, 37, 440	155809:13.13.12841 ;	806, 781, 115	411241:97.9973 ;
769, 767, 32	520609:37.18061 ;	806, 563, 333	177109:241.7417 ;
787, 781, 56	13.38197:760993 ;	806, 517, 357	37.4561:61.31069 ;
793, 743, 160	325009:457.2377 ;	806, 277, 437	165877:97.23497 ;
793, 71, 456	157.1069:2337481 ;	818, 769, 161	13.27061:61.97.193 ;
811, 757, 168	109.3061:13.73.1213 ;	818, 143, 465	109.1621:37.66457 ;
817, 719, 224	268993:13.13.8089 ;	842, 839, 41	13.46957:457.1789 ;
817, 433, 400	13.12853:97.21937 ;	842, 481, 399	178693:2194441 ;
823, 529, 364	229.769:61.32353 ;	854, 827, 123	37.12421:13.83833 ;
829, 229, 460	178021:13.189697 ;	854, 229, 475	13.14557:2623141 ;
853, 829, 116	13.36241:109.9769 ;	854, 733, 253	273157:13.157.769 ;
859, 709, 280	97.2593:13.61.2137 ;	854, 13, 493	37.5281:97.28057 ;
871, 863, 68	594829:37.61.421 ;	866, 503, 407	189517:13.157.1129 ;
871, 479, 420	61.3121:157.15193 ;	866, 359, 455	188941:229.11149 ;
877, 299, 476	196501:541.4993 ;	878, 803, 205	356989:13.110533 ;
883, 851, 136	109.4357:13.91957 ;	878, 709, 299	97.2593:37.49369 ;
889, 647, 352	13.13.1321:157.13381 ;	914, 767, 287	337.877:73.25609 ;
889, 793, 232	333049:181.8677 ;	914, 47, 527	223549:13.239509 ;
907, 107, 520	37.61.97:3048769 ;	926, 923, 43	613.1213:433.2269 ;
919, 433, 468	241.877:37.61.1237 ;	926, 397, 483	73.2953:13.223009 ;
931, 803, 272	13.25309:37.49957 ;	938, 911, 129	13.43597:61.21193 ;
931, 419, 480	13.73.229:2898601 ;	938, 649, 391	238213:13.187597 ;
937, 599, 416	421.541:2566513 ;	938, 937, 25	13.13.4801:193.4933 ;
949, 227, 532	235069:313.10429 ;	938, 431, 481	61.3613:709.4129 ;
949, 733, 348	273157:37.60937 ;	962, 887, 215	443629:1684609 ;
961, 97, 552	246217:13.37.7129 ;	962, 121, 551	241.1021:3426433 ;
967, 767, 340	337.877:2268229 ;	962, 913, 175	512269:1522369 ;
973, 971, 36	13.193.337:709.1489 ;	962, 719, 369	268993:2398861 ;
973, 949, 124	630901:1354813 ;	974, 613, 437	244669:13.61.3529 ;
991, 23, 572	263077:3664189 ;	974, 349, 525	13.13.1429:73.45337 ;
997, 947, 180	73.7573:13.125353 ;	998, 877, 275	13.73.421:1321.1549 ;
1009, 913, 248	451897:13.97.1549 ;	998, 851, 301	13.28201:1069.2029 ;
1021, 923, 252	193.2389:2004829 ;	1022, 659, 451	13.13.1609:3037453 ;
1027, 541, 504	263953:733.4597 ;	1022, 347, 555	13.20533:97.157.241 ;
1027, 1021, 64	37.23509:97.13033 ;	1022, 853, 325	364909:2357809 ;
1033, 649, 464	13.21157:3150913 ;	1022, 61, 589	337.829:13.299317 ;
1039, 313, 572	278149:1213.3169 ;	1046, 803, 387	13.25309:2757829 ;
1051, 997, 192	13.61.769:193.9433 ;	1046, 179, 595	13.37.601:229.17569 ;
1057, 479, 544	13.13.1657:37.73.1381 ;	1082, 793, 425	333049:769.4021 ;
1057, 193, 600	294649:13.315373 ;	1082, 241, 609	13.23581:997.4273 ;
1063, 671, 476	13.22441:109.30529 ;	1094, 1093, 27	397.2797:13.37.2677 ;
1069, 853, 372	364909:13.61.3469 ;	1094, 587, 533	409.733:61.73.853 ;
1087, 1079, 76	951061:181.7993 ;	1106, 1103, 47	13.82609:1385809 ;
1093, 851, 396	13.28201:73.40597 ;	1106, 481, 575	307261:541.7621 ;

Trin-Aurifeuillian Sextans (T).

y, x, z	L : M	y, x, z	L : M
1099, 1067, 152	673.1153:13.37.3697 ;	1106, 1081, 135	601.1381:13.132757 ;
1099, 949, 320	459961:1381.1861 ;	1106, 743, 473	325009:13.266449 ;
1117, 59, 644	13.61.421:4649941 ;	1118, 1091, 141	73.73.157:61.29221 ;
1123, 611, 544	316201:313.12721 ;	1118, 757, 475	109.3061:3519949 ;
1129, 407, 608	13.61.409:4442929 ;	1118, 1069, 189	723181:313.6361 ;
1141, 803, 468	356989:37.157.613 ;	1118, 251, 629	97.3373:61.73.1021 ;
1141, 541, 580	13.25057:97.44293 ;	1142, 1067, 235	13.51133:2274949 ;
1147, 1019, 304	13.42061:61.43261 ;	1142, 181, 651	13.26557:37.193.673 ;
1147, 421, 616	13.25717:4573633 ;	1154, 1033, 297	97.5857:13.201889 ;
1153, 769, 496	13.27061:433.8737 ;	1154, 71, 665	13.27397:757.6553 ;
1159, 191, 660	355261:13.380377 ;	1178, 671, 559	313.1117:4300633 ;
1159, 1153, 68	13.86209:1593589 ;	1178, 503, 615	348949:13.361213 ;
1171, 877, 448	13.73.421:3547177 ;	1178, 1031, 329	241.2281:13.109.2029 ;
1183, 983, 380	109.4441:97.32797 ;	1178, 1009, 351	37.61.229:2997721 ;
1183, 1129, 204	313.2557:2248333 ;	1202, 1199, 49	13.98101:1628701 ;
1201, 1199, 40	13.100237:229.6949 ;	1202, 673, 575	37.9817:13.346933 ;
1213, 37, 700	13.30313:5488669 ;	1214, 1187, 147	13.229.337:2073997 ;
1231, 1081, 340	13.13.37.97:3117781 ;	1214, 373, 667	13.29173:73.157.457 ;
1237, 1213, 140	1069429:13.241.673 ;	1226, 983, 423	109.4441:13.276589 ;
1249, 1151, 280	746041:13.218797 ;	1226, 143, 703	400069:5571337 ;
1261, 683, 612	398557:5028949 ;	1238, 1163, 245	13.37.1669:109.24061 ;
1261, 661, 620	37.10753:1669.3049 ;	1238, 949, 459	459961:61.229.277 ;
1267, 467, 680	193.2113:13.13.61.541 ;	1262, 1261, 29	1485373:1704961 ;
1267, 781, 576	411241:13.229.1609 ;	1262, 587, 645	13.37.829:5282689 ;
1273, 409, 696	13.31981:5731801 ;	1274, 1249, 145	1131961:2240341 ;
1273, 1177, 280	13.60493:2925049 ;	1274, 407, 697	61.6829:97.59221 ;
1279, 887, 532	443629:13.397.877 ;	1274, 913, 513	451897:4373269 ;
1291, 277, 728	337.1297:37.97.1693 ;	1274, 313, 713	423097:37.181.877 ;
1297, 239, 736	13.109.313:6171073 ;	1286, 1237, 203	994249:13.61.3229 ;
1303, 1009, 476	37.61.229:13.97.3361 ;	1286, 923, 517	193.2389:37.120277 ;
1321, 1079, 440	37.15733:829.4909 ;	1322, 1201, 319	13.60601:601.5521 ;
1327, 1319, 84	13.193.577:2116501 ;	1322, 1079, 441	37.15733:4080133 ;
1333, 611, 684	445141:5915773 ;	1346, 1177, 377	715777:661.5689 ;
1333, 1117, 420	13.48193:3985669 ;	1346, 23, 777	61.73.109:13.433.1201 ;
1339, 1307, 168	61.19717:2552449 ;	1358, 851, 611	109.4357:5453341 ;
1339, 827, 608	37.12421:5344249 ;	1358, 491, 731	469153:13.494257 ;
1351, 383, 748	13.36313:61.107137 ;	1358, 829, 621	13.36241:5531041 ;
1351, 1, 780	489061:6811741 ;	1358, 517, 725	13.157.229:6374689 ;
1369, 1031, 520	241.2281:1777.2713 ;	1382, 1019, 539	13.42061:5016181 ;
1381, 1357, 148	13.104593:2586037 ;	1382, 299, 779	500713:37.313.601 ;
1387, 1259, 336	864361:13.281581 ;	1406, 1403, 53	73.24133:13.169909 ;
1387, 661, 704	13.37021:457.13873 ;	1406, 781, 675	13.38197:193.32077 ;
1393, 143, 800	517249:7203649 ;	1406, 1117, 493	13.48193:313.15289 ;
1393, 529, 744	13.157.241:73.91921 ;	1406, 181, 805	13.13.3109:61.277.433 ;
1399, 913, 612	512269:13.73.5953 ;	1418, 1391, 159	37.38113:2762953 ;
1417, 1033, 560	97.5857:5329249 ;	1418, 457, 775	13.97.409:7109449 ;
1417, 1321, 296	241.2401:61.57853 ;	1442, 1367, 265	13.87973:1213.2833 ;
1423, 1223, 420	409.1861:13.334393 ;	1442, 1081, 551	13.13.37.97:5373793 ;
1429, 253, 812	97.5557:13.577009 ;	1442, 1441, 31	13.277.541:73.97.313 ;
1447, 1441, 76	13.181.757:2441053 ;	1442, 767, 705	520609:61.108529 ;

Trin-Aurifeuillian Sextans (T).

y, x, z	L : M	y, x, z	L : M
1453, 1451, 44	37.61.853:13.177601 ;	1454, 1429, 155	37.40813:13.220177 ;
1459, 109, 840	37.15373:13.609397 ;	1454, 947, 637	73.7573:181.33757 ;
1471, 1417, 228	1313629:3325957 ;	1466, 1417, 217	1336057:37.87697 ;
1477, 1427, 220	13.10393:1201.2749 ;	1466, 383, 817	558457:13.73.8161 ;
1477, 1261, 444	805573:109.157.277 ;	1478, 1331, 371	13.61.1201:4242421 ;
1483, 683, 760	13.42373:409.17881 ;	1478, 109, 851	583753:13.13.48109 ;
1489, 911, 680	13.43597:97.68473 ;	1502, 1381, 341	13.37.2221:337.12289 ;
1501, 1403, 308	13.88513:3924517 ;	1502, 179, 861	13.61.757:8359633 ;
1501, 443, 828	13.61.733:8038237 ;	1514, 1511, 55	2051461:13.61.3217 ;
1519, 793, 748	37.15601:7394509 ;	1514, 673, 783	575077:13.73.8101 ;
1519, 1369, 380	1008901:13.344017 ;	1526, 1499, 165	1654981:13.243517 ;
1531, 517, 832	73.8209:37.337.661 ;	1526, 997, 667	13.61.769:37.181537 ;
1543, 191, 884	37.109.157:8817253 ;	1526, 1357, 403	970969:229.20353 ;
1549, 373, 868	37.16921:13.668713 ;	1526, 1283, 477	733.1129:13.399613 ;
1561, 839, 760	13.46957:7728601 ;	1538, 863, 735	594829:13.567493 ;
1561, 1177, 592	715777:13.337.1429 ;	1538, 671, 799	13.45697:193.41281 ;
1567, 599, 836	13.47857:37.229249 ;	1574, 949, 725	630901:7477801 ;
1579, 1067, 672	13.51133:7031257 ;	1574, 613, 837	421.1489:13.37.17737 ;
1591, 1583, 92	13.162889:2995789 ;	1586, 1223, 583	409.1861:97.193.337 ;
1591, 1297, 532	*229.3673:13.455353 ;	1586, 1439, 385	37.30493:4791901 ;
1603, 1571, 184	733.2437:13.37.7393 ;	1586, 1297, 527	229.3673:337.17377 ;
1603, 1597, 80	73.109.277:13.228733 ;	1622, 1261, 589	805573:229.28549 ;
1621, 1283, 572	733.1129:6390829 ;	1634, 1391, 495	978541:5831521 ;
1627, 1573, 240	577.2857:3991369 ;	1634, 1633, 33	13.193189:2834989 ;
1651, 1523, 368	1309369:4954777 ;	1634, 767, 833	37.18061:8834989 ;
1669, 1069, 740	723181:13.625657 ;	1646, 1621, 165	13.152017:37.97453 ;
1687, 1487, 460	1021.1129:13.73.6121 ;	1658, 1609, 231	37.47569:13.541.577 ;
1687, 1201, 684	13.60601:7711261 ;	1658, 1151, 689	746041:157.48409 ;
1693, 1669, 164	769.2749:13.290761;	1706, 1177, 713	13.60493:73.110749 ;
1729, 1727, 48	1237.2221:97.33457 ;	1718, 1549, 429	157.8233:181.31573 ;
1729, 1633, 328	661.2437:61.82189 ;	1742, 1739, 59	2736673:3353341 ;
1741, 1163, 748	13.37.1669:8616397 ;	1754, 1727, 177	37.73.829:13.315529 ;
1747, 1453, 560	13.81373:37.187237 ;	1754, 1129, 775	313.2557:13.688957 ;
1753, 1703, 240	1983649:37.73.1669 ;	1766, 1259, 715	864361:1549.5449 ;
1777, 1679, 336	181.9421:13.37.10993 ;	1778, 1703, 295	397.4657:4995889 ;
1783, 1391, 644	978541:7868053 ;	1778, 1489, 561	13.85621:37.191833 ;
1789, 1573, 492	1286149:6567277 ;	1814, 1667, 413	1554757:13.465373 ;
1843, 1357, 720	970969:13.673.1021 ;	1814, 1453, 627	13.81373:109.72313 ;
1849, 1607, 528	73.18169:13.241.2293 ;	1838, 1837, 35	3188929:13.274993 ;
1861, 1837, 172	13.193.1033:73.61813 ;	1862, 1619, 531	13.13.13.613:349.20857 ;
1873, 1489, 656	13.85621:8485201 ;	1862, 1859, 61	3137461:3818953 ;
1879, 1871, 100	337.8893:13.317257 ;	1874, 1847, 183	2583517:13.37.9649 ;
1891, 1859, 200	1117.2293:4830481 ;	1898, 1823, 305	193.11113:5618149 ;
1897, 1559, 624	1215553:2557.3253 ;	1898, 1777, 385	1854889:109.57241 ;
1897, 1801, 344	1995913:13.73.6229 ;	1922, 1753, 455	73.22173:61.113749 ;
1939, 1811, 400	1912921:13.505117 ;	1922, 1559, 649	1215553:13.541.1237 ;
1939, 1933, 88	73.44809:13.37.8929 ;	1934, 1787, 427	73.24793:13.520369 ;
1957, 1741, 516	1599181:13.97.6073 ;	1982, 1739, 549	1568173:13.97.6421 ;
1963, 1909, 264	157.15973:397.14149 ;	1982, 1693, 595	13.111733:109.78241 ;
2011, 1861, 440	1970401:13.313.1789 ;	2054, 2053, 37	3995029:4451017 ;

Continued at foot of page 194.

* The values of y are continuous to $y = \omega = 1591$, and $y = \epsilon = 1586$. Afterwards only those values of y are included which give L and M $\not> 9.10^6$.

Dimorph Sextans (D).

$$D = (x^6+y^6)\div(x^2+y^2) = (x^6+z^6)\div(x^2+z^2) = L.M; \quad [L, M \not> 9.10^6].$$

$$x^2 = y^2+z^2; \qquad L = x^2-yz, \quad M = x^2+yz.$$

x, y, z	L : M	x, y, z	L : M
5, 3, 4	13:37;	317, 75, 308	13·5953:73·1693;
13, 5, 12	109:229;	325, 323, 36	93997:37·3169;
17, 15, 8	13.13:409;	325, 253, 204	54013:97·1621;
25, 7, 24	457:13·61;	337, 175, 288	181.349:13.12613;
29, 21, 20	421:13·97;	349, 299, 180	157·433:175621;
37, 35, 12	13·73:1789;	353, 225, 272	63409:13.14293;
41, 9, 40	1321:13·157;	365, 27, 364	123397:143053;
53, 45, 28	1549:13·313;	365, 357, 76	13.8161:160357;
61, 11, 60	3061:13·337;	373, 275, 252	69829:13.16033;
65, 63, 16	3217:5233;	377, 135, 352	37·2557:61·3109;
65, 33, 56	2377:6073;	377, 345, 152	89689:194569;
73, 55, 48	2689:13·613;	389, 189, 340	13·37·181:193·1117;
85, 13, 84	6133:8317;	397, 325, 228	37·37·61:231709;
85, 77, 36	61.73:13·769;	401, 399, 40	241.601:13.13597;
89, 39, 80	4801:61.181;	409, 391, 120	37·3253:13·16477;
97, 65, 72	4729:73·193;	421, 29, 420	13.12697:189421;
101, 99, 20	8221:13·937;	425, 87, 416	97·1489:157·1381;
109, 91, 60	6421:17341;	425, 297, 304	13.6949:270913;
113, 15, 112	13·853:14449;	433, 145, 408	181.709:13.18973;
125, 117, 44	10477:20773;	445, 203, 396	13.9049:278413;
137, 105, 88	13·733:37·757;	445, 437, 84	13.12409:234733;
145, 143, 24	73·241:37·661;	449, 351, 280	277·373:299881;
145, 17, 144	13.1429:23473;	457, 425, 168	13·97.109:280249;
149, 51, 140	15061:13·37·61;	461, 261, 380	113341:13.23977;
157, 85, 132	13·1033:35869;	481, 319, 360	109.1069:346201;
169, 119, 120	14281:42841;	481, 31, 480	216481:246241;
173, 165, 52	37·577:97·397;	485, 483, 44	213973:13.109.181;
181, 19, 180	13·37·61:97·373;	485, 93, 476	13·37·397:277.1009;
185, 57, 176	13.1861:44257;	493, 475, 132	13.13873:305749;
185, 153, 104	18313:181.277;	493, 155, 468	170509:315589;
193, 95, 168	61.349:13·4093;	505, 217, 456	97.1609:13·73·373;
197, 195, 28	33349:44269;	505, 377, 336	37·3469:381697;
205, 187, 84	26317:13·4441;	509, 459, 220	37·4273:13·27697;
205, 133, 156	21277:62773;	521, 279, 440	13.11437:394201;
221, 171, 140	37·673:73·997;	533, 435, 308	37·4057:418069;
221, 21, 220	44221:193·277;	533, 525, 92	235789:61·5449;
229, 221, 60	39181:65701;	541, 341, 420	13.11497:349·1249;
233, 105, 208	37·877:76129;	545, 33, 544	279073:13·24229;
241, 209, 120	61.541:13·6397;	545, 513, 184	97·2089:13·30109;
257, 255, 32	13·61·73:74209;	557, 165, 532	13.109.157:398029;
265, 247, 96	193·241:93937;	565, 403, 396	61.2617:478813;
265, 23, 264	64153:13·5869;	565, 493, 276	13·73·193:109.4177;
269, 69, 260	54421:73·1237;	569, 231, 520	203641:443881;
277, 115, 252	13·3673:37·2857;	577, 575, 48	305329:13·27733;
281, 231, 160	97·433:13·37·241;	593, 465, 368	73·2473:13·40213;
289, 161, 240	37·1213:13·9397;	601, 551, 240	228961:13·37957;
293, 285, 68	13·5113:105229;	613, 35, 612	37·61·157:13·30553;
305, 207, 224	13·37·97:139393;	617, 105, 608	13·24373:444529;
305, 273, 136	55897:157·829;	625, 527, 336	213553:13·43669;
313, 25, 312	37·2437:105769;	629, 429, 460	198301:61.9721;

Dimorph Sextans (D).

x , y , z	L : M	x , y , z	L : M
629, 621, 100	13.25657:397.1153;	949, 301, 900	629701:241.4861;
641, 609, 200	13.37.601:73.7297;	953, 615, 728	61.7549:1129.1201;
653, 315, 572	73.3373:606589;	965, 387, 884	577.1021:1273333;
661, 589, 300	13.37.541:601.1021;	965, 957, 124	37.21961:13.80761;
673, 385, 552	13.18493:61.10909;	977, 945, 248	337.2137:13.91453;
677, 675, 52	423229:61.8089;	985, 473, 864	561553:13.73.1453;
685, 667, 156	365173:573277;	985, 697, 696	485113:13.111949;
685, 37, 684	443917:13.109.349;	997, 925, 372	13.49993:1338109;
689, 111, 680	399241:73.7537;	1009, 559, 840	548521:1487641;
689, 561, 400	193.1297:61.73.157;	1013, 45, 1012	13.241.313:61.17569;
697, 455, 528	37.6637:109.6661;	1021, 779, 660	73.7237:13.119737;
697, 185, 672	193.1873:13.46933;	1025, 1023, 64	13.75781:73.15289;
701, 651, 260	61.5281:660661;	1025, 897, 496	109.5557:61.24517;
709, 259, 660	157.2113:13.51817;	1033, 1015, 192	13.13.13.397:1261969;
725, 627, 364	297397:433.1741;	1037, 315, 988	764149:109.12721;
725, 333, 644	311173:13.56929;	1037, 645, 812	13.42433:1599109;
733, 725, 108	277.1657:13.47353;	1049, 999, 320	780721:13.313.349;
745, 407, 624	301057:808993;	1061, 861, 620	591901:13.127657;
745, 713, 216	401017:13.54541;	1069, 731, 780	572581:61.28081;
757, 595, 468	97.3037:709.1201;	1073, 975, 448	714529:1093.1453;
761, 39, 760	549481:37.16453;	1073, 495, 952	61.11149:13.13.9601;
769, 481, 600	277.1093:879961;	1093, 1085, 132	37.157.181:13.102913;
773, 195, 748	451669:373.1993;	1097, 585, 928	660529:37.109.433;
785, 783, 56	13.44029:660073;	1105, 817, 744	613177:37.49429;
785, 273, 736	73.5689:817153;	1105, 1073, 264	61.15373:1504297;
793, 775, 168	37.13477:433.1753;	1105, 943, 576	677857:1764193;
793, 665, 432	341569:916129;	1105, 47, 1104	1169137:1272913;
797, 555, 572	61.5209:952669;	1109, 141, 1100	673.1597:13.106537;
809, 759, 280	13.33997:867001;	1117, 235, 1092	991069:37.109.373;
821, 429, 700	97.3853:541.1801;	1129, 329, 1080	13.70717:37.44053;
829, 629, 540	13.26737:241.4261;	1145, 903, 704	675313:13.149749;
841, 41, 840	13.73.709:741721;	1145, 423, 1064	37.23269:13.135469;
845, 123, 836	97.6301:181.4513;	1153, 1025, 528	788209:13.37.3889;
845, 837, 116	616933:61.13297;	1157, 1155, 68	37.34057:1417189;
853, 205, 828	13.13.3301:897349;	1157, 765, 868	157.4297:2002669;
857, 825, 232	13.37.1129:925849;	1165, 1147, 204	673.1669:13.122401;
865, 703, 504	13.157.193:1102537;	1165, 517, 1044	709.1153:13.337.433;
865, 287, 816	13.39541:109.9013;	1181, 1131, 340	61.16561:1779301;
877, 805, 348	241.2029:13.80713;	1189, 611, 1020	790501:2036941;
881, 369, 800	13.36997:109.9829;	1189, 989, 660	13.58537:2066461;
901, 899, 60	13.97.601:865741;	1193, 855, 832	711889:2134609;
901, 451, 780	37.12433:1163581;	1201, 49, 1200	829.1669:13.37.3121;
905, 663, 616	410617:349.3517;	1205, 1107, 476	13.71161:73.27109;
905, 777, 464	13.13.2713:1179553;	1205, 147, 1196	1276213:1627837;
925, 43, 924	13.62761:895357;	1213, 245, 1188	13.90793:1762429;
925, 533, 756	109.4153:313.4021;	1217, 705, 992	13.60133:1009.2161;
929, 129, 920	109.6829:13.13.37.157;	1229, 1221, 140	109.12289:13.13.9949;
937, 215, 912	13.52453:73.14713;	1237, 1075, 612	872269:13.37.4549;
941, 741, 580	455701:397.3313;	1241, 441, 1160	13.61.1297:2051641;
949, 851, 420	181.3001:337.3733;	1241, 1209, 280	541.2221:37.50773;

Dimorph Sextans (D).

x , y , z	L M	x , y , z	L M
1249, 799, 960	13.181.337:37.109.577;	1565, 1173, 1036	1093.1129:13.61.4621;
1261, 539, 1140	975661:1093.2017;	1585, 1007, 1224	1279657:13.288061;
1261, 1189, 420	61.17881:37.56473;	1585, 1457, 624	1603057:3421393;
1277, 1035, 748	856549:13.184993;	1597, 715, 1428	1529389:3571429;
1285, 637, 1116	373.2521:37.63841;	1601, 1599, 80	2435281:37.72733;
1285, 893, 924	826093:13.13.14653;	1609, 1591, 240	61.97.373:13.228517;
1289, 1161, 560	13.77797:2311681;	1613, 1275, 988	1342069:1117.3457;
1297, 1295, 72	193.8233:13.136573;	1621, 1421, 780	1519261:181.20641;
1301, 51, 1300	1626301:97.18133;	1625, 57, 1624	337.7561:73.37441;
1313, 735, 1088	313.2953:97.26017;	1625, 1113, 1184	73.18121:1021.3877;
1313, 255, 1288	37.37717:2052409;	1637, 285, 1612	2220349:3139189;
1321, 1271, 360	13.97.1021:2202601;	1649, 1551, 560	13.142357:3587761;
1325, 987, 884	883117:2628133;	1649, 399, 1600	2080801:13.258277;
1325, 357, 1276	61.21313:13.37.4597;	1657, 935, 1368	13.37.3049:4024729;
1345, 1247, 504	1180537:13.97.1933;	1669, 1219, 1140	13.107377:277.15073;
1345, 833, 1056	397.2341:13.206821;	1681, 1519, 720	13.13.37.277:3919441;
1361, 561, 1240	1156681:13.195997;	1685, 627, 1564	13.142969:3819853;
1369, 1081, 840	13.74317:2782201;	1685, 1677, 164	1249.2053:37.73.1153;
1373, 1365, 148	577.2917:97.21517;	1693, 1045, 1332	61.24169:13.327553;
1381, 931, 1020	13.73.1009:109.26209;	1697, 1665, 328	2333689:13.263533;
1385, 663, 1216	1112017:73.37321;	1709, 741, 1540	1779541:181.22441;
1385, 1353, 296	13.313.373:829.2797;	1717, 1325, 1092	241.6229:61.109.661;
1405, 53, 1404	1069.1777:109.18793;	1717, 1645, 492	2138749:13.289033;
1405, 1333, 444	13.106321:73.35149;	1721, 1479, 880	13.127717:4263361;
1409, 159, 1400	1762681:13.169837;	1733, 1155, 1292	13.13.8941:193.23293;
1417, 1175, 792	1077289:2938489;	1741, 59, 1740	2928421:13.193.1249;
1417, 265, 1392	61.97.277:37.64237;	1745, 177, 1736	109.25117:13.257869;
1429, 371, 1380	13.37.3181:2554021;	1745, 1617, 656	37.53629:13.315829;
1433, 1305, 592	13.98533:73.38713;	1753, 295, 1728	13.37.73.73:3582769;
1445, 1443, 76	157.12601:2197693;	1765, 1763, 84	13.13.97.181:61.61.877;
1445, 477, 1364	13.110569:2738653;	1765, 413, 1716	37.193.337:3823933;
1453, 1435, 228	13.37.3709:1069.2281;	1769, 1431, 1040	157.10453:4617601;
1465, 1127, 936	397.2749:61.97.541;	1769, 969, 1480	877.1933:13.351037;
1465, 583, 1344	13.37.2833:157.18661;	1777, 1265, 1248	1579009:4736449;
1469, 1419, 380	1618741:1129.2389;	1781, 531, 1700	61.37201:1777.2293;
1469, 1269, 740	1218901:3097021;	1781, 1581, 820	1875541:241.18541;
1481, 969, 1120	13.85237:1597.2053;	1789, 1739, 420	2470141:13.61.4957;
1489, 689, 1320	1307641:97.32233;	1801, 649, 1680	13.73.2269:37.117133;
1493, 1395, 532	1486909:13.13.17581;	1825, 767, 1656	2060473:2017.2281;
1513, 55, 1512	13.169693:37.97.661;	1825, 1537, 984	37.157.313:13.13.28657;
1513, 1225, 888	13.92413:3376969;	1853, 885, 1628	241.8269:13.374953;
1517, 795, 1292	1274149:13.256033;	1853, 1845, 172	13.239713:37.101377;
1517, 165, 1508	349.5881:73.181.193;	1861, 61, 1860	3349861:13.73.3769;
1525, 1363, 684	1393333:13.73.3433;	1865, 183, 1856	13.241429:337.11329;
1525, 1517, 156	2088973:2562277;	1865, 1833, 344	181.15733:61.193.349;
1537, 385, 1488	13.137653:2935249;	1873, 305, 1848	1657.1777:13.97.3229;
1537, 1505, 312	37.51157:109.25981;	1877, 1485, 1148	13.61.2293:5227909;
1549, 901, 1260	73.17317:13.271897;	1885, 1643, 924	2035093:61.83137;
1553, 495, 1472	1683169:13.37.6529;	1885, 1003, 1596	1952437:5154013;
1565, 1323, 836	1343197:13.13.109.193;	1885, 427, 1836	97.28549:541.8017;

Dimorph Sextans (D).

x, y, z	L	M	x, y, z	L	M
1885, 1813, 516	2617717:	313·14341 ;	2225, 1647, 1496	2486713:	13·13·73·601 ;
1889, 1311, 1360	37·73·661:	13·411637 ;	2225, 2193, 376	13·433·733:	577·1009 ;
1901, 549, 1820	2614621:	241·19141 ;	2237, 1995, 1012	13·277·829:	7023109 ;
1913, 1785, 688	2431489:	13·13·28921 ;	2245, 67, 2244	13·13·28933:	73·97·733 ;
1921, 671, 1800	13·13·37·397:	181·27061 ;	2245, 2173, 564	229·16657:	13·349·1381 ;
1921, 1121, 1560	1941481:	313·17377 ;	2249, 201, 2240	1777·2593:	5508241 ;
1933, 1595, 1092	313·6373:	277·19777 ;	2249, 1449, 1720	61·42061:	997·7573 ;
1937, 1935, 88	3581689:	3922249 ;	2257, 335, 2232	13·334333:	757·7717 ;
1937, 1425, 1312	1882369:	673·8353 ;	2257, 1105, 1968	97·30097:	229·31741 ;
1945, 1927, 264	13·61·4129:	541·7933 ;	2269, 469, 2220	13·315937:	6189541 ;
1945, 793, 1776	97·24481:	5191393 ;	2273, 2145, 752	313·11353:	397·17077 ;
1949, 1749, 860	13·176497:	109·48649 ;	2281, 1769, 1440	13·37·5521:	829·9349 ;
1961, 1911, 440	3004681:	97·48313 ;	2285, 1947, 1196	601·4813:	1297·5821 ;
1961, 1239, 1520	733·2677:	13·440677 ;	2285, 603, 2204	13·299401:	1801·3637 ;
1973, 915, 1748	2293309:	13·241·1753 ;	2293, 1235, 1932	2871829:	7643869 ;
1985, 1887, 616	2777833:	13·13·109·277 ;	2297, 1575, 1672	13·203293:	7909609 ;
1985, 63, 1984	1597·2389:	13·312709 ;	2305, 2303, 96	5091937:	13·425701 ;
1993, 1705, 1032	37·59797:	13·440893 ;	2305, 737, 2184	3703417:	6922633 ;
1997, 315, 1972	3366829:	13·354553 ;	2309, 2109, 940	13·73·3529:	1741·4201 ;
2005, 1037, 1716	2240533:	5799517 ;	2329, 2279, 480	4330321:	13·13·38569 ;
2005, 1357, 1476	13·155161:	61·98737 ;	2329, 871, 2160	337·10513:	7305601 ;
2017, 1855, 792	13·199933:	229·24181 ;	2333, 1365, 1892	2860309:	8025469 ;
2029, 2021, 180	13·288697:	4480621 ;	2341, 1891, 1380	2870701:	13·13·47869 ;
2041, 1159, 1680	2218561:	73·83737 ;	2353, 2255, 672	4021249:	157·44917 ;
2041, 2009, 360	3442441:	37·229·577 ;	2353, 2065, 1128	3207289:	7865929 ;
2045, 693, 1924	2848693:	5515357 ;	2357, 1005, 2132	1597·2137:	37·208057 ;
2045, 1653, 1204	13·168601:	229·26953 ;	2377, 1495, 1848	37·73·1069:	8412889 ;
2053, 1475, 1428	13·241·673:	337·18757 ;	2381, 69, 2380	13·423457:	313·18637 ;
2069, 819, 1900	2724661:	37·73·2161 ;	2389, 1139, 2100	3315421:	13·623017 ;
2081, 1281, 1640	13·171517:	6431401 ;	2393, 345, 2368	13·377653:	61·107269 ;
2089, 1961, 720	13·97·2341:	5775841 ;	2405, 1827, 1564	61·47977:	8641453 ;
2105, 1767, 1144	97·24841:	109·59197 ;	2405, 2397, 196	5314213:	73·85669 ;
2105, 1593, 1376	2239057:	13·673·757 ;	2405, 483, 2356	709·6553:	157·44089 ;
2113, 65, 2112	4327489:	1009·4561 ;	2405, 2013, 1316	3134917:	8433133 ;
2117, 2115, 92	97·193·229:	13·359713 ;	2417, 2385, 392	4906969:	13·37·73·193 ;
2117, 195, 2108	37·110017:	61·80209 ;	2425, 2183, 1056	13·229·1201:	8185873 ;
2125, 2107, 276	3934093:	13·37·10597 ;	2425, 1273, 2064	3253153:	13·61·10729 ;
2125, 1403, 1596	181·12577:	13·229·2269 ;	2437, 2365, 588	13·181·1933:	37·198097 ;
2129, 1071, 1840	13·197077:	6503281 ;	2441, 759, 2320	*4197601:	13·229·2593 ;
2137, 455, 2088	109·33181:	5516809 ;	2465, 2337, 784	4244017:	13·181·3361 ;
2141, 2091, 460	13·278617:	5545741 ;	2465, 1407, 2024	3228457:	13·37·18553 ;
2153, 585, 2072	3423289:	5847529 ;	2465, 897, 2296	4016713:	601·13537 ;
2161, 1711, 1320	37·65173:	13·61·8737 ;	2473, 2135, 1248	37·37·2521:	577·15217 ;
2165, 2067, 644	193·17389:	6018373 ;	2501, 2499, 100	6005101:	13·661·757 ;
2165, 1197, 1804	13·337·577:	157·43609 ;	2501, 2301, 980	1801·2221:	8509981 ;
2173, 715, 2052	3254749:	6189109 ;	2509, 2491, 300	73·75997:	109·64609 ;
2173, 1525, 1548	13·37·4909:	7082629 ;	2521, 71, 2520	13·37·12841:	6534361 ;
2197, 2035, 828	3141829:	6511789 ;	2525, 213, 2516	13·449209:	337·20509 ;
2213, 2205, 188	13·61·5653:	5311909 ;	2525, 1173, 2236	349·10753:	541·16633 ;
2221, 1829, 1260	13·181·1117:	1153·6277 ;	2533, 355, 2508	181·30529:	13·277·2029 ;

* The values of x are continuous to $x = 2441$. Afterwards only those values of x are included which give L and M $\not> 9.10^6$.

x , y , z	L M	x , y , z	L M
2545, 497, 2496	601·8713:61·126517 ;	2677, 365, 2652	997·6217:229·35521 ;
2549, 2451, 700	313·15277:13·61·10357 ;	2689, 511, 2640	13·61·7417:8579761 ;
2561, 639, 2480	61·73·1117:37·97·2269 ;	2705, 2703, 104	7035913:997·7621 ;
2581, 2419, 900	1213·3697:13·679897 ;	2713, 2695, 312	6519529:157·52237 ;
2581, 781, 2460	109·157·277:13·660217 ;	2729, 2679, 520	6054361:37·337·709 ;
2605, 2597, 204	13·481249:7315813 ;	2813, 75, 2812	7702069:13·241·2593 ;
2617, 2585, 408	13·277·1609:7903369 ;	2813, 2805, 212	7318309:13·13·50341 ;
2665, 73, 2664	6907753:7296697 ;	2917, 2915, 108	13·61·10333:8823709 ;
2669, 219, 2660	6541021:13·37·37·577 ;		

☞ *This Table, pages* 190–194, *contains all Dimorph Sextans* (D) *with* L, M $\not>$ 9.10^6.

Sext-Aurifeuillian Sextans (S), *Species* ii. (*Continued from page* 184.)

ξ, η	x , y	L M	ξ, η	x , y	L M
5, 29	75, 1682	1684609:13·364753 ;	3, 32	27, 2048	13·243517:5556121 ;
7	147	1364773:13·73·6133 ;	5	75	109·24061:13·515293 ;
9	243	409·2689:13·37·14821 ;	7, 32	147, 2048	1069·2029:109·181·409 ;
11, 29	363, 1682	61·14401:313·27817 ;	1, 34	3, 2312	193·25357:349·16729 ;
1, 31	3, 1922	3353341:97·41953 ;	3	27	13·315529:6964813 ;
3	27	2762953:13·379849 ;	5, 34	75, 2312	1213·2833:109·181·421 ;
5	75	2274949:193·31033 ;	1, 35	3, 2450	5509429:13·503053 ;
7	147	73·25609:13·558241 ;	3, 35	27, 2450	13·37·9649:61·127249 ;
9, 31	243, 1922	13·37·3181:1213·7237 ;	1, 37	3, 2738	1321·5233:13·625369 ;
1, 32	3, 2048	3818953:13·37·61·157 ;			

☞ *This Table, pages* 183, 184, *and on this page, shows all Sext-Aurifeuillians* (S′′) *with* $x = 3\xi^2$, $y = 2\eta^2$, *giving* L, M $\not>$ 9.10^6.

Trin-Aurifeuillian Sextans (T). (*Continued from page* 189.)

y , x , z	L M	y , x , z	L M
2029, 2027, 52	3808429:4441477 ;	2066, 2041, 185	3224401:5517661 ;
2053, 2003, 260	2816269:6018949 ;	2078, 2029, 259	1657·1753:13·471841 ;
2077, 1979, 364	97·25189:1069·6529 ;	2114, 2111, 65	97·41953:193·25357 ;
2119, 2113, 92	3930709:5100397 ;	2114, 1993, 407	2384749:13·580549 ;
2143, 2089, 276	709·4297:13·61·8317 ;	2126, 2099, 195	601·5641:5877661 ;
2191, 2183, 108	13·317353:5545357 ;	2138, 1969, 481	2179993:8350261 ;
2191, 2041, 460	73·33037:8458861 ;	2186, 2039, 455	193·12517:673·12457 ;
2203, 2171, 216	61·58453:37·97·1789 ;	2246, 2243, 67	13·37·61·157:5509429 ;
2221, 2197, 188	3786229:37·97·1753 ;	2258, 2231, 201	3858193:13·181·2797 ;
2251, 2123, 432	13·13·16033:8544169 ;	2282, 2207, 335	3250789:13·337·1789 ;
2257, 2161, 376	13·228637:337·23929 ;	2282, 2281, 39	73·67741:61·89821 ;
2353, 2351, 56	5150713:5941321 ;	2294, 2269, 195	97·41593:157·42793 ;
2377, 2327, 280	229·16981:7882009 ;	2306, 2257, 273	3652609:7429837 ;
2413, 2389, 196	541·8353:13·61·9277 ;	2522, 2519, 71	349·16729:1321·5233 ;
2503, 2497, 100	5544109:13·541993 ;	2522, 2521, 41	6055321:6675733 ;
2527, 2519, 116	577·9613:13·561961 ;	2534, 2507, 213	13·379849:8176489 ;
2527, 2473, 300	13·37·9109:8930029 ;	2534, 2509, 205	37·109·1237:2137·3793 ;
2539, 2507, 232	13·73·5101:8375137 ;	2546, 2497, 287	13·421·829:37·241117 ;
2701, 2699, 60	13·109·4813:157·49633 ;	2666, 2663, 73	13·503053:7707397 ;
2707, 2701, 104	2137·3049:373·21997 ;	2774, 2773, 43	13·409·1381:97·83077 ;

☞ *This Table, pages* 185–189 *and on this page, contains all Trin-Aurifeuillian Sextans* (T) *with* L, M $\not>$ 9.10^6.

Elements (ξ, η, ζ) *of Bin-Aurifeuillian Chains* [B].

$$B_r = (x_r^6 + y_r^6) \div (x_r^2 + y_r^2) = L_r . M_r\,; \qquad x_r = \xi_r^2, \quad y_r = 2\eta_r^2\,; \qquad M_r = L_{r+1}.$$

$$\xi_r^2 - 2\eta_r^2 = (\bar{1})^r . \zeta, \ [\zeta \text{ const.}]\,; \qquad \xi_{r+1} = \xi_r + 2\eta_r, \quad \eta_{r+1} = \xi_r + \eta_r.$$

For L_r, M_r ($\not> 9.10^6$), see pp. 172–179.

$r=$ / ζ	1 (ξ, η)	2 (ξ, η)	3 (ξ, η)	4 (ξ, η)	5 (ξ, η)
1	1, 1	3, 2	7, 5	17,12	41,29
7	$\bar{1}$, 2	3, 1	5, 4	13, 9	31,22
	1, 2	5, 3	11, 8	27,19	65,46
17	$\bar{1}$, 3	5, 2	9, 7	23,16	55,39
	1, 3	7, 4	15,11	37,26	89,63
23	$\bar{3}$, 4	5, 1	7, 6	19,13	45,32
	3, 4	11, 7	25,18	61,43	
31	$\bar{1}$, 4	7, 3	13,10	33,23	79,56
	1, 4	9, 5	19,14	47,33	
41	$\bar{3}$, 5	7, 2	11, 9	29,20	69,49
	3, 5	13, 8	29,21	71,50	
47	$\bar{5}$, 6	7, 1	9, 8	25,17	59,42
	5, 6	17,11	39,28	95,67	
49	$\bar{1}$, 5	9, 4	17,13	43,30	
	1, 5	11, 6	23,17	57,40	
71	$\bar{1}$, 6	11, 5	21,16	53,37	
	1, 6	13, 7	27,20	67,47	
73	$\bar{5}$, 7	9, 2	13,11	35,24	
	5, 7	19,12	43,31		
79	$\bar{7}$, 8	9, 1	11,10	31,21	
	7, 8	23,15	53,38		
89	$\bar{3}$, 7	11, 4	19,15	49,34	
	3, 7	17,10	37,27	91,64	
97	$\bar{1}$, 7	13, 6	25,19	63,44	
	1, 7	15, 8	31,23	77,54	
103	$\bar{5}$, 8	11, 3	17,14		
	5, 8	21,13	47,34		
113	$\bar{7}$, 9	11, 2	15,13		
	7, 9	25,16	57,41		
119	$\bar{9}$,10	11, 1	13,12		
	9,10	29,19	67,48		
119	$\bar{3}$, 8	13, 5	23,18		
	3, 8	19,11	41,30		
127	$\bar{1}$, 8	15, 7	29,22		
	1, 8	17, 9	35,26		
137	$\bar{5}$, 9	13, 4	21,17		
	5, 9	23,14	51,37		
151	$\bar{7}$,10	13, 3	19,16		
	7,10	27,17	61,44		
161	$\bar{9}$,11	13, 2	17,15		
	9,11	31,20	71,51		

$r=$ / ζ	1 (ξ, η)	2 (ξ, η)	3 (ξ, η)
161	$\bar{1}$, 9	17, 8	33,25
	1, 9	19,10	39,29
167	$\overline{11}$,12	13, 1	15,14
	11,12	35,23	81,58
191	$\bar{3}$,10	17, 7	31,24
	3,10	23,13	49,36
193	$\bar{7}$,11	15, 4	23,19
	7,11	29,18	65,47
199	$\bar{1}$,10	19, 9	37,28
	1,10	21,11	43,32
217	$\overline{11}$,13	15, 2	19,17
	11,13	37,24	85,61
217	$\bar{5}$,11	17, 6	29,23
	5,11	27,16	59,43
223	$\overline{13}$,14	15, 1	17,16
	13,14	41,27	95,68
233	$\bar{3}$,11	19, 8	35,27
	3,11	25,14	53,39
239	$\bar{7}$,12	17, 5	27,22
	7,12	31,19	69,50
241	$\bar{1}$,11	21,10	41,31
	1,11	23,12	47,35
257	$\bar{9}$,13	17, 4	25,21
	9,13	35,22	79,57
263	$\bar{5}$,12	19, 7	33,26
	5,12	29,17	63,46
271	$\overline{11}$,14	17, 3	23,20
	11,14	39,25	89,64
281	$\overline{13}$,15	17, 2	21,19
	13,15	43,28	99,71
287	$\overline{15}$,16	17, 1	19,18
	15,16	47,31	
287	$\bar{1}$,12	23,11	45,34
	1,12	25,13	51,38
289	$\bar{7}$,13	19, 6	31,25
	7,13	33,20	73,53
311	$\bar{9}$,14	19, 5	29,24
	9,14	37,23	83,60
313	$\bar{5}$,13	21, 8	37,29
	5,13	31,18	67,49
329	$\overline{11}$,15	19, 4	27,23
	11,15	41,26	93,67

$r=$ / ζ	1 (ξ, η)	2 (ξ, η)	3 (ξ, η)
329	$\bar{3}$,13	23,10	43,33
	3,13	29,16	61,45
337	$\bar{1}$,13	25,12	49,37
	1,13	27,14	55,41
343	$\overline{13}$,16	19, 3	25,22
	13,16	45,29	
353	$\overline{15}$,17	19, 2	23,21
	15,17	49,32	
359	$\overline{17}$,18	19, 1	21,20
	17,18	51,35	
367	$\bar{5}$,14	23, 9	41,32
	5,14	33,19	71,52
383	$\bar{3}$,14	25,11	47,36
	3,14	31,17	65,48
391	$\overline{11}$,16	21, 5	31,26
	11,16	43,27	97,70
391	$\bar{1}$,14	27,13	53,40
	1,14	29,15	59,44
401	$\bar{7}$,15	23, 8	39,31
	7,15	37,22	81,59
409	$\overline{13}$,17	21, 4	29,25
	13,17	47,30	
431	$\bar{9}$,16	23, 7	37,30
	9,16	41,25	91,66
433	$\overline{17}$,19	21, 2	25,23
	17,19	55,36	
439	$\overline{19}$,20	21, 1	23,22
	19,20	59,39	
449	$\bar{1}$,15	29,14	57,43
	1,15	31,16	63,47
457	$\overline{11}$,17	23, 6	35,29
	11,17	45,28	
463	$\bar{7}$,16	25, 9	43,34
	7,16	39,23	85,62
479	$\overline{13}$,18	23, 5	33,28
	13,18	49,31	
487	$\bar{5}$,16	27,11	49,38
	5,16	37,21	79,58
497	$\overline{15}$,19	23, 4	31,27
	15,19	53,34	
497	$\bar{9}$,17	25, 8	41,33
	9,17	43,26	95,69

Continuation for $\zeta = 1$.	$r =$	6	7	8	9	10
	$\xi, \eta =$	99,70	239,169	577,408	1393,485	8119,5741

Elements (ξ, η, ζ) *of Sext-Aurifeuillian Chains.*

$$N = (x_r^6 + y_r^6) \div (x_r^2 + y_r^2) = L_r . M_r ; \quad M_r = L_{r+1}.$$

$$\left.\begin{array}{llll} r = \omega ; & x_r = 3\xi_r^2, & y_r = 2\eta_r^2 ; & 3\xi_r^2 - 2\eta_r^2 = \\ r = \epsilon ; & x_r = \xi_r^2, & y_r = 6\eta_r^2 ; & \xi_r^2 - 6\eta_r^2 = \end{array}\right\} \zeta \text{ (const.)}.$$

Each ζ (> 1) gives two Chains.

For L_r, M_r $(\not> 9.10^6)$, see pp. 179–184 and 194 [Argt. ξ, η].

$r = \omega$, $\xi_{r+1} = 3\xi_r + 2\eta_r$, $\eta_{r+1} = \xi_r + \eta_r$; $r = \epsilon$, $\xi_{r+1} = \xi_r + 2\eta_r$, $\eta_{r+1} = \xi_r + 3\eta_r$.

$r =$ / ζ	1 / ξ, η	3 / ξ, η	5 / ξ, η	2 / ξ, η	4 / ξ, η	6 / ξ, η
1	1, 1	9,11	89,109	5, 2	49, 20	485, 198
$\overline{5}$	$\bar{1}$, 2	3, 4	31, 38	1, 1	17, 7	169, 69
	1, 2	13,16	129,158	7, 3	71, 29	703, 287
19	$\bar{3}$, 2	7, 8	67, 82	5, 1	37, 15	365, 149
	3, 2	23,28	227,278	13, 5	125, 51	1237, 505
$\overline{23}$	$\bar{3}$, 5	5, 7	53, 65	1, 2	29, 12	289, 118
	3, 5	35,43	347,425	19, 8	191, 78	1891, 772
25	$\bar{3}$, 1	11,13	107,131	7, 2	59, 24	583, 238
	3, 1	19,23	187,229	11, 4	103, 42	1019, 416
$\overline{29}$	$\bar{1}$, 4	11,14	111,136	5, 3	61, 25	605, 247
	1, 4	21,26	209,256	11, 5	115, 47	1139, 465
43	$\bar{5}$, 4	9,10	85,104	7, 1	47, 19	463, 189
	5, 4	41,50	405,496	23, 9	223, 91	2207, 901
$\overline{47}$	$\bar{1}$, 5	15,19	151,185	7, 4	83, 34	823, 336
	1, 5	25,31	249,305	13, 6	137, 56	1357, 554
$\overline{53}$	$\bar{5}$, 8	7,10	75, 92	1, 3	41, 17	309, 167
	5, 8	57,70	565,692	31,13	311,127	3077,1257
67	$\bar{5}$, 2	17,20	165,202	11, 3	91, 37	
	5, 2	33,40	325,398	19, 7	179, 73	
$\overline{71}$	$\bar{3}$, 7	13,17	133,163	5, 4	73, 30	
	3, 7	43,53	427,523	23,10	235, 96	
73	$\bar{5}$, 1	21,25		13, 4	113, 46	
	5, 1	29,35		17, 6	157, 64	
$\overline{95}$	$\bar{1}$, 7	23,29		11, 6	127, 52	
	1, 7	33,41		17, 8	191, 74	
$\overline{95}$	$\bar{7}$,11	9,13		1, 4	53, 22	
	7,11	79,97		43,18	431,176	
97	$\bar{7}$, 5	15,17		11, 2	79, 32	
	7, 5	55,67		31,12	299,122	
$\overline{101}$	$\bar{3}$, 8	17,22		7, 5	95, 39	
	3, 8	47,58		25,11	257,105	

$r =$ / ζ	1 / ξ, η	3 / ξ, η	2 / ξ, η
115	$\bar{7}$, 4	19, 22	13, 3
	7, 4	51, 62	29,11
115	$\bar{9}$, 8	13, 14	11, 1
	9, 8	77, 94	43,17
$\overline{125}$	$\bar{1}$, 8	27, 34	13, 7
	1, 8	37, 46	19, 9
139	$\bar{7}$, 2	27, 32	17, 5
	7, 2	43, 52	25, 9
145	$\bar{9}$, 7	17, 19	13, 2
	9, 7	73, 89	41,16
145	$\bar{7}$, 1	31, 37	19, 6
	7, 1	39, 47	23, 8
$\overline{149}$	$\bar{9}$,14	11, 16	1, 5
	9,14	101,124	55,23
163	$\overline{11}$,10	15, 16	13. 1
	11,10	95,116	53,21
$\overline{167}$	$\bar{5}$,11	19, 25	7, 6
	5,11	69, 85	37,16
$\overline{173}$	$\bar{3}$,10	25, 32	11, 7
	3,10	55, 68	29,13
$\overline{191}$	$\bar{7}$,13	17, 23	5, 6
	7,13	87,107	47,20
193	$\bar{9}$, 5	25, 29	17, 4
	9, 5	65, 79	37,14
$\overline{197}$	$\bar{1}$,10	35, 44	17, 9
	1,10	45, 56	23,11
211	$\bar{9}$, 4	29, 34	19, 5
	9, 4	61, 74	35,13
$\overline{215}$	$\overline{11}$,17	13, 19	1, 6
	11,17	123,151	67,28
$\overline{215}$	$\bar{3}$,11	29, 37	13, 8
	3,11	59, 73	31,14

Elements (x, y, z) *of Trin-Aurifeuillian Chains.*

$$N_r = (x_r^6 + y_r^6) \div (x_r^2 + y_r^2) = L_r . M_r; \quad M_r = L_{r+1}; \quad y_r^2 - 3z_r^2 = x_r^2 = \text{constant}.$$

$$y_{r+1} = 2y_r \pm 3z_r; \quad z_{r+1} = y_r \pm 2z_r.$$

For values of L_r, M_r ($\not> 9.10^6$), see pp. 185–189 and 194.

$r=$	1	2	3	4	5	6	7
x	y, z	y, z	y, z	y, z	y, z	y, z	y, z
1	2, 1	7, 4	26, 15	97, 56	362, 209	1351, 780	5042,2911
$\overline{11}$	13, 4	38,21	139, 80	518,299	1933,1116	7214,4165	
	$\overline{13}$, 4	14, 5	43, 24	158, 91	589, 340	2198,1269	
13	14, 3	37,20	134, 77	499,288	1862,1075		
	$\overline{14}$, 3	19, 8	62, 35	229,132	854, 493		
$\overline{23}$	26, 7	73,40	266,153	991,572	3698,2135		
	$\overline{26}$, 7	31,12	98, 55	361,208	1346, 777		

$r=$	1	2	3	4
x	y, z	y, z	y, z	y, z
37	38, 5	91, 48	326,187	1213, 700
	$\overline{38}$, 5	61, 28	206,117	763, 440
$\overline{47}$	49, 8	122, 65	439,252	1634, 943
	$\overline{49}$, 8	74, 33	247,140	914, 527
$\overline{59}$	62,11	157, 84	566,325	2107,1216
	$\overline{62}$,11	91, 40	302,171	1117, 644
61	67,16	182, 99	661,380	2462,1421
	$\overline{67}$,16	86, 35	277,156	1022, 589
$\overline{71}$	79,20	218,119	793,456	2954,1705
	$\overline{79}$,20	98, 39	313,176	1154, 665
73	74, 7	169, 88	602,345	2239,1292
	$\overline{74}$, 7	127, 60	434,247	1609, 928
$\overline{83}$	86,13	211,112	758,435	2821,1628
	$\overline{86}$,13	133, 60	446,253	1651, 952
97	103,20	266,143	961,552	3578,2065
	$\overline{103}$,20	146, 63	481,272	1778,1025
$\overline{107}$	109,12	254,133	907,520	3374,1947
	$\overline{109}$,12	182, 85	619,352	2294,1323
109	133,44	398,221	1459,840	5438,3139
	$\overline{133}$,44	134, 45	403,224	1478, 851
121	122, 9	271,140	962,551	3577,2064
	$\overline{122}$, 9	217,104	746,425	2767,1596
$\overline{131}$	158,51	469,260	1718,989	6403,3696
	$\overline{158}$,51	163, 56	494,275	1813,1044
$\overline{143}$	146,17	343,180	1226,703	4561,2632
	$\overline{146}$,17	241,112	818,465	3031,1748
$\overline{143}$	151,28	386,207	1393,800	5186,2993
	$\overline{151}$,28	218, 95	721,408	2666,1537

$r=$	1	2	3
x	y, z	y, z	y, z
157	181,52	518,285	1891,1088
	$\overline{181}$,52	206, 77	643, 360
167	194,57	559,308	2042,1175
	$\overline{194}$,57	217, 80	674, 377
169	194,55	553,304	2018,1161
	$\overline{194}$,55	223, 84	698, 391
$\overline{179}$	182,19	421,220	1502, 861
	$\overline{182}$,19	307,144	1046, 595
181	182,11	397,204	1406, 805
	$\overline{182}$,11	331,160	1142, 651
$\overline{191}$	193,16	434,225	1543, 884
	$\overline{193}$,16	338,161	1159, 660
193	199,28	482,255	1729, 992
	$\overline{199}$,28	314,143	1057, 600
$\overline{227}$	259,72	734,403	2677,1540
	$\overline{259}$,72	302,115	949, 532
229	278,91	829,460	3038,1749
	$\overline{278}$,91	283, 96	854, 475
$\overline{239}$	247,36	602,319	2161,1240
	$\overline{247}$,36	386,175	1297, 736
241	266,65	727,396	2642,1519
	$\overline{266}$,65	337,136	1082, 609
251	278,69	763,416	2774,1595
	$\overline{278}$,69	349,140	1118, 629

Elements $(r, \eta;\ x, y, z)$ *of*
Simple Bin-Aurifeuillian (B) *cum Dimorph* (D) *Chains.*

$$\mathrm{B}_r = (1^6 + y_r'^6) \div (1^2 + y_r'^2) = \mathrm{L}_r'.\mathrm{M}_r'; \qquad \eta_r = r, \quad y_r' = 2\eta_r^2 = 2r^2.$$

$$\mathrm{D}_r = (x_r^6 + y_r^6) \div (x_r^2 + y_r^2) = (x_r^6 + z_r^6) \div (x_r^2 + z_r^2) = \mathrm{L}_r.\mathrm{M}_r.$$

$$x = (r+1)^2 + r^2, \quad y = 2r+1, \quad z = x-1.$$

$\mathrm{M}_r' = \mathrm{L}_r$, $\mathrm{M}_r = \mathrm{L}_{r+1}'$. [For L_r', M_r', see pp. 172–179, Argt. η. For L_r, $\mathrm{M}_r \not> 9.10^6$, see pp. 190–194, Argt. x.]

r	x	y	r	x	y	r	x	y	r	x	y
1	5	3	34	2 381	69	67	9 113	135	100	20 201	201
2	13	5	35	2 521	71	68	9 385	137	101	20 605	203
3	25	7	36	2 665	73	69	9 661	139	102	21 013	205
4	41	9	37	2 813	75	70	9 941	141	103	21 425	207
5	61	11	38	2 965	77	71	10 225	143	104	21 841	209
6	85	13	39	3 121	79	72	10 513	145	105	22 261	211
7	113	15	40	3 281	81	73	10 805	147	106	22 685	213
8	145	17	41	3 445	83	74	11 101	149	107	23 113	215
9	181	19	42	3 613	85	75	11 401	151	108	23 545	217
10	221	21	43	3 785	87	76	11 705	153	109	23 981	219
11	265	23	44	3 961	89	77	12 013	155	110	24 421	221
12	313	25	45	4 141	91	78	12 325	157	111	24 865	223
13	365	27	46	4 325	93	79	12 641	159	112	25 313	225
14	421	29	47	4 513	95	80	12 961	161	113	25 765	227
15	481	31	48	4 705	97	81	13 285	163	114	26 221	229
16	545	33	49	4 901	99	82	13 613	165	115	26 681	231
17	613	35	50	5 101	101	83	13 945	167	116	27 145	233
18	685	37	51	5 305	103	84	14 281	169	117	27 613	235
19	761	39	52	5 513	105	85	14 621	171	118	28 085	237
20	841	41	53	5 725	107	86	14 965	173	119	28 561	239
21	925	43	54	5 941	109	87	15 313	175	120	29 041	241
22	1 013	45	55	6 161	111	88	15 665	177	121	29 525	243
23	1 105	47	56	6 385	113	89	16 021	179	122	30 013	245
24	1 201	49	57	6 613	115	90	16 381	181	123	30 505	247
25	1 301	51	58	6 845	117	91	16 745	183	124	31 001	249
26	1 405	53	59	7 081	119	92	17 113	185	125	31 501	251
27	1 513	55	60	7 321	121	93	17 485	187	126	32 005	253
28	1 625	57	61	7 565	123	94	17 861	189	127	32 513	255
29	1 741	59	62	7 813	125	95	18 241	191	128	33 025	257
30	1 861	61	63	8 065	127	96	18 625	193	160	51 521	321
31	1 985	63	64	8 321	129	97	19 013	195			
32	2 113	65	65	8 581	131	98	19 405	197	200	80 401	401
33	2 245	67	66	8 845	133	99	19 801	199	201	81 205	403

Elements (ξ, η, x) *of Bin-Aurifeuillian* (B) *cum Dimorph* (D) *Chains, Class* i.

$\mathrm{B}_\rho = (x_\rho^6 + y_\rho^6) \div (x_\rho^2 + y_\rho^2) = \mathrm{L}_\rho . \mathrm{M}_\rho$; $\rho = \omega$, $\xi_\rho = \xi$ (const.), $\eta_{\rho+2} = \eta_\rho + \xi$; $x_\rho = \xi^2$ (const.), $y_\rho = 2\eta_\rho^2$.

$\mathrm{D}_r = (x_r^6 + y_r^6) \div (x_r^2 + y_r^2) = \mathrm{L}_r . \mathrm{M}_r$; $r = \rho + 1 = \epsilon$, $u_r = u$ (const.), $t_{r+2} = t_r + 2u$; $x_r = \frac{1}{2}(t_r^2 + u^2)$.

$\xi = u$, $\mathrm{M}_\rho = \mathrm{L}_r$, $\mathrm{M}_r = \mathrm{L}_{\rho+2}$; [For $\mathrm{L}_\rho, \mathrm{M}_\rho, \mathrm{L}_r, \mathrm{M}_r$ ($\not> 9.10^6$), see pp. 174–179, 190–194].

ρ, ξ, η	t, x
1, 3, 1	5, 17
3 4	11, 65
5 7	17, 149
7 10	23, 269
9 13	29, 425
11 16	35, 617
13 19	41, 845
15 22	47, 1109
17 25	53, 1409
19 28	59, 1745
21 31	65, 2117
23 34	71, 2525
25, 3, 37	77, 2969
1, 3, 2	7, 29
3 5	13, 89
5 8	19, 185
7 11	25, 317
9 14	31, 485
11 17	37, 689
13 20	43, 929
15 23	49, 1205
17 26	55, 1517
19 29	61, 1865
21 32	67, 2249
23, 3, 35	73, 2669
1, 5, 1	7, 37
3 6	17, 157
5 11	27, 377
7 16	37, 697
9 21	47, 1117
11 26	57, 1637
13 31	67, 2257
15, 5, 36	77, 2977
1, 5, 2	9, 53
3 7	19, 193
5 12	29, 433
7 17	39, 773
9 22	49, 1213
11 27	59, 1753
13 32	69, 2393
15, 5, 37	79, 3133
1, 5, 3	11, 73

ρ, ξ, η	t, x
3, 5, 8	21, 233
5 13	31, 493
7 18	41, 853
9 23	51, 1313
11 28	61, 1873
13, 5, 33	71, 2533
1, 5, 4	13, 97
3 9	23, 277
5 14	33, 557
7 19	43, 937
9 24	53, 1417
11 29	63, 1997
13, 5, 34	73, 2677
1, 7, 1	9, 65
3 8	23, 289
5 15	37, 709
7 22	51, 1325
9 29	65, 2137
11, 7, 36	79, 3145
1, 7, 2	11, 85
3 9	25, 337
5 16	39, 785
7 23	53, 1429
9, 7, 30	67, 2269
1, 7, 3	13, 109
3 10	27, 389
5 17	41, 865
7 24	55, 1537
9, 7, 31	69, 2405
1, 7, 4	15, 137
3 11	29, 445
5 18	43, 949
7 25	57, 1649
9, 7, 32	71, 2545
1, 7, 5	17, 169
3 12	31, 505
5 19	45, 1037
7 26	59, 1765
9, 7, 33	73, 2689
1, 7, 6	19, 205

ρ, ξ, η	t, x
3, 7, 13	33, 569
5 20	47, 1129
7 27	61, 1885
9, 7, 34	75, 2837
1, 9, 1	11, 101
3 10	29, 461
5 19	47, 1145
7, 9, 28	65, 2153
1, 9, 2	13, 125
3 11	31, 521
5 20	49, 1241
7, 9, 29	67, 2285
1, 9, 4	17, 185
3 13	35, 653
5 22	53, 1445
7, 9, 31	71, 2561
1, 9, 5	19, 221
3 14	37, 725
5 23	55, 1553
7, 9, 32	73, 2705
1, 9, 7	23, 305
3 16	41, 881
5 25	59, 1781
7, 9, 34	77, 3005
1, 9, 8	25, 353
3 17	43, 965
5 26	61, 1901
7, 9, 35	79, 3161
1, 11, 1	13, 145
3 12	35, 673
5 23	57, 1685
7, 11, 34	79, 3181
1, 11, 2	15, 173
3 13	37, 745
5 24	59, 1801
7, 11, 35	81, 3341
1, 11, 3	17, 205
3 14	39, 821
5, 11, 25	61, 1921

ρ, ξ, η	t, x
1, 11, 4	19, 241
3 15	41, 901
5, 11, 26	63, 2045
1, 11, 5	21, 281
3 16	43, 985
5, 11, 27	65, 2173
1, 11, 6	23, 325
3 17	45, 1073
5, 11, 28	67, 2305
1, 11, 7	25, 373
3 18	47, 1165
5, 11, 29	69, 2441
1, 11, 8	27, 425
3 19	49, 1261
5, 11, 30	71, 2581
1, 11, 9	29, 481
3 20	51, 1361
5, 11, 31	73, 2725
1, 11, 10	31, 541
3 21	53, 1465
5, 11, 32	75, 2873
1, 13 1	15, 197
3 14	41, 925
5, 13, 27	67, 2329
1, 13, 2	17, 229
3 15	43, 1009
5, 13, 28	69, 2465
1, 13, 3	19, 265
3 16	45, 1097
5, 13, 29	71, 2605
1, 13, 4	21, 305
3 17	47, 1189
5, 13, 30	73, 2749
1, 13, 5	23, 349
3 18	49, 1285
5, 13, 31	75, 2897
1, 13, 6	25, 397
3 19	51, 1385
5, 13, 32	77, 3049

Elements (ξ, η, x) *of Bin-Aurifeuillian* (B) *cum Dimorph* (D) *Chains, Class* ii.

$B_\rho = (x_\rho^6 + y_\rho^6) \div (x_\rho^2 + y_\rho^2) = L_\rho . M_\rho$;	$D_r = (x_r^6 + y_r^6) \div (x_r^2 + y_r^2) = L_r . M_r$;
$\rho = \omega$, $\eta_\rho = \eta$ (const.), $\xi_{\rho+2} = \xi_\rho + 2\eta$;	$r = \rho + 1 = \epsilon$, $u_r = u$ (const.), $t_{r+2} = t_r + 2u$;
$x_\rho = \xi_\rho$, $y_\rho = 2\eta^2$ (const.).	$x_r = (t_r^2 + u^2)$.

$\eta = u$, $M_\rho = L_r$, $M_r = L_{\rho+2}$; [For L_ρ, M_ρ, L_r, M_r ($\not> 9.10^6$), see pp. 174–179, 190–194].

ρ, ξ, η	t, x
1, 1, 1	2, 5
3, 3	4, 17
5, 5	6, 37
7, 7	8, 65
9, 9	10, 101
11, 11	12, 145
13, 13	14, 197
15, 15	16, 257
17, 17	18, 325
19, 19	20, 401
21, 21	22, 485
23, 23	24, 577
25, 25	26, 677
27, 27	28, 785
29, 29	30, 901
31, 31	32, 1025
33, 33	34, 1157
35, 35	36, 1297
37, 37	38, 1445
39, 39	40, 1601
41, 41	42, 1765
43, 43	44, 1937
45, 45	46, 2117
47, 47	48, 2305
49, 49	50, 2501
51, 51	52, 2705
53, 53, 1	54, 2917
1, 1, 2	3, 13
3, 5	7, 53
5, 9	11, 125
7, 13	15, 229
9, 17	19, 365
11, 21	23, 533
13, 25	27, 733
15, 29	31, 965
17, 33	35, 1229
19, 37	39, 1525
21, 41	43, 1853
23, 45	47, 2213
25, 49	51, 2605
27, 53, 2	55, 3029
1, 3, 2	5, 29
3, 7, 2	9, 85

ρ, ξ, η	t, x
5, 11, 2	13, 173
7, 15	17, 293
9, 19	21, 445
11, 23	25, 629
13, 27	29, 845
15, 31	33, 1093
17, 35	37, 1373
19, 39	41, 1685
21, 43	45, 2029
23, 47	49, 2405
25, 51, 2	53, 2813
1, 1, 3	4, 25
3, 7	10, 109
5, 13	16, 265
7, 19	22, 493
9, 25	28, 793
11, 31	34, 1165
13, 37	40, 1609
15, 43	46, 2125
17, 49, 3	52, 2713
1, 5, 3	8, 73
3, 11	14, 205
5, 17	20, 409
7, 23	26, 685
9, 29	32, 1033
11, 35	38, 1453
13, 41	44, 1945
15, 47	50, 2509
17, 53, 3	56, 3145
1, 1, 4	5, 41
3, 9	13, 185
5, 17	21, 457
7, 25	29, 857
9, 33	37, 1385
11, 41	45, 2041
13, 49, 4	53, 2825
1, 3, 4	7, 65
3, 11	15, 241
5, 19	23, 545
7, 27	31, 977
9, 35	39, 1537
11, 43, 4	47, 2225

ρ, ξ, η	t, x
13, 51, 4	55, 3041
1, 5, 4	9, 97
3, 13	17, 305
5, 21	25, 641
7, 29	33, 1105
9, 37	41, 1697
11, 45, 4	49, 2417
1, 7, 4	11, 137
3, 15	19, 377
5, 23	27, 745
7, 31	35, 1241
9, 39	43, 1865
11, 47, 4	51, 2617
1, 1, 5	6, 61
3, 11	16, 281
5, 21	26, 701
7, 31	36, 1321
9, 41	46, 2141
11, 51, 5	56, 3161
1, 3, 5	8, 89
3, 13	18, 349
5, 23	28, 809
7, 33	38, 1469
9, 43, 5	48, 2329
1, 7, 5	12, 169
3, 17	22, 509
5, 27	32, 1049
7, 37	42, 1789
9, 47, 5	52, 2729
1, 9, 5	14, 221
3, 19	24, 601
5, 29	34, 1181
7, 39	44, 1961
9, 49, 5	54, 2941
1, 1, 6	7, 85
3, 13	19, 397
5, 25	31, 997
7, 37	43, 1885
9, 49, 6	55, 3061

ρ, ξ, η	t, x
1, 5, 6	11, 157
3, 17	23, 565
5, 29	35, 1261
7, 41, 6	47, 2245
1, 7, 6	13, 205
3, 19	25, 661
5, 31	37, 1405
7, 43, 6	49, 2437
1, 11, 6	17, 325
3, 23	29, 877
5, 35	41, 1717
7, 47, 6	53, 2845
1, 1, 7	8, 113
3, 15	22, 533
5, 29	36, 1345
7, 43, 7	50, 2549
1, 3, 7	10, 149
3, 17	24, 625
5, 31	38, 1493
7, 45, 7	52, 2753
1, 5, 7	12, 193
3, 19	26, 725
5, 33	40, 1649
7, 47, 7	54, 2965
1, 9, 7	16, 305
3, 23	30, 949
5, 37	44, 1985
7, 51, 7	58, 3413
1, 11, 7	18, 373
3, 25	32, 1073
5, 39, 7	46, 2165
1, 13, 7	20, 449
3, 27	34, 1205
5, 41, 7	48, 2353
1, 1, 8	9, 145
3, 17	25, 689
5, 33	41, 1745
7, 49, 8	57, 3313

☞ *Continued at top of page* 201.

Elements (ξ, η, x) *of Bin-Aurifeuillian* (B) *cum Dimorph* (D), *Chains, Class* ii.

(*Continued from page* 200.)

ρ, ξ, η	t, x	ρ, ξ, η	t, x	ρ, ξ, η	t, x	ρ, ξ, η	t, x
1, 3, 8	11, 185	1, 9, 8	17, 353	1, 15, 8	23, 593	1, 7, 9	16, 337
3, 19	27, 793	3, 25	33, 1153	3, 31	39, 1585	3, 25	34, 1237
5, 35, 8	43, 1913	5, 41, 8	49, 2465	5, 47, 8	55, 3089	5, 43, 9	52, 2785
1, 5, 8	13, 233	1, 11, 8	19, 425	1, 1, 9	10, 181	1, 11, 9	20, 481
3, 21	29, 905	3, 27	35, 1289	3, 19	28, 865	3, 29	38, 1525
5, 37, 8	45, 2089	5, 43, 8	51, 2665	5, 37, 9	46, 2197	5, 47, 9	56, 3217
1, 7, 8	15, 289	1, 13, 8	21, 505	1, 5, 9	14, 277	1, 13, 9	22, 565
3, 23	31, 1025	3, 29	37, 1433	3, 23	32, 1105	3, 31	40, 1681
5, 39, 8	47, 2273	5, 45, 8	53, 2873	5, 41, 9	50, 2581	5, 49, 9	58, 3445
						1, 17, 9	26, 757
						3, 35, 9	44, 2017

Elements of Sexto-Trin-Aurifn. 6-*tan Chains* (S, T) [*Species* i, *Class* 1°].

$$T_\rho = (x_\rho^6 + y_\rho^6) \div (x_\rho^2 + y_\rho^2) = L_\rho . M_\rho ; \qquad t_\rho = t \text{ (const.)}, \quad u_{\rho+2} = u_\rho + 2t ;$$

$$y_\rho = \tfrac{1}{2}(t_\rho^2 + 3u_\rho^2), \quad x_\rho = \tfrac{1}{2}(t_\rho^2 - 3u_\rho^2), \quad z_\rho = t_\rho u_\rho ; \qquad y_\rho^2 - 3z_\rho^2 = x_\rho^2 .$$

$$S_r = (x_r^6 + y_r^6) \div (x_r^2 + y_r^2) = L_r . M_r ; \qquad x_r = \xi_r^2 = \xi^2 \text{ (const.)}, \quad y_r = 6\eta_r^2 .$$

$$\rho = \omega, \quad r = \rho + 1 = \epsilon ; \qquad t = \xi \text{ (const.)}, \quad u_\rho = \eta_r + \eta_{r-2}, \quad \eta_r = \tfrac{1}{4}(u_\rho + u_{\rho+2}).$$

$M_\rho = L_r, \;\; M_r = L_{\rho+2}$; { For $L_\rho, M_\rho \not> 9.10^6$, see pp. 185–189, 194, Argt. y; For $L_r, M_r \not> 9.10^6$, see pp. 179–182, Argt. ξ, η.

ρ, u, y	ξ, η
1, $\bar{3}$, 26	5, 1
3, 7, 86	6
5, 17, 446	11
7, 27, 1106	16
9, 37, 2066	5, 21
1, $\bar{1}$, 14	5, 2
3, 9, 134	7
5, 19, 554	12
7, 29, 1274	17
9, 39, 2294	5, 22
1, 1, 14	5, 3
3, 11, 194	8
5, 21, 674	13
7, 31, 1454	18
9, 41, 2534	5, 23
1, 3, 26	5, 4
3, 13, 266	9
5, 23, 806	14
7, 33, 1646	5, 19
1, $\bar{5}$, 62	7, 1
3, 9, 146	8
5, 23, 818	15
7, 37, 2078	7, 22

ρ, u, y	ξ, η
1, $\bar{3}$, 38	7, 2
3, 11, 206	9
5, 25, 962	16
7, 39, 2306	7, 23
1, $\bar{1}$, 26	7, 3
3, 13, 278	10
5, 27, 1118	17
7, 41, 2546	7, 24
1, 1, 26	7, 4
3, 15, 362	11
5, 29, 1286	18
7, 43, 2798	7, 25
1, 3, 38	7, 5
3, 17, 458	12
5, 31, 1466	7, 19
1, 5, 62	7, 6
3, 19, 566	13
5, 33, 1658	7, 20
1, $\bar{9}$, 182	11, 1
3, 13, 314	12
5, 35, 1898	11, 23
1, $\bar{7}$, 134	11, 2
3, 15, 398	13
5, 37, 2114	11, 24

ρ, u, y	ξ, η
1, $\bar{5}$, 98	11, 3
3, 17, 494	14
5, 39, 2342	11, 25
1, $\bar{3}$, 74	11, 4
3, 19, 602	15
5, 41, 2582	11, 26
1, $\bar{1}$, 62	11, 5
3, 21, 722	16
5, 43, 2834	11, 27
1, 1, 62	11, 6
3, 23, 854	11, 17
1, 3, 74	11, 7
3, 25, 998	11, 18
1, 5, 98	11, 8
3, 27, 1154	11, 19
1, 7, 134	11, 9
3, 29, 1322	11, 20
1, 9, 182	11, 10
3, 31, 1502	11, 21
1, $\overline{11}$, 266	13, 1
3, 15, 422	13, 14

ρ, u, y	ξ, η
1, $\bar{9}$, 206	13, 2
3, 17, 518	13, 15
1, $\bar{7}$, 158	13, 3
3, 19, 626	13, 16
1, $\bar{5}$, 122	13, 4
3, 21, 746	13, 17
1, $\bar{3}$, 98	13, 5
3, 23, 878	13, 18
1, $\bar{1}$, 86	13, 6
3, 25, 1022	13, 19
1, 1, 86	13, 7
3, 27, 1178	13, 20
1, 3, 98	13, 8
3, 29, 1346	13, 21
1, 5, 122	13, 9
3, 31, 1526	13, 22
1, 7, 158	13, 10
3, 33, 1718	13, 23
1, 9, 206	13, 11
3, 35, 1922	13, 24
1, 11, 266	13, 12
3, 37, 2138	13, 25

Elements of Sexto-Trin-Aurifeuillian Sextan Chains (S *and* T)
[*Species* i, *Class* 2°].

$$T_\rho = (x_\rho^6 + y_\rho^6) \div (x_\rho^2 + y_\rho^2) = L_\rho . M_\rho\,; \qquad t_{\rho+2} = t_\rho + 6u, \quad u_\rho = u\,(\text{const.})\,;$$

$$y_\rho = (t_\rho^2 + 3u_\rho^2), \quad x_\rho = (t_\rho^2 - 3u_\rho^2), \quad z_\rho = 2t_\rho u_\rho\,; \qquad y_\rho^2 - 3z_\rho^2 = x_\rho^2.$$

$$S_r = (x_r^6 + y_r^6) \div (x_r^2 + y_r^2) = L_r . M_r\,; \qquad x_r = \xi_r^2, \quad y_r = 6\eta_r^2, \quad \eta_r = \eta\,(\text{const.}).$$

$$\rho = \omega, \quad r = \rho + 1 = \epsilon\,; \qquad u = \eta\,(\text{const.}), \quad t_\rho = \tfrac{1}{2}(\xi_{r-2} + \xi_r), \quad \xi_r = \tfrac{1}{2}(t_\rho + t_{\rho+2}).$$

$$M_\rho = L_r, \quad M_r = L_{\rho+2}\,;$$

For $L_\rho, M_\rho \not> 9.10^6$, see pp. 185–189, 194, Argt. y;

For $L_r, M_r \not> 9.10^6$, see pp. 179–182, Argt. ξ, η.

ρ, t, y	ξ, η
1, $\bar{2}$, 7	1, 1
3, 4, 19	7
5, 10, 103	13
7, 16, 259	19
9, 22, 487	25
11, 28, 787	31
13, 34, 1159	37
15, 40, 1603	43
17, 46, 2119	49
19, 52, 2707	55, 1
1, 2, 7	5, 1
3, 8, 67	11
5, 14, 199	17
7, 20, 403	23
9, 26, 679	29
11, 32, 1027	35
13, 38, 1447	41
15, 44, 1939	47
17, 50, 2503	53, 1
1, $\bar{5}$, 37	1, 2
3, 7, 61	13
5, 19, 373	25
7, 31, 973	37
9, 43, 1861	49, 2
1, $\bar{1}$, 13	5, 2

ρ, t, y	ξ, η
3, 11, 133	17, 2
5, 23, 541	29
7, 35, 1237	41
9, 47, 2221	53, 2
1, 1, 13	7, 2
3, 13, 181	19
5, 25, 637	31
7, 37, 1381	43
9, 49, 2413	55, 2
1, 5, 37	11, 2
3, 17, 301	23
5, 29, 853	35
7, 41, 1693	47, 2
1, 8, 91	1, 3
3, 10, 127	19
5, 28, 811	37
7, 46, 2143	55, 3
1, $\bar{4}$, 43	5, 3
3, 14, 223	23
5, 32, 1051	41
7, 50, 2503	59, 3
1, $\bar{2}$, 31	7, 3
3, 16, 283	25
5, 34, 1183	43, 3

ρ, t, y	ξ, η
7, 52, 2731	61, 3
1, 2, 31	11, 3
3, 20, 427	29
5, 38, 1471	47, 3
1, 4, 43	13, 3
3, 22, 511	31
5, 40, 1627	49, 3
1, 8, 91	17, 3
3, 26, 703	35
5, 44, 1963	53, 3
1, $\overline{14}$, 271	1, 5
3, 16, 331	31
5, 46, 2191	61, 5
1, $\bar{8}$, 139	7, 5
3, 22, 559	37
5, 52, 2779	67, 5
1, $\bar{4}$, 91	11, 5
3, 26, 751	41, 5
1, $\bar{2}$, 79	13, 5
3, 28, 859	43, 5
1, 2, 79	17, 5
3, 32, 1099	47, 5

ρ, t, y	ξ, η
1, 4, 91	19, 5
3, 34, 1231	49, 5
1, 8, 139	23, 5
3, 38, 1519	53, 5
1, 14, 271	29, 5
3, 44, 2011	59, 5
1, $\overline{20}$, 547	1, 7
3, 22, 631	43, 7
1, $\overline{16}$, 403	5, 7
3, 26, 823	47, 7
1, $\overline{10}$, 247	11, 7
3, 32, 1171	53, 7
1, 8, 211	13, 7
3, 34, 1303	55, 7
1, $\bar{4}$, 163	17, 7
3, 38, 1591	59, 7
1, $\bar{2}$, 151	19, 7
3, 40, 1747	61, 7
1, 2, 151	23, 7
3, 44, 2083	65, 7
1, 4, 163	25, 7
3, 46, 2263	67, 7

Elements of Sexto-Trin-Aurifeuillian Sextan Chains (S *and* T)
[*Species* ii, *Class* 1°].

$$S_\rho = (x_\rho^6 + y_\rho^6) \div (x_\rho^2 + y_\rho^2) = L_\rho . M_\rho\,;$$

$$x_\rho = 3\xi_\rho^2 \;\; [\xi_\rho = \xi\,(\text{const.})], \quad y_\rho = 2\eta_\rho^2, \quad \eta_{\rho+2} = \eta_\rho + 3\xi.$$

$$T_r = (x_r^6 + y_r^6) \div (x_r^2 + y_r^2) = L_r . M_r\,; \qquad t_{r+2} = t_r + 6u, \quad u_r = u\,(\text{const.})\,;$$

$$y_r = \tfrac{1}{2}(t_r^2 + 3u_r^2), \quad x_r = \tfrac{1}{2}(t_r^2 - 3u_r^2), \quad z_r = t_r u_r, \quad y_r^2 - 3z_r^2 = x_r^2.$$

$$\rho = \omega, \quad r = \rho + 1 = 2\,; \qquad u = \xi\,(\text{const.}), \quad t_r = (\eta_\rho + \eta_{\rho+2}), \quad \eta_\rho = \tfrac{1}{4}(t_r + t_{r-2}).$$

$$M_\rho = L_r, \quad M_r = L_{\rho+2}\,;$$

For $L_\rho, M_\rho \not> 9.10^6$, see pp. 183, 184, 194, Argt. ξ, η;
For $L_r, M_r \not> 9.10^6$, see pp. 185-189, 194, Argt. y.

ρ, ξ, η	t, y
1, 1, 1	5, 14
3 4	11, 62
5 7	17, 146
7 10	23, 266
9 13	29, 422
11 16	35, 614
13 19	41, 842
15 22	47, 1106
17 25	53, 1406
19 28	59, 1742
21 31	65, 2114
23 34	71, 2522
25, 1, 37	77, 2966
1, 1, 2	7, 26
3 5	13, 86
5 8	19, 182
7 11	25, 314
9 14	31, 482
11 17	37, 686
13 20	43, 926
15 23	49, 1202
17 26	55, 1514
19 29	61, 1862
21, 1, 32	67, 2246

ρ, ξ, η	t, y
23, 1, 35	73, 2666
1, 3, 1	11, 74
3 10	29, 434
5 19	47, 1118
7, 3, 28	65, 2126
1, 3, 2	13, 98
3 11	31, 494
5 20	49, 1214
7, 3, 29	67, 2258
1, 3, 4	17, 158
3 13	35, 626
5 22	53, 1418
7, 3, 31	71, 2534
1, 3, 5	19, 194
3 14	37, 698
5 23	55, 1526
7, 3, 32	73, 2678
1, 3, 7	23, 278
3 16	41, 854
5 25	59, 1754
7, 3, 34	77, 2978

ρ, ξ, η	t, y
1, 3, 8	25, 326
3 17	43, 938
5 26	61, 1874
7, 3, 35	79, 3134
1, 5, 1	17, 182
3 16	47, 1142
5, 5, 31	77, 3002
1, 5, 2	19, 218
3 17	49, 1238
5, 5, 32	79, 3158
1, 5, 4	23, 302
3 19	53, 1442
5, 5, 34	83, 3482
1, 5, 7	29, 458
3, 5, 22	59, 1778
1, 5, 8	31, 518
3, 5, 23	61, 1898
1, 5, 11	37, 722
3, 5, 26	67, 2282
1, 5, 13	41, 878
3, 5, 28	71, 2558

ρ, ξ, η	t, y
1, 5, 14	43, 962
3, 5, 29	73, 2702
1, 7, 1	23, 338
3, 7, 22	65, 2186
1, 7, 2	25, 386
3, 7, 23	67, 2318
1, 7, 4	29, 494
3, 7, 25	71, 2594
1, 7, 5	31, 554
3, 7, 26	73, 2738
1, 7, 8	37, 758
3, 7, 29	79, 3194
1, 7, 10	41, 914
3, 7, 31	83, 3518
1, 7, 11	43, 998
1, 7, 32	85, 3686
1, 9, 1	29, 542
1, 9, 28	83, 3566
1, 9, 2	31, 602
1, 9, 29	85, 3734

Elements of Sexto-Trin-Aurifeuillian Sextan Chains (S *and* T)
[*Species* ii, *Class* 2°].

$$S_\rho = (x_\rho^6 + y_\rho^6) \div (x_\rho^2 + y_\rho^2) = L_\rho . M_\rho \,; \qquad x_\rho = 3\xi_\rho^2, \quad y_\rho = 2\eta_\rho^2, \quad \eta_\rho = \eta \text{ (const.)}.$$

$$T_r = (x_r^6 + y_r^6) \div (x_r^2 + y_r^2) = L_r . M_r \,;$$

$$\xi_{\rho+2} = \xi_\rho' + 2\eta, \quad t_r = t \text{ (const.)}, \quad u_{r+2} = u_r + 2t\,;$$

$$y_r = (t_r^2 + 3u_r^2), \quad x_r = (t_r \sim 3u_r^2), \quad z_r = t_r u_r, \quad y_r^2 - 3z_r^2 = x_r^2.$$

$$\rho = \omega, \quad r = \rho + 1 = 2, \quad t = \eta \text{ (const.)}, \quad u_r = \tfrac{1}{2}(\xi_\rho + \xi_{\rho+2}), \quad \xi_\rho = \tfrac{1}{2}(u_r + u_{r-2}).$$

$$M_\rho = L_r, \quad M_r = L_{\rho+2}\,;$$

For $L_\rho, M_\rho \not> 9.10^6$, see pp. 183, 184, 194, Argt. ξ, η;

For $L_r, M_r \not> 9.10^6$, see pp. 185–189, 194, Argt. y.

ρ, ξ, η	u, y
1, 1, 1	2, 13
3, 3	4, 49
5, 5	6, 109
7, 7	8, 193
9, 9	10, 301
11, 11	12, 433
13, 13	14, 589
15, 15	16, 769
17, 17	18, 973
19, 19	20, 1201
21, 21	22, 1453
23, 23	24, 1729
25, 25	26, 2029
27, 27	28, 2353
29, 29	30, 2701
31, 31	32, 3073
33, 33, 1	34, 3469
1, 1, 2	3, 31
3, 5	7, 151
5, 9	11, 367
7, 13	15, 679
9, 17	19, 1087
11, 21	23, 1591
13, 25	27, 2191
15, 29, 2	31, 2887
1, 3, 2	5, 79
3, 7, 2	9, 247

ρ, ξ, η	u, y
5, 11, 2	13, 511
7, 15	17, 871
9, 19	21, 1327
11, 23	25, 1879
13, 27, 2	29, 2527
1, 1, 4	5, 91
3, 9	13, 523
5, 17	21, 1339
7, 25	29, 2539
9, 33, 4	37, 4123
1, 3, 4	7, 163
3, 11	15, 691
5, 19	23, 1603
7, 27, 4	31, 2899
1, 5, 4	9, 259
3, 13	17, 883
5, 21	25, 1891
7, 29, 4	33, 3283
1, 7, 4	11, 379
3, 15	19, 1099
5, 23	27, 2203
7, 31, 4	35, 3691
1, 1, 5	6, 133
3, 11	16, 793
5, 21, 5	26, 2053

ρ, ξ, η	u, y
1, 3, 5	8, 217
3, 13	18, 997
5, 23, 5	28, 2377
1, 7, 5	12, 457
3, 17	22, 1477
5, 27, 5	32, 3097
1, 9, 5	14, 613
3, 19	24, 1753
5, 29, 5	34, 3493
1, 1, 7	8, 241
3, 15, 7	22, 1501
1, 3, 7	10, 349
3, 17, 7	24, 1777
1, 5, 7	12, 481
3, 19, 7	26, 2077
1, 9, 7	16, 817
3, 23, 7	30, 2749
1, 11, 7	18, 1021
3, 25, 7	32, 3121
1, 13, 7	20, 1249
3, 27, 7	34, 3517
1, 1, 8	9, 307
3, 17, 8	25, 1939

ρ, ξ, η	u, y
1, 3, 8	11, 427
3, 19, 8	27, 2251
1, 5, 8	13, 571
3, 21, 8	29, 2587
1, 7, 8	15, 739
3, 23, 8	31, 2947
1, 9, 8	17, 931
3, 25, 8	33, 3331
1, 11, 8	19, 1147
3, 27, 8	35, 3739
1, 13, 8	21, 1387
3, 29, 8	37, 4171
1, 15, 8	23, 1651
3, 31, 8	39, 4627
1, 1, 10	11, 463
3, 21, 10	31, 2983
1, 3, 10	13, 607
3, 23, 10	33, 3367
1, 1, 11	12, 553
3, 23, 11	34, 3589
1, 3, 11	14, 709
3, 25, 11	36, 4009

Trinomial Power-form Sextans (H_n).

$$H_n = \mu\,(1^4 + 14y^{2n} + y^{4n}) = \mu\,(x'^6 + y'^6) \div (x'^2 + y'^2).$$

$$x' = \lambda\,(y^n - 1), \quad y' = \lambda\,(y^n + 1);$$

$$\lambda = 1, \ \ \mu = 1, \ \text{ when } \ y = \epsilon; \qquad \lambda = \tfrac{1}{2}, \ \ \mu = \tfrac{1}{16}, \ \text{ when } \ y = \omega.$$

y ; n	x', y'	H_n	y ; n	x', y'	H_n
2; 1	1, 3	73;	3; 1	1, 2	13;
2	3, 5	13.37;	2	4, 5	13.37;
3	7, 9	4993;	3	13, 14	97.349;
4	15, 17	13.13.409;	4	40, 41	13.157.1321;
5	31, 33	1062913;	5	121, 122	13.13.109.11533;
6	63, 65	3217.5233;	5; 1	2, 3	61;
7	127, 129	268664833? †	2	12, 13	109.229;
8	255, 257	13.61.73.74209;	3	62, 63	15272461? †
6; 1	5, 7	1801;	7; 1	3, 4	193;
2	35, 37	13.73.1789;	2	24, 25	457 : 13.61;
3	215, 217	337.6461233;	3	171, 172	865183393;
10; 1	9, 11	13.877;	14; 1	13, 15	41161;
2	99, 101	13.937.8221;	2	195, 197	33549 : 44269;

☞ *See also page* 171, *at foot.*

Factors (p) *of* H_n *in above for* $y = 2, 3$.

$$n = m\xi + n_0, \quad n' = m'\xi + n_0'; \qquad n + n' = \tfrac{1}{2}\xi \ (\text{if } \xi = \epsilon), = \xi \ (\text{if } \xi = \omega).$$
$$[y^\xi \equiv +1 \ (\text{mod } p)].$$

$y = 2$	$p =$	13	37	61	73	97	181	193	277	313	349	373	409	421
	$n_0 =$	2	2	8	1	9	36	20	16	29	78	54	4	76
	$n_0' =$	4	16	22	8	15	54	28	30	49	96	132	98	134
$y = 3$	$p =$	13	37	97	109	157	&c.	241	277	349	601	&c.		
	$n_0 =$	1	2	3	5	4		22	33	3	32			
	$n_0' =$	2	7	21	22	35		38	36	84	33			

Base-Sextan, $N_0 = (x_0^4 - x_0^2 y_0^2 + y_0^4) = (h^4 - h^2k^2 + k^4)$.

Ineffective Characteristics $[C = -1, -\frac{1}{2}, +1]$.

	$x_0, \quad y_0$	a_0, A_0, A_0'	C	$x_0, \quad y_0$	a_0, A_0, A_0'	C
ii	$h, \quad k$	$a_0 = h^2 - k^2$	$C'' = -1$	$k, \quad h$	$a_0 = k^2 - h^2$	$C'' = -1$
iii	$h, \quad k = \epsilon$	$A_0 = h^2 - \frac{1}{2}k^2$	$C''' = -1/2$	$k, \quad h = \epsilon$	$A_0 = k^2 - \frac{1}{2}h^2$	$C''' = -1/2$
iv	$h, \quad k$	$A_0' = h^2 + k^2$	$C^{iv} = +1$	$k, \quad h$	$A_0' = k^2 + \frac{1}{2}h^2$	$C^{iv} = +1$

Equivalent and Reciprocal Characteristics (E, R).

	$x_0 \quad , \quad y_0$	Data	C	C	
ii	$h \quad , \quad k$	$a_0 = k^2 - h^2$	$C'' = (k^2 - 2h^2)/k^2$,	$C'' = (h^2 - \frac{3}{2}k^2)/h^2$	R
iii	$k = \epsilon, \quad h$	$A_0 = h^2 - \frac{1}{2}k^2$	$C''' = (h^2 - \frac{3}{2}k^2)/h^2$,	$C''' = (k^2 - 2h^2)/k^2$	
iii	$k = \epsilon, \quad h$	$A_0 = \frac{1}{2}k^2 - h^2$	$C''' = -(h^2 + \frac{1}{2}k^2)/h^2$,	$C''' = -(2h^2 + k^2)/k^2$	R
iv	$h \quad , \quad k$	$A_0' = -(h^2 + k^2)$	$C^{iv} = -(2h^2 + k^2)/k^2$,	$C^{iv} = -(h^2 + \frac{1}{2}k^2)/h^2$	
i	$h = 1, \quad k = \omega$	$P_0 = \frac{1}{2}(k^4 - k^2) + 1$	$C' = \frac{1}{2}(k^2 - 1)$,	$C' = -(k^2 + 3)/2k^2$	E
iii	$h = 1, \quad k = \omega$	$A_0 = -\frac{1}{2}(1 + k^2)$	$C''' = -(k^2 + 3)/2k^2$,	$C''' = \frac{1}{2}(k^2 - 1)$	

Sextan Factorisants.

N_0	Ref. No.	x_0, y_0	P_0, Q_0	$z_0, \quad C'$	Factorisant.	Serial.
	1	1, 2	7 , 6	3 , 3/2	$(2x)^2 + 5(\frac{1}{2}y)^2 = z^2$	x ;
13	i 3	2, 1	7 , 6	6 , 3	$7x^2 + 2(2y)^2 = z^2$	x, x ; y
	4	2, 1	$\overline{7}$, 6	6 , −11	$-21x^2 + 30(2y)^2 = z^2$	x, x ; z, z
61	i 1	3, 2	31 , 30	15, 11/2	$3(2x)^2 + 13(3y)^2 = z^2$	x, x ; y, y
	3	2, 3	31 , 30	10, 3	$7x^2 + 2(2y)^2 = z^2$	x, x ; y, y
73	i 1	1, 3	37 , 36	12, 4	$(3x)^2 + 15y^2 = z^2$	x, x ;
193	i 1	3, 4	97 , 96	24, 11/2	$3(2x)^2 + 13(3y)^2 = z^2$	x, x ; y, y
193	i 3	4, 3	97 , 96	32, 9	$19x^2 + 5(4y)^2 = z^2$	x, x ; y, y
481	i 1	5, 3	241, 240	80, 24	$(7x)^2 + 23(5y)^2 = z^2$	x, x ;
481	i 3	4, 5	241, 240	48, 9	$19x^2 + 5(4y)^2 = z^2$	x, x ; y, y
13.97	i 1	1, 6	55 , 42	7 , 3/2	$(2x)^2 + 5(\frac{1}{2}y)^2 = z^2$	x, x ;
13.157	i 1	7, 3	85 , 72	24, 4	$(3x)^2 + 15y^2 = z^2$	x, x ;
		x_0, y	a_0, b_0	$z_0, \quad C''$		
	2	1, 2	3 , 2	1 , 1/2	$-2x^2 + 3(\frac{1}{2}y)^2 = z^2$	x ; z
13	ii 3	2, 1	$\overline{3}$, 2	2 , −7	$13x^2 - 3(4y)^2 = z^2$	y, y ; z
	5	1, 2	2 , 3	3/2, 1/4	$-\frac{3}{2}x^2 + 15(\frac{1}{4}y)^2 = z^2$	x ; z
	8	2, 1	$\overline{2}$, 3	3 , −6	$11x^2 - 35y^2 = z^2$	y, y ; z
		x_0, y_0	A_0', B_0'	$z_0, \quad C^{iv}$		
13	iv 4	2, 1	$\overline{5}$, 2	2 , −9	$-\frac{17}{3}x^2 + \frac{5}{3}(4y)^2 = z^2$	x, x ; z
	8	2, 1	$\overline{4}$, 1	1 , −8	$-5x^2 + 21y^2 = z^2$	x, x ; z

Primary Characteristics (C) *of Simple Sextans* (N_0).

$$N_0 = (x_0^4 - x_0^2 y_0^2 + y_0^4) = (1 - k^2 + k^4).$$

Ref. No.	x_0, y_0	P_0	Q_0	z_0	C'	E, R
i 1	1, k	$\frac{1}{2}(k^4-k^2)+1$,	$\frac{1}{2}(k^4-k^2)$	$\frac{1}{2}(k^3-k)$,	$\frac{1}{2}(k^2-1)$	C_2'''
2	1, k	$-[\frac{1}{2}(k^4-k^2)+1]$,	$\frac{1}{2}(k^4-k^2)$	$\frac{1}{2}(k^3-k)$,	$-[\frac{1}{2}(k^4-k^2)+2]/k^2$	
3	k, 1	$\frac{1}{2}(k^4-k^2)+1$,	$\frac{1}{2}(k^4-k^2)$	$\frac{1}{2}(k^4-k^2)$,	$\frac{1}{2}(k^4-3k^2+1)$	
4	k, 1	$-[\frac{1}{2}(k^4-k^2)+1]$,	$\frac{1}{2}(k^4-k^2)$	$\frac{1}{2}(k^4-k^2)$,	$-[\frac{1}{2}(k^4+k^2)+1]$	
	x_0, y_0	a_0 ,	b_0	z_0 ,	C''	
ii 1	1, k	$1-k^2$,	k	1 ,	-1	I
2	1, k	k^2-1 ,	k	1 ,	$(k^2-2)/k^2$	C_3'''
3	k, 1	$1-k^2$,	k	k ,	$1-2k^2$	
4	k, 1	k^2-1 ,	k	k ,	-1	I
5	1, k	k ,	$1\sim k^2$	$(1\sim k^2)/k$,	$(k-1)/k^2$	
6	1, k	$-k$,	$1\sim k^2$	$(1\sim k^2)/k$,	$-(k+1)/k^2$	
7	k, 1	k ,	$k^2\sim 1$	$(k^2\sim 1)/k$,	$k-k^2$	
8	k, 1	$-k$,	$k^2\sim 1$	$(k^2\sim 1)/k$,	$-(k+k^2)$	
	x_0, y_0	A_0 ,	B_0	z_0 ,	C'''	
iii ($k=\epsilon$) 1	1, $k=\epsilon$	$1-\frac{1}{2}k^2$,	$\frac{1}{2}k^2$	$\frac{1}{2}k$,	$-1/2$	I
2	1, $k=\epsilon$	$\frac{1}{2}k^2-1$,	$\frac{1}{2}k^2$	$\frac{1}{2}k$,	$(\frac{1}{2}k^2-2)/k^2$	
3	$k=\epsilon$, 1	$1-\frac{1}{2}k^2$,	$\frac{1}{2}k^2$	$\frac{1}{2}k^2$,	$1-\frac{3}{2}k^2$	C_2''
4	$k=\epsilon$, 1	$\frac{1}{2}k^2-1$,	$\frac{1}{2}k^2$	$\frac{1}{2}k^2$,	$-(1+\frac{1}{2}k^2)$	C_2^{iv}
iii ($k=\omega$) 1	1, $k=\omega$	$\frac{1}{2}(1+k^2)$,	$\frac{1}{2}(1\sim k^2)$	$\frac{1}{2}(1\sim k^2)/k$,	$\frac{1}{2}(k^2-1)/k^2$	
2	1, $k=\omega$	$-\frac{1}{2}(1+k^2)$,	$\frac{1}{2}(1\sim k^2)$	$\frac{1}{2}(1\sim k^2)/k$,	$-\frac{1}{2}(k^2+3)/k^2$	C_1'
3	$k=\omega$, 1	$\frac{1}{2}(k^2+1)$,	$\frac{1}{2}(k^2\sim 1)$	$\frac{1}{2}(k^2\sim 1)/k$,	$\frac{1}{2}(1-k^2)$	
4	$k=\omega$, 1	$-\frac{1}{2}(k^2+1)$,	$\frac{1}{2}(k^2\sim 1)$	$\frac{1}{2}(k^2\sim 1)/k$,	$-\frac{1}{2}(1+3k^2)$	
	x_0, y_0	A_0' ,	B_0'	z_0 ,	C^{iv}	
iv 1	1, k	$1+k^2$,	k	1 ,	1	I
2	1, k	$-(1+k^2)$,	k	1 ,	$-(k^2+2)/k^2$	C_4'''
3	k, 1	k^2+1 ,	k	k ,	1	I
4	k, 1	$-(k^2+1)$,	k	k ,	$-(2k^2+1)/k^2$	
iv Extra 5	1, k	$(2-3k+2k^2)$,	$(1-2k+k^2)$	$(1-2k+k^2)/k$,	$(1-3k+2k^2)/k^2$	
6	1, k	$-(2-3k+2k^2)$,	$(1-2k+k^2)$	$(1-2k+k^2)/k$,	$-(3-3k+2k^2)/k^2$	
7	k, 1	$(2-3k+2k^2)$,	$(1-2k+k^2)$	$(1-2k+k^2)$,	$(2-3k+k^2)$	
8	k, 1	$-(2-3k+2k^2)$,	$(1-2k+k^2)$	$(1-2k+k^2)$,	$-(2-3k+3k^2)$	

Characteristics (C) *of Sextan Primes* (N_0).

$N = 13 = 1^4 - 1^2.2^2 + 2^4$. $N_0 = 61 = 3^4 - 3^2.2^2 + 2^4$.

Ref. No.	x_0, y_0	P_0, Q_0	z_0, C'	E	R	R	x_0, y_0	P_0, Q_0	z_0, C'	E	R	R
1	1, 2	7, 6	3, 3/2				3, 2	31, 30	15, 11/2			
i 2	1, 2	$\bar{7}$, 6	3, −2	0	0	−2	3, 2	$\overline{31}$, 30	15, −10			−10/9
3	2, 1	7, 6	6, 3			−3/4	2, 3	31, 30	10, 3			−3/4
4	2, 1	$\bar{7}$, 6	6, −11		3/4		2, 3	$\overline{31}$, 30	10, −35/9			
	x_0, y_0	a_0, b_0	z_0, C''				x_0, y_0	a_0, b_0	z_0, C''			
1	1, 2	$\bar{3}$, 2	1, −1	1	−1/2		3, 2	5, 6	3, −1	1	−1/2	
2	1, 2	3, 2	1, 1/2	−5/4	−5		3, 2	$\bar{5}$, 6	3, −7/2	−1/4	1/3	
3	2, 1	$\bar{3}$, 2	2, −7				2, 3	5, 6	2, 1/9			
ii 4	2, 1	3, 2	2, −1	1	−1/2		2, 3	$\bar{5}$, 6	2, −1	1	−1/2	
5	1, 2	2, 3	3/2, 1/4	−3/2	−3		3, 2	6, 5	5/2, −3/4			3
6	1, 2	$\bar{2}$, 3	3/2, −3/4			3	3, 2	$\bar{6}$, 5	5/2, −15/4			
7	2, 1	2, 3	3, −2	0	0	−2	2, 3	6, 5	5/3, 2/9			
8	2, 1	$\bar{2}$, 3	3, −6				2, 3	$\bar{6}$, 5	5/3, −10/9			−10
	x_0, y_0	A_0, B_0	z_0, C'''				x_0, y_0	A_0, B_0	z_0, C'''			
1	1, 2	$\bar{1}$, 2	1, −1/2		−1	1	3, 2	7, 2	1, −1/2		−1	1
iii 2	1, 2	1, 2	1, 0	−2	−2	0	3, 2	$\bar{7}$, 2	1, −4			−4/3
3	2, 1	$\bar{1}$, 2	2, −5		1/2	−5/4	2, 3	7, 2	2/3, 1/3		−7/2	−1/4
4	2, 1	1, 2	2, −3		1/4	−3/2	2, 3	$\bar{7}$, 2	2/3, −11/9			−11/2
	x_0, y_0	A'_0, B'_0	z_0, C^{iv}				x_0, y_0	A'_0, B'_0	z_0, C^{iv}			
1	1, 2	5, 2	1, 1	−1	−1/2		3, 2	13, 6	3, 1	−1	−1/2	
2	1, 2	$\bar{5}$, 2	1, −3/2	1/4	−3		3, 2	$\overline{13}$, 6	3, −11/2			−11/9
3	2, 1	5, 2	2, 1	−1	−1/2		2, 3	13, 6	2, 1	−1	−1/2	
iv 4	2, 1	$\bar{5}$, 2	2, −9				2, 3	$\overline{13}$, 6	2, −17/9			
5	1, 2	4, 1	1/2, 3/4		−11		3, 2	8, 1	1/2, −1/4	−7/2		1/3
6	1, 2	$\bar{4}$, 1	1/2, −5/4	1/2		−5	3, 2	$\bar{8}$, 1	1/2, −17/4			
7	2, 1	4, 1	1, 0	−2	−2	0	2, 3	8, 1	1/3, 4/9			
8	2, 1	$\bar{4}$, 1	1, −8				2, 3	8, 1	1/3, −4/3			−4

☞ The cases $C'' = -1$, $C''' = -1/2$, $C^{iv} = +1$ are *ineffective*.
The cases $C' = -2$, $C'' \equiv -2$, $C''' = 0$, $C^{iv} = 0$ give *Trin-Aurifeuillians*.

Characteristics (C) *of Composite Sextans* (N_0).

$$N_0 = 1261 = 13.97 = 1^4 - 1^2.6^2 + 6^4.$$

$N_0 = 1261$ (Primary). $N_0 = 13.97$ (Secondary).

	Ref. No.	x_0, y_0	P_0, Q_0	z_0, C'	E, R, I	P_0, Q_0	z_0, C'	R
i	1	1, 6	631, 630	105, 35/2		55, 42	7, 3/2	7
	2	1, 6	$\overline{631}$, 630	105, −158/9		$\overline{55}$, 42	7, −14/9	
	3	6, 1	631, 630	630, 595		55, 42	42, 19	7/6
	4	6, 1	$\overline{631}$, 630	630, −667		$\overline{55}$, 42	42, −91	
		x_0, y_0	a_0, b_0	z_0, C''		a_0, b_0	z_0, C''	
ii	1	1, 6	35, 6	1, 17/18	−53,	19, 30	5, 1/2	−5
	2	1, 6	$\overline{35}$, 6	1, −1	1, −1/2, I	$\overline{19}$, 30	5, −5/9	
	3	6, 1	35, 6	6, −1	1, −1/2, I	19, 30	30, −17	5/6
	4	6, 1	$\overline{35}$, 6	6, −71		$\overline{19}$, 30	30, −55	
	5	1, 6	6, 35	35/6, 5/36		30, 19	19/6, 29/36	
	6	1, 6	$\overline{6}$, 35	35/6, −7/36		$\overline{30}$, 19	19/6, −31/36	
	7	6, 1	6, 35	35, −30		30, 19	19, −6	
	8	6, 1	$\overline{6}$, 35	35, −42		$\overline{30}$, 19	19, −66	
		x_0, y_0	A_0, B_0	z_0, C'''		A_0, B_0	z_0, C'''	
iii	1	1, 6	17, 18	3, 4/9		31, 10	5/3, 5/6	−17
	2	1, 6	$\overline{17}$, 18	3, −1/2	−1, 1, I	$\overline{31}$, 10	5/3, −8/9	
	3	6, 1	17, 18	18, −19	−19/18,	31, 10	10, −5	1/2
	4	6, 1	$\overline{17}$, 18	18, −53	17/18,	$\overline{31}$, 10	10, −67	
		x_0, y_0	A'_0, B'_0	z_0, C^{iv}		A'_0, B'_0	z_0, C^{iv}	
iv	1	1, 6	37, 6	1, 1	−1, −1/2, I	43, 14	7/3, 7/6	19
	2	1, 6	$\overline{37}$, 6	1, −19/18	−19,	$\overline{43}$, 14	7/3, −11/9	
	3	6, 1	37, 6	6, 1	−1, −1/2, I	43, 14	14, 7	3/2
	4	6, 1	$\overline{37}$, 6	6, −73		$\overline{43}$, 14	14, −79	

Simple Sextan Chains, $N_r = (1^4 - y_r^2 + y_r^4) = L_r . M_r$.

$y_{-1} = 0,\ \ y_1 = 1,\ \ y_{r+1} = y_0^2 . y_r - y_{r-1}$; $\qquad C' = \frac{1}{2}(y_0^2 - 1)$.

$L_0 = 1,\ \ M_0 = 13$; $\qquad M_{r-1} = 1 + y_{r-1} y_r = L_r,\ \ M_r = 1 + y_r y_{r+1} = L_{r+1}$.

r	0	1	2	3	4	5	6
x, y	1, 2	1, 6	1, 16	1, 42	1, 110	1, 288	1, 754
M	13 ;	97 ;	673 ;	4621 ;	13·2437 ;	37·5869 ;	73·20389 ;
r	7	8					
x, y	1, 1974	1, 5168					
M	13·277·2833 ;	69923041 ;					

r	0	1	2	3	4
x, y	1, 3	1, 24	1, 189	1, 1488	1, 11715
M	73 ;	13·349 ;	281233 ;	13·37·36241 ;	5281·204601 ;
x, y	1, 4	1, 60	1, 896	1, 13380	1, 199804
M	241 ;	37·1453 ;	37·457·709 ;	61·97·193·2341 ;	
x, y	1, 5	1, 120	1, 2875	1, 68880	1, 1697245
M	601 ;	345001 ;	13·109·139753 ;	37·457·1993·3373 ;	

r	0	1	2	3
x, y	1, 6	1, 210	1, 7344	1, 256830
M	13·97 ;	109·14149 ;	6961·270961 ;	13·157 ;
x, y	1, 7	1, 336	1, 16121	1, 773472
M	13·181 ;	157·34501 ;	1621·7692253 ;	
x, y	1, 8	1, 504	1, 31744	1, 1999368
M	37·109 ;	15998977 ;	13· §	
x, y	1, 9	1, 720	1, 57591	1, 4606560
M	6481 ;	41465521 ;	§	
x, y	1, 10	1, 990	1, 98000	1, 9701010
M	9901 ;	13·7463077 ;	5521·172196881 ;	
x, y	1, 11	1, 1320	1, 158389	
M	13·1117 ;	6301·33181 ;		

Sextan Chains and Series.

i.—(3). $C' = 3$; $z^2 - 2(2y)^2 = 7x^2 = 7.2^2$; $3^2 - 2.2^2 = +1$; $x = x_0 = 2$.

x-Chn. 1	x, y	2, 1	2, 9	2, 53	2, 309	2, 1801
	L	1 ;	13 ;	13·37 ;	16381 ;	556513 ;
	M	13 ;	13·37 ;	16381 ;	556513 ;	18905101 ;
x-Chn. 2	x, y	2, 3	2, 19	2, 111	2, 647	2, 3771
	L	1 ;	61 ;	2113 ;	71821 ;	97·25153 ;
	M	61 ;	2113 ;	71821 ;	97·25153 ;	13·73·87337 ;

i.—(3). $C' = 3$; $z^2 - 7x^2 = 8y^2 = 3.1^2$; $8^2 - 7.3^2 = +1$; $y = y_0 = 1$.

y-Series	x, y, z	2, 1, 6	34, 1, 90	542, 1, 1434	8638, 1, 22854
	L	1 ;	1069 ;	241.1213 ;	13.409.14029 ;
	M	13 ;	1249 ;	295201 ;	13·73·78649 ;

i.—(4). $C' = -11$; $z^2 - 30(2y)^2 = -21x^2 = -21.2^2$; $11^2 - 30.2^2 = +1$; $x = x_0 = 2$.

x-Chn. 1	x, y, z	2, 1, 6	2, 17, 186	2, 373, 4086	x-Chn. 2	2, 5, 54	2, 109, 1194	2, 2393, 26214
	L	1 ;	13 ;	6337 ;		1 ;	541 ;	97·2689 ;
	M	13 ;	6337 ;	13·234961 ;		541 ;	97·2689 ;	13·9670849 ? †

ii.—(2). $C'' = 1/2$; $z^2 - 3(\frac{1}{2}y)^2 = -2x^2 = -2.1^2$; $2^2 - 3.1^2 = +1$; $x = x_0 = 1$.

x-Series	x, y, z	1, 2, 1	1, 6, 5	1, 22, 19	1, 82, 71	1, 306, 265	1, 1142, 989
	a, b	3, 2	35, 6	483, 22	6723, 82	93635, 306	1304163, 1142
	a, b	3, 2	19, 30	243, 418	3363, 5822	46819, 81090	652083, 1129438
	α, β	1, 0	3, 2	11, 6	41, 24	153, 88	571, 330
	α′, β′	3, 2	9, 4	33, 20	123, 70	459, 266	1713, 988
	L	1 ;	13 ;	157 ;	37·61 ;	31153 ;	13·33457 ;
	M	13 ;	97 ;	1489 ;	20029 ;	13·21649 ;	3910513 ;

ii.—(3). $C'' = -7$; $z^2 - 13x^2 = -3(4y)^2 = -48.1^2$; $649^2 - 13.180^2 = +1$;
$y = y_0 = 1$.

y-Series 1	x, y, z	2, 1, 2	938, 1, 3382	y-Series 2	1658, 1, 5978
	a, b	3, 2	879843, 938		2748963, 1658
	a, b	3, 2	879837, 3382		2748957, 5978
	α, β	1, 0	1, 720		1, 720
	α′, β′	3, 2	3, 1222		3, 3818
	L	1 ;	13·39877 ;		13·39877 ;
	M	13 ;	1493293 ;		14577133 ;

Sextan Chains and Series (*continued*).

ii.—(5). $C'' = 1/4$; $z^2 - 15(\frac{1}{4}y)^2 = -\frac{3}{2}x^2 = -\frac{3}{2}.1^2$; $4^2 - 15.1^2 = +1$; $x = x_0 = 1$.

x-Series	x, y, z	1, 2, 3	1, 14, 27	1, 110, 213	1, 866, 1677	1, 6818, 13203
	a, **b**	2, 3	14, 195	110, 12099	866, 749955	6818, 46485123
	a, b	2, 3	50, 189	3026, 11715	187490, 726141	11621282, 45009027
	α, β	0, 1	1, 6	6, 49	49, 384	384, 3025
	α', β'	2, 3	3, 32	32, 243	243, 1922	1922, 15123
	L	1 ;	37 ;	2437 ;	277.541 ;	13.715237 ;
	M	13 ;	1033 ;	13.4621 ;	3753133 ;	193.1204141 ;

ii.—(8). $C'' = -6$; $z^2 - 11x^2 = -35y^2 = -35.1^2$; $10^2 - 11.3^2 = +1$; $y = y_0 = 1$.

y-Series 1	x, y, z	2, 1, 3	29, 1, 96	578, 1, 1917	y-Series 2	11, 1, 36	218, 1, 723	4349, 1, 14424
	a, b	$\bar{2}$, 3	840, 29	334083, 578		120, 11	47523, 218	18913800, 4349
	a, b	$\bar{2}$, 3	96, 835	1917, 334078		36, 115	723, 47518	14424, 18913795
	α, β	1, 0	13, 12	249, 250		3, 2	51, 50	1007, 1008
	α', β'	$\bar{3}$, 2	31, 36	667, 672		21, 26	473, 468	9389, 9384
	L	1 ;	313 ;	13.61.157 ;		13 ;	5101 ;	97.20929 ;
	M	13 ;	37.61 ;	37.24229 ;		1117 ;	442753 ;	13.13554829 ;

iv.—(4). $C^{iv} = -9$; $z^2 - \frac{5}{3}(4y)^2 = -\frac{17}{3}x^2 = -\frac{17}{3}.2^2$; $31^2 - \frac{80}{3}.6^2 = +1$; $x = x_0 = 2$.

x-Series 1	x, y, z	2, 1, 2	2, 43, 222	x-Series 2	2, 19, 98
	A′, B′	$\bar{5}$, 2	1853, 86		365, 38
	A', B'	$\bar{5}$, 2	16637, 9546		3245, 1862
	γ, δ	1, 0	43, 22		19, 10
	γ', δ'	$\bar{5}$, 2	215, 112		95, 48
	L	1 ;	397 ;		61 ;
	M	13 ;	13.661 ;		2113 ;

iv.—(8). $C^{iv} = -8$; $z^2 - 21y^2 = -5x^2 = -5.2^2$; $55^2 - 21.12^2 = +1$; $x = x_0 = 2$.

x-Series 1	x, y, z	2, 1, 1	2, 67, 307	x-Series 2	2, 43, 197
	A′, B′	$\bar{4}$, 1	4493, 134		1853, 86
	A', B'	$\bar{4}$, 1	10109, 5230		4163, 2154
	γ, δ	1, 0	49, 18		33, 11
	γ', δ'	$\bar{4}$, 1	149, 52		94, 35
	L	1 ;	1429 ;		661 ;
	M	13 ;	73.193 ;		13.397 ;

Sextans.—2ic Partitions of L, M.

i.—(1)

L	1;	13;	97;	673;	4621;	13.2437;	37.5869;
a, b	1, 0	3, 2	9, 4	23, 12	61, 30	159, 80	417, 208
A, B	1, 0	1, 2	7, 4	25, 4	17, 38	127, 72	249, 72
A′, B′	2, 1	4, 1	10, 1	26, 1	68, 1	178, 1	466, 1

i.—(3)

x-Chain 1

L	1;	13;	13.37;	16381;	556513;	18905101;
a, b	1, 0	3, 2	15, 16	91, 90	527, 528	3075, 3074
A, B	1, 0	1, 2	7, 12	41, 70	239, 408	1393, 2378
A′, B′	2, 1	4, 1	22, 1	128, 1	746, 1	4348, 1

x-Chain 2

L	1;	61;	2113;	71821;	97.25153;
a, b	1, 0	5, 6	33, 32	189, 190	1105, 1104
A, B	1, 0	7, 2	41, 12	239, 70	1393, 408
A′, B′	2, 1	8, 1	46, 1	268, 1	1562, 1

i.—(3)

L	1;	1069;	241.1213;	13.409.14029;
a, b	1, 0	13, 30	222, 493	7872, 3553
A, B	1, 0	31, 6	511, 102	8161, 1632
A′, B′	2, 1	37, 10	571, 106	9025, 1512
M	13;	1249;	295201;	13.73.78649;
a, b	3, 2	15, 32	224, 495	7874, 3555
A, B	1, 2	7, 20	103, 308	1633, 4898
A′, B′	4, 1	41, 12	623, 176	9232, 1879

i.—(4)

	x-Chain 1				*x*-Chain 2		
L	1;	13;	6337;	13.234961;	1;	541;	97.2689;
a, b	1, 0	3, 2	71, 36	1563, 782	1, 0	21, 10	457, 228
A, B	1, 0	1, 2	23, 44	505, 966	1, 0	23, 2	505, 44
A′, B′	2, 1	4, 1	97, 32	2140, 713	2, 1	28, 9	625, 208

ii.—(2)

L	1;	13;	157;	37.61;	31153;	13.33457;
a, b	1, 0	3, 2	11, 6	41, 24	153, 88	571, 330
A, B	1, 0	1, 2	7, 6	23, 24	89, 88	329, 330
A′, B′	2, 1	4, 1	13, 2	55, 16	185, 32	692, 121
M	13;	97;	1489;	20029;	13.21649;	
a, b	3, 2	9, 4	33, 20	123, 70	459, 266	
A, B	1, 2	7, 4	17, 20	73, 70	263, 266	
A′, B′	4, 1	10, 1	41, 8	221, 98	557, 98	

ii.—(5)

L	1;	37;	2437;	277.541;	13.715237;	M	13;	1033;	13.4621;	3753133;
a, b	0,1	1,6	6,49	49,384	384, 3025	a, b	2,3	3,32	32, 243	243, 1922
A, B	1,0	5,2	43,14	335,112	2641, 880	A, B	1, 2	29,8	211, 72	1679, 558
A′, B′	2,1	7,2	55,14	433,112	3409, 880	A′, B′	4, 1	35,8	275, 72	2165, 558

ii.—(8)

y-Series 1

L	1;	313;	13.61.157;	M	13;	37.61;	37.24229;
a, b	1, 0	13, 12	249, 250	a, b	3, 2	31, 36	667, 672
A, B	1, 0	11, 8	349, 30	A, B	1, 2	47, 4	901, 168
A′, B′	2, 1	19, 4	283, 31	A′, B′	4, 1	55, 16	1010, 203

ii.—(8)

y-Series 2

L	13;	5101;	97.20929;	M	13.1117;	442753;	13.13554829;
a, b	3, 2	51, 50	1007, 1008	a, b	21, 26	473, 468	9389, 9384;
A, B	1, 2	49, 30	1369, 228	A, B	61, 60	191, 368	
A′, B′	2, 1	76, 15	1519, 304	A′, B′	122, 11	710, 143	

Factorisants of Class i *of Trinomial Sextans.*

$$N = x^4+14x^2y^2+y^4; \qquad (2C'-14)\,x^2+(C'^2-1)\,y^2 = z^2.$$

Ex.	N_0	x_0, y_0	P_0, Q_0	C', z_0	Factorisant.	Serial.
1	16.13	1, 3	28, 24	3, 8	$-8x^2+8y^2=z^2$	x
2	16.13	3, 1	28, 24	19, 24	$6(2x)^2+10(6y)^2=z^2$	$x, x;\ y$
3	16.13	1, 3	$\overline{17}$, 9	$\bar{2}$, 3	$-2(3x)^2+3y^2=z^2$	x
4	16.13	3, 1	17, 9	8, 9	$2x^2+7(3y)^2=z^2$	$x, x;\ y$
5	16.61	1, 5	$\overline{124}$, 120	$\bar{5}$, 24	$-24x^2+24y^2=z^2$	x
6	73	1, 2	37, 36	9, 18	$(2x)^2+5(4y)^2=z^2$	x
7	481	1, 4	241, 240	15, 60	$(4x)^2+14(4y)^2=z^2$	x
8	13.37	1, 4	25, 12	3/2, 3	$-11x^2+5(\frac{1}{2}y)^2=z^2$	$x;\ y, y$
9	13.37	4, 1	25, 12	9, 12	$(2x)^2+5(4y)^2=z^2$	x, x

Chain Examples from above Factorisants.

Ex. 1. $C'=3$;
$$y^2-2(\tfrac{1}{4}z)^2 = x^2 = +1.$$

x, y, z	1,3,8	1,17,48	1,99,280	1,577,1632
x', y'	1, 2	8, 9	49, 50	288, 289
$\frac{1}{4}$L	1 ;	13 ;	421 ;	14281 ;
$\frac{1}{4}$M	13 ;	421 ;	14281 ;	485113 ;

Ex. 5. $C'=-5$;
$$y^2-6(\tfrac{1}{12}z)^2 = x^2 = +1.$$

x, y, z	1,5,24	1,49,240	1,485,2376
x', y'	2, 3	24, 25	242, 243
$\frac{1}{4}$L	1 ;	61 ;	13.457 ;
$\frac{1}{4}$M	61 ;	13.457 ;	37.15733 ;

Ex. 3. $C'=-2$; $(\frac{1}{3}z)^2-3(\frac{1}{3}y)^2=-2x^2=-2$.

x, y, z	1, 3, 3	1, 9, 15	1, 33, 57	1, 123, 213	1, 459, 795	1, 1713, 2967
x', y'	1, 2	4, 5	16, 17	61, 62	229, 230	856, 857
l	1 ;	13 ;	37 ;	2029 ;	7057 ;	13.30241 ;
m	13 ;	37 ;	2029 ;	7057 ;	13.30241 ;	61.22441 ;

Ex. 6. $C'=9$;
$$(\tfrac{1}{2}z)^2-5(2y)^2 = x^2 = +1.$$

x, y, z	1,2,18	1,36,322	1,646,5778
x', y'	1, 3	35, 37	645, 647
L	1 ;	73 ;	13.1789 ;
M	73 ;	13.1789 ;	7488433 ;

Ex. 7. $C'=15$;
$$(\tfrac{1}{4}z)^2-14y^2 = x^2 = +1.$$

x, y, z	1,4,60	1,120,1796	1,3596,53820
x', y'	3, 5	119, 121	3595, 3597
L	1 ;	13.37 ;	431521 ;
M	13.37 ;	431521 ;	387504961 ? †

Ex. 9. $C'=9$; $(\frac{1}{2}z)^2-5(2y)^2=x^2=+16$.

Chain 1	x, y, z	4, 1, 12	4, 21, 188	4, 377, 3372
	x', y'	3, 5	17, 25	373, 381
	L	13 ;	37 ;	7933 ;
	M	37 ;	7933 ;	97.26293 ;
Chain 2	x, y, z	4, 3, 28	4, 55, 492	4, 987, 8828
	x', y'	1, 7	51, 59	983, 991
	L	13 ;	181 ;	13.4177 ;
	M	181 ;	13.4177 ;	17480773 ? †

Simple Duodecimans, $N = (y^{12} + 1^{12}) \div (y^4 + 1^4)$.
[All divisors ≮ 100,000 cast out.]

y	N		y	N	
1	1;		51	56737.806668273;	
2	241;		52	73.24793.29537569;	
3	6481;		53		
4	97.673;		54	97.337.433.5108113;	
5	390001;		55	193.	
6	1678321;		56	73.	
7	73.193.409;		57		
8	433.38737;		58	20161.33409.190129;	
9	97.577.769;		59		
10	99990001;	Lo, R	60	80449.2087802049;	
11	10657.20113;	B	61	1297.	
12	193.2227777;	B	62		
13	815702161;		63		
14	1475750641;		64	577.487824887233;	La, Ll
15	2562840001;		65		
16	193.22253377;	La	66	73.15217.324115321;	
17	73.1321.72337;		67	1489.	
18			68		
19	4297.3952393;		69		
20	31177.821113;		70	457.5689.221733937;	
21	73.518118697;		71	87553.7375572577;	
22			72	6529.	
23	937.83575993;		73	97.	
24	97.1134793633;		74	5857.	
25			75		
26			76		
27			77		
28	7321.51605161;		78	81409.	
29	9001.55576681;		79	4729.88681.3677889;	
30	73.		80	73.60937.377151601;	
31			81		
32	241; 4562284561;	La	82	28081.	
33	15313.91844017;		83		
34			84	1753.	
35	5737.392517673;		85	1873.	
36	5953.473896897;		86		
37			87		
38	409.18217.583537;		88	97.	
39			89		
40			90	73.937.1201.52400401;	
41			91	19681.	
42			92	94777.	
43	73.97.313.5273617;		93	97.	
44			94	73.	
45	35569.472746529;		95	9649.73897.1129986241;	
46			96	337.29473.726299281;	
47			97	5281.	
48	8929.3155927329;		98	3361.	
49			99		
50			100		

Simple Duodecimans (continued).

y	N	y	N
101	97.23529.3329497177;	151	97.
102		152	
103	73.	153	73.
104		154	4177.23929.3165022057?
105	2137.	155	3049.
106	97.	156	337.
107		157	2161.
108		158	21961.
109		159	
110		160	1657.2137.2689.45106801;
111	457.	161	
112		162	
113	241.	163	73.937.
114	31489.	164	769.
115		165	337.
116	73.	166	
117	409.1801.12289.3879121;	167	73.
118		168	313.673.
119		169	1009.
120	241.	170	97.
121	97.241.1777.1106131489;	171	769.
122	2017.	172	337.2377.89017.10702257;
123		173	4657.
124	5233.49201.217094497;	174	
125	73.	175	
126		176	73.1033.5857.2084504977?
127		177	193.313.937.35521.479137;
128	241; 3361.88959882481; Ll	178	2161.
129	73.4657.	179	
130	42457.44809.42877777;	180	21169.
131	313.9001.	181	193.337.2113.45433.184489;
132	601.2689.4969.11477761;	182	313.1609.
133	433.25633.8821161889?	183	409.21673.
134	2377.	184	
135	1033.	185	97.1489.
136	313.	186	193.8929.
137	1249.4729.	187	
138	193.10513.66529.974401;	188	1609.
139	73.457.7057.21961.26953;	189	73.12409.
140	97.433.	190	97.6217.96697.29124817;
141		191	
142		192	
143	2593.	193	74209.
144	7681.40609.592734049;	194	
145	313.7537.12073.6860977;	195	577.
146		196	19489.
147	20113.	197	
148		198	73.97.
149	6841.	199	
150		200	193.1489.

Duodecimans, $\mathrm{N} = (x^{12} + y^{12}) \div (x^4 + y^4)$; $[x$ and $y > 1]$.

x, y	N	x, y	N	x, y	N
3, 2	5521;	11, 8	1129.151609;	13, 18	73.121061257? †
5	380881;	15	241.9843601? †	19, 18	1609.2593.3433;
7	337.16993;	17, 8	73.91101817? †	7, 20	4561.5529841;
9	42942001;	3, 10	73.1358857;	5, 3	346561;
11	97.601.3673;	7	97.97.8689;	7	5576881;
13	73.11168137? †	9	73.1060777;	11	8209.25969;
15, 2	241.10631041? †	11	167948881? ‡	13	433.1878577;
3, 4	51361;	19, 10	457.5449.6337;	17	73.95465737? †
5	73.4057;	5, 12	73.5117977;	19, 3	1201.14132401? †
7	1993.2617;	7	313.1233097;	7, 5	4654801;
9	41432641;	11	73.97.48121;	9	39336721;
11, 4	73.2885977;	17, 12	4969.1141849;	11, 5	205598881? ‡
5, 6	1009.1249;	9, 14	601.1009.2089;	9, 7	73.452857;
7	4332721;	11, 14	769.1466449;	11	184970641? ‡
11	197063761? ‡	3, 16	‡	13, 7	313.2405497;
13, 6	337.1321.1753;	5	‡	11, 9	409.394489;
3, 8	577.28513;	7	‡	13, 9	3529.190249;
5	2857.5113;	9	97.1249.32257;	17, 11	193.30918577? †
7	12707521;	11	‡	19, 13	73.192838297? †
9, 8	1489.22129;	15, 16	313.11310217? †	19, 15	313.337.122761;

Simple 24-mans, $\mathrm{N} = (1^{24} + y^{24}) \div (1^8 + y^8)$.

[All divisors < 100,000 cast out.]

y	N	y	N
1	1;	17	
2	97.673;	18	1153;
3	97.577.769;	19	241.577.1009.4657.14929.29569;
4	193.22253377; L	20	
5		21	673.193.433.
6	5953.473896897;	22	241.
7		23	
8	577.487824887233; Ll	24	33409.
9		25	97.
10	*	26	
11	97.241.1777.1106131489;	27	
12	7681.	28	193.
13	1009.	29	673.
14	19489.	30	
15	3169.	31	97.
16		32	97.673.
		33	3313.

* No factor < 120,000.

$N = (y^{32}+1),\ (y^{64}+1),\ (y^{48}+1)\div(y^{16}+1),\ (y^{96}+1)\div(y^{32}+1).$

y	$N = y^{32}+1$		$N = y^{64}+1$	
1	2 ;		2 ;	
2	641 . 6700417 ;	E	27417 . 67280421310721 ;	L
3	2 .	§	2 .	§
4	27417 . 67280421310721 ;	L		¶
5	2 . 641 .	§	2 . 769 .	§
6	2753 .	§		‡
7	2 .	§	2 .	‡
8	$(2^{32}+1)$; &c.	¶	$(2^{64}+1)$; 769 .	¶
9	2 .	§	2 . 257 .	§
10	19841 .	¶		¶
11	2 . 193 . 257 .	§	2 .	‡
12	769 .	§		‡
13	2 . 193 .	†	2 . 257 .	†
14		†		†
15	2 .	†	2 .	†
16		¶		¶
17	2 .	†	2 . 769 .	†
18		†	257 . 769 .	†
19	2 .	†	2 .	†
20	193 . 577 . 641 .	†		†

y	$N = (y^{48}+1)\div(y^{16}+1)$		$N = (y^{96}+1)\div(y^{32}+1)$	
1	1 ;		1 ;	
2	193 . 22253377 ;	L		¶
3		§	7297 .	§
4		¶	769 .	¶
5	97 .	§	193 .	§
6	193 . 8641 .	§		‡
7	97 .	§	2113 .	‡
8	193 . 22253377 ;	¶	3457 .	¶
9		§		§
10	97 .	¶	193 . 769 .	¶
11	577 .	§	2689 .	‡
12	7489 .	§	1153 . 2689 . 4993 .	‡
13	97 .	†		
14	97 .	†	577 .	
15	97 .	†		
16		¶		¶
17	97 .	†		
18	193 .	†		
19		†		
20		†	6529 .	

¶ No more divisors < 100,000.

One Root (y) *of* $y^{\epsilon}+1\equiv 0$ *and* $(y^{3\epsilon}+1)\div(y^{\epsilon}+1)\equiv 0$ (mod p and p^{κ});
$[\epsilon = 8, 16, 32, 64]$.

Supplement to Tables, pages 93–96.

☞ *Roots marked * are Least Roots.*

$y^{16}+1\equiv 0$		$y^{32}+1\equiv 0$	$y^{64}+1\equiv 0$	$\frac{y^{24}+1}{y^{8}+1}\equiv 0$		$\frac{y^{48}+1}{y^{16}+1}\equiv 0$	$\frac{y^{96}+1}{y^{32}+1}\equiv 0$
p , y	p , y	p , y	p , y	p , y	p , y	p , y	p , y
1153, 512	5441, * 47	1153, 291	1153, 343	1009, * 13	5281, * 72	1153, * 8	1153, * 12
1217, * 21	5569, 2036	1217, 307	1409, 387	1153, * 18	5521, 2515	1249, 76	2113, * 7
1249, 315	5857, 257	1409, 415	2689, 362	1201, 431	5569, * 50	2017, 180	2689, * 11
1409, * 10	5953, 231	1601, 674	3329, 1795	1249, 469	5857, 342	2113, 49	3457, * 8
1601, 109	6113, 140	2113, 343	3457, 1000	1297, 125	5953, * 6	2593, 1118	4801, 525
1697, * 6	6337, * 68	2689, 717	4481, 1732	1489, 575	6337, * 64	2689, 121	4993, * 12
1889, * 31	6529, 530	2753, * 6	4993, 2518	1777, * 11	6481, * 82	3169, 1008	5569, 901
2017, 500	6689, 3236	3137, 300	6529, 2338	1873, 276	6529, 3225	3361, 277	5953, 1240
2081, 888	6977, 2201	3329, 447	7297, 3585	2017, 128	6577, 240	3457, 64	6337, 601
2113, 679	7297, 729	3457, 512	7681, 243	2113, 288	6673, 1431	4129, 1455	6529, 517
2273, 1173	7393, 2093	4289, 1076	7937, 1306	2161, 1098	6961, 2268	4513, 1778	7297, * 3
2593, * 5	7457, 2542	4481, 2035	9473, 28	2593, 98	7057, 2757	4801, 814	7489, 2381
2657, * 13	7489, 1728	4673, 2204	9601, 988	2689, 1196	7297, * 81	4993, 144	7873, 3566
2689, * 21	7649, 3685	4801, 985	9857, 2187	2833, 298	7393, 1590	5281, 1786	8641, 475
2753, * 36	7681, 2132	4993, 1556		3121, 1012	7489, * 52	5569, 1273	9601, 906
3041, 1519	7841, 1082	5441, 1638		3169, * 15	7537, 1345	5857, 1879	
3137, 973	7873, 516	5569, 241		3217, 990	7681, * 12	5953, 1726	
3169, 233	7937, 3884	5953, 2840		3313, * 33	7873, 905	6337, * 8	
3329, 69	8161, 2993	6337, 2580		3361, 486	8017, 2226	6529, 400	
3361, 1515	8353, 1516	6529, 267		3457, 639	8161, 1142	7297, * 9	
3457, 588	8513, * 46	6977, 3055		3697, 869	8209, 1069	7393, 1447	
3617, 1084	8609, 1522	7297, * 27		3793, 1132	8353, 1115	7489, * 12	
4001, 1011	8641, * 40	7489, 1384		3889, * 71	8641, * 36	7873, 1461	
4129, 1902	8737, 4024	7681, 2399		4129, 1152	8689, 2184	8161, 2520	
4289, 254	8929, 308	7873, 176		4177, 204	8737, 677	8353, 3395	
4481, * 50	9281, 2727	7937, 819		4273, 601	8929, 506	8641, * 6	
4513, 409	9377, * 62	8513, 2187		4513, 498	9601, * 50	8737, 983	
4673, 2304	9473, 1089	8641, 2448		4561, 1724	9649, 655	8929, 4271	
4801, 423	9601, 578	9281, 2958		4657, * 19	9697, 2148	9601, 4751	
4993, 170	9697, 2477	9473, 33		4801, * 58		9697, 3825	
5153, 2169	9857, 660	9601, 1134		4993, 764			
5281, 2319	97^2, 4337	9857, 2324		5233, 678	97^2, * 53		

Additional Quartans and Sextans, [N > 9.10⁶, but $\ngtr 10^7$].

Supplement to Tables, pages 125, 170; [x and $y > 1$].

Quartans, N = (x^4+y^4)						Sextans, N = $(x^6+y^6)\div(x^2+y^2)$					
x, y	N	x, y	N	x, y	$\frac{1}{2}$N	x, y	N	x, y	N	x, y	N
55, 2	17.137.3929;	49, 44	1033.9209;	65, 21	89.101377;	55, 2	9138541;	3, 56	421.23293;	55, 3	9123481;
55, 4	1801.5081;	47, 46	2713.3449;	65, 23	17.533249;	55, 4	37.37.61.109;	5, 56	13.750517;	55, 7	13.37.97.193;
55, 6	1697.5393;	41, 50	9075761;	65, 27	17.577.937;	55, 6	13.13.73.733;	9, 56	9587041;	57, 17	13.746197;
55, 8	17.538513;	43, 50	17.233.2441;	65, 29	9278953;	55, 8	13.689317; *	11, 56	13.728437;	57, 23	13.61.11497;
55, 12	89.103049;	37, 52	673.13649;	65, 31	41.228953;	57, 14	9957613;	13, 56	397.23509;	59, 29	13.73.10429;
55, 14	9189041;	29, 54	73.281.449;	65, 33	41.232153;	57, 16	13.37.20353;	15, 56	13.706117;	59, 31	73.132817;
55, 16	9216161;	31, 54	9426577;	65, 37	9862393;	57, 20	109.86389;	17, 56	61.241.613;	59, 33	157.60589;
55, 18	9255601;	3, 56	1217.8081;	63, 41	17.449.1217;	57, 22	13.709057;	55, 56	433.21937;	59, 35	1249.7489;
55, 24	233.40697;	5, 56	41.97.2473;	63, 43	9585881;	59, 30	937.10453;	23, 58	13.755137;	59, 37	9226033;
55, 26	17.353.1601;	9, 56	73.113.1193;	61, 47	73.128257;	59, 32	373.25741;	25, 58	13.397.1861;	59, 39	9136201;
55, 28	97.100673;	11, 56	17.593.977;	61, 49	257.38153;	59, 34	97.97213;	27, 58	13.722737;	59, 41	9091561;
53, 34	9226817;	13, 56	97.101681;	59, 51	9441281;	59, 36	13.109.6553;	31, 58	9007213;	59, 43	9099793;
53, 36	41.233417;	15, 56	409.24169;	57, 53	9223241;	59, 38	13.705841;	49, 58	13.692641;	59, 45	9168961;
53, 38	17.586801;	17, 56	9918017;	57, 55	9853313;	59, 40	13.700597;	51, 58	13.717841;	59, 47	9307513;
51, 40	353.26417;	19, 56	1913.5209;			59, 42	13.61.73.157;	53, 58	13.193.3889;	59, 49	13.37.19801;
						59, 44	73.125017;	37, 60	373.26557;	59, 51	13.181.4177;
						59, 46	37.249433;	41, 60	9734161;	57, 53	37.251893;
						59, 48	9405553;	43, 60	13.13.57529;	57, 55	13.61.12457;
						59, 50	193.50077;	47, 60	109.90709;		
						57, 52	313.29017;				

Dimorph Binomial Cubics.

$$N = x^3 + y^3 = x'^3 + y'^3 = \lambda\lambda'.LM.$$

$$\lambda l^2 - \lambda' l'^2 = \tfrac{1}{12}(\lambda'^3 - \lambda^3);$$

$$x = \tfrac{1}{2}(\lambda + 2l), \quad y = \tfrac{1}{2}(\lambda - 2l); \qquad x' = \tfrac{1}{2}(\lambda' + l'), \quad y' = \tfrac{1}{2}(\lambda' - l');$$

$$\lambda = x + y, \quad \lambda' = x' + y'.$$

Ex.—$\lambda = 1$, $\lambda' = 7$; $(2l)^2 - 7(2l')^2 = 114$.

	$2l$, $2l'$	x , y ; x' , y'	$\lambda\lambda'$	L : M
Series 1°	11, 1	6, $\overline{5}$; 4, 3	7 ;	1:13 ;
	67, 25	34, $\overline{33}$; 16, $\overline{9}$	7 ;	13:37 ;
	1061, 401	531, $\overline{530}$; 204, $\overline{199}$	7 ;	103:1171 ;
	16909, 6391	8455, $\overline{8454}$; 3199, $\overline{3192}$	7 ;	49·67:731·43 ;
	269483, 101855	134742, $\overline{134741}$; 50931, $\overline{50924}$	7 ;	26161:297421 ;
Series 2°	109, 41	55, $\overline{54}$; 24, $\overline{17}$	7 ;	19:67 ;
	1733, 655	867, $\overline{866}$; 331, $\overline{324}$	7 ;	151:2131 ;
	27619, 10439	13810, $\overline{13809}$; 5223, $\overline{5216}$	7 ;	4813:16981 ;
	440271, 166369	220136, $\overline{220135}$; 83188, $\overline{83181}$	7 ;	7·5479:7·77323 ;

Trimorph Binomial Cubics.

$$N = x^3 + y^3 = x'^3 + y'^3 = x''^3 + y''^3 = K.M.$$

$$\lambda = x + y, \quad \lambda' = x' + y', \quad \lambda'' = x'' + y'';$$

$$K = \text{L.C.M. of } \lambda, \lambda', \lambda''.$$

x , y ;	x' , y' ;	x'' , y''	K ;	M
12, $\overline{10}$;	9, $\overline{1}$;	8, 6	8.7 ;	13 ;
34, $\overline{33}$;	16, $\overline{9}$;	15, $\overline{2}$	7·13 ;	37 ;
27, $\overline{24}$;	19, $\overline{10}$;	18, 3	9·7 ;	3·31 ;
76, $\overline{72}$;	40, 12 ;	33, 31	64·13 ;	79 ;
89, $\overline{86}$;	41, $\overline{2}$;	40, 17	3·13·19 ;	3·31 ;
53, $\overline{29}$;	50, $\overline{8}$;	44, 34	8.3.7·13 ;	3·19 ;
96, $\overline{90}$;	54, $\overline{12}$;	53, 19	8.9.7 ;	3·103 ;
67, $\overline{51}$;	58, $\overline{30}$;	54, 22	16.7·19 ;	79 ;
58, $\overline{22}$;	57, $\overline{9}$;	54, 30	16.9.7 ;	3·61 ;
213, $\overline{210}$;	69, 42 ;	61, 56	9·13·37 ;	3·31 ;
134, $\overline{116}$;	95, $\overline{23}$;	102, $\overline{60}$	8.9.7 ;	3·13·43 ;
358, $\overline{354}$;	115, $\overline{3}$;	114, 34	16.4·37 ;	367 ;

Elements of Dimorph, $\mathrm{N} = x^3 + y^3 = x'^3 + y'^3$.

$x + y = \lambda, \quad x' + y' = \lambda'; \qquad x^3 + y^3 = \lambda . \mathrm{Z}, \quad x'^3 + y'^3 = \lambda' . \mathrm{Z}'$.

$(\frac{1}{2}\lambda - x) = l, \quad (\frac{1}{2}\lambda' - x') = l'; \quad$ then $\quad \lambda . l^2 - \lambda' . l'^2 = \frac{1}{12}(\lambda^3 - \lambda'^3)$.

$\mathrm{X} = m + x, \quad \mathrm{Y} = m + y; \qquad \mathrm{X}' = m + x', \quad \mathrm{Y}' = m + y'$.

$m = \frac{1}{3}(2\mathrm{Z}/\lambda' - \lambda - \lambda') = \frac{1}{3}(2\mathrm{Z}'/\lambda - \lambda - \lambda')$ gives $\mathbf{N} = \mathrm{X}^3 + \mathrm{Y}^3 = \mathrm{X}'^3 + \mathrm{Y}'^3$.

x,	y;	x',	y'	λ,	λ';	$2l$,	$2l'$;	m	X,	Y;	X',	Y'
6,	$\bar{5}$;	4,	3	1,	7;	11,	1;	6	12,	1;	10,	9
6,	$\bar{4}$;	5,	3	2,	8;			3	9,	$\bar{1}$;	8,	6
6,	$\bar{3}$;	5,	4	3,	9;			2/3	20,	$\bar{7}$;	17,	14
12,	$\overline{10}$;	9,	$\bar{1}$	2,	8;			27	39,	17;	36,	26
12,	$\bar{9}$;	10,	$\bar{1}$	3,	9;			62/3	98,	35;	92,	59
9,	$\bar{8}$;	6,	1	1,	7;	17,	5;	18	27,	10;	24,	19
9,	$\bar{6}$;	8,	1	3,	9;			26/3	53,	8;	50,	29
20,	$\overline{17}$;	14,	7	3,	21;			74/3	134,	23;	116,	95
20,	$\overline{14}$;	17,	7	6,	24;			43/3	103,	1;	94,	64
34,	$\overline{33}$;	16,	$\bar{9}$	1,	7;	67,	25;	318	352,	285;	334,	309
34,	$\overline{16}$;	33,	$\bar{9}$	18,	24;			121/3	223,	73;	220,	94
34,	9;	33,	16	43,	49;			−18	16,	$\bar{9}$;	15,	$\bar{2}$
55,	$\overline{54}$;	24,	$\overline{17}$	1,	7;	109,	41;	846	901,	792;	870,	829
55,	$\overline{24}$;	54,	$\overline{17}$	31,	37;			66	121,	42;	120,	49
55,	17;	54,	24	72,	78;			−89/3	76,	$\overline{38}$;	73,	$\overline{17}$
16,	$\overline{15}$;	9,	$\bar{2}$	1,	7;	31,	11;	66	82,	51;	75,	64
16,	2;	15,	9	18,	24;			−23/3	25,	$\overline{17}$;	22,	4
25,	$\overline{22}$;	17,	4	3,	21;			134/3	209,	68;	185,	146
25,	$\bar{4}$;	22,	17	21,	39;			−22/3	53,	$\overline{34}$;	44,	29
19,	$\overline{18}$;	10,	3	1,	13;	37,	7;	48	67,	30;	58,	51
19,	$\overline{10}$;	18,	3	9,	21;			32/3	89,	2;	86,	41
19,	$\bar{3}$;	18,	10	16,	28;			−9/2	29,	$\overline{15}$;	27,	11
67,	$\overline{58}$;	51,	$\overline{30}$	9,	21;			1088/3	1289,	914;	1241,	998
67,	$\overline{51}$;	58,	$\overline{30}$	16,	28;			471/2	605,	369;	587,	411
89,	$\overline{86}$;	41,	$\bar{2}$	3,	39;			1136/3	1403,	878;	1259,	1130
89,	$\overline{41}$;	86,	$\bar{2}$	48,	84;			367/6	901,	121;	883,	355
29,	$\overline{27}$;	15,	11	2,	26;			51	40,	12;	33,	31
29,	$\overline{11}$;	27,	15	18,	42;			1/3	44,	$\overline{16}$;	41,	23
41,	$\overline{40}$;	17,	2	1,	19;	81,	15;	166	69,	42;	61,	56
41,	$\overline{17}$;	40,	2	24,	42;			61/3	184,	10;	181,	67
41,	$\bar{2}$;	40,	17	39,	57;			−34/3	89,	$\overline{40}$;	86,	17
54,	$\overline{53}$;	19,	12	1,	31;	107,	7;	174	228,	121;	193,	186
54,	$\overline{19}$;	53,	12	35,	65;			54/5	324,	$\overline{41}$;	319,	114
54,	$\overline{12}$;	53,	19	42,	72;			−11/3	151,	$\overline{47}$;	148,	46
71,	$\overline{70}$;	23,	14	1,	37;	141,	9;	256	327,	186;	279,	270
71,	$\overline{23}$;	70,	14	48,	84;			79/6	505,	$\overline{59}$;	499,	163
71,	$\overline{14}$;	70,	23	57,	93;			−16/3	197,	$\overline{58}$;	194,	53
115,	$\overline{114}$;	34,	3	1,	37;	229,	31;	696	811,	582;	730,	699
115,	$\overline{34}$;	114,	3	81,	117;			344/9	1379,	38;	1370,	371
115,	$\bar{3}$;	114,	34	112,	148;			−51/2	179,	$\overline{57}$;	177,	17

Dimorph Trinomial Quartic Forms, $N = f(x, y) = f(x', y')$.

$$f(x, y) = (x^4 - kx^2y^2 + y^4), \quad [k = a^2 + b^2].$$

$$x = x'; \quad y^2 + y'^2 = kx^2 = kx'^2; \quad \tau^2 - kv^2 = -1; \quad t, u \text{ arbitrary}.$$

$$y = (\tau t^2 \pm 2kvtu + k\tau u^2), \quad x = (vt^2 \pm 2\tau tu + kvu^2) = x', \quad y' = t^2 - ku^2.$$

$$N = f(x, y) = f(x', y') = L.M; \qquad L = (x^2 - yy'), \quad M = (x^2 + yy').$$

Special Solutions (below), $y' = t^2 - ku^2 = \tau^2 - kv^2 = -1$.

i. $y = t = \tau, \quad x = u = v = x', \quad y' = -1$; [if $u^2 > t$].

ii. $y = (t^3 + 3ktu^2), \quad x = (ku^3 + 3t^2u) = x', \quad y' = -1$; [if $u^2 < t$].

iii. $y = (4\kappa^3 + 3\kappa), \quad x = (4\kappa^2 + 1) = x', \quad y' = -1$; [if $k = \kappa^2 + 1, \ t = \kappa, \ u = 1$].

Least solutions ($y, \ x = x', \ y' = -1$), for N positive, for every $k = a^2 + b^2 \not> 101$ and $\neq \square$.

k	y ,	x	L : M
1	$y' \neq -1$,	$x = x'$	(N is a Dimorph Sextan (see pp. 190–194))
2	† 7,	5	2.9 : 32 ;
5	38,	17	251 : 3.109 ;
5	682,	305	3.30781 : 83.1129 ;
10	117,	37	4.313 : 2.743 ;
10	4443,	1405	2.23.47.911 : 4.494617 ;
13	18,	5	7 : 43 ;
17	268,	65	3.1319 : 4493 ;
26	515,	101	2.29.167 : 3.19.47 ;
29	70,	13	9.11 : 239 ;
34	. ,	.	no solution with $x = x'$
37	882,	145	20143 : 19.1153 ;
41	131168,	20485	
50	1393,	197	8.3.1559 : 2.20101 ;
53	182,	25	443 : 3.269 ;
58	99,	13	2.5.7 : 4.67 ;
61	29718,	3805	
65	2072,	257	63977 : 81.29.29 ;
73	1068,	125	14557 : 16693 ;
74	318157,	36985	
82	2943,	325	2.51341 : 8.41.331 ;
85	378,	41	1303 : 29.71 ;
89	500,	53	2309 : 3.1103 ;
97	5604,	569	49.43.151 : 5.19.3467 ;
101	4030,	401	9.17419 : 164831 ;

† $k = 2$ has a more general solution;

$$N = (x^2 \sim y^2)^2 = (x'^2 \sim y'^2)^2 = (4a_1a_2b_1b_2)^2;$$

$$x = (a_1a_2 + b_1b_2), \quad y = (a_1a_2 - b_1b_2), \quad x' = (a_1b_2 + b_1a_2), \quad y' = (a_1b_2 - b_1a_2).$$

Dimorph Trinomial Quartic Forms, $N = f(x, y) = f(x', y')$.

$$f(x, y) = (ax^4 - x^2y^2 + ay^4); \quad [a = \alpha^2 + \beta^2].$$

$$x = a\xi = x'; \qquad y^2 + y'^2 = a\xi^2; \qquad \tau^2 - a\upsilon^2 = -1; \qquad t, u \text{ arbitrary.}$$

$$y = (\tau t^2 \pm 2a\upsilon tu + a\tau u^2), \quad x = a(\upsilon t^2 \pm 2\tau tu + a\upsilon u^2) = x', \quad y' = t^2 - au^2.$$

$$N = f(x, y) = f(x', y') = aL.M; \qquad L = x^2 - yy', \quad M = x^2 + yy'.$$

Special Cases (below). $y' = t^2 - au^2 = \tau^2 - a\upsilon^2 = -1$; [For every $a = \alpha^2 + \beta^2 \not> 101$ and $\neq \square$].

Initial Solution. $y = t = \tau$, $x = au = a\upsilon = x'$, $y' = -1$; [y, ξ as in Pellian Equation]. y, ξ are successive solutions of the Pellian Equation.

a	y , x	L : M
1	$y' \neq -1$, $x = x'$	{N is a Dimorph Sextan} {(see pp. 190–194)}
2	7, 10	3.31 : 107;
	41, 58	3323 : 3.5.227;
	239, 338	5.151.151 : 3.31.1231;
	1393, 1970	3.1293169 : 173.22441;
5	2, 5	23 : 27;
	38, 85	7187 : 27.269;
	682, 1525	81.28703 : 67.34721;
10	3, 10	97 : 103;
	117, 370	43.3181 : 181.757;
13	18, 65	7.601 : 4243;
17	4, 17	3.5.19 : 293;
	268, 1105	3.149.2731 : 271.4409;
26	5, 26	11.61 : 3.227;
	515, 2626	11.31.73.277 : 3.433.5309;
29	70, 377	3.13.3643 : 53.2683;
34	. , .	no solution with $x = x'$
37	6, 37	29.47 : 125.11;
	882, 5365	17.113.14983 : 11.19.137723;
41	32, 205	49.857 : 9.4673;
50	7, 50	9.277 : 23.109;
	1393, 9850	
53	182, 1325	1755443 : 3.585269;
58	99, 754	43.13219 : 5.113723;
61	29718, 232105	
65	8, 65	4217 : 3.17.83;
73	1068, 9125	
74	43, 370	3.2401.19 : 136943;
82	9, 82	5.17.79 : 6733;
85	378, 3485	11.13.13.47.139 : 263.46181;
89	500, 4717	19.1171031 : 3.263.28201;
97	5604, 55193	
101	10, 101	3.43.79 : 10211;

Dimorph Trinomial Quartic Forms, $N = F(X, Y) = F(X', Y')$.

$$F(X, Y) = (X^4 + KX^2Y^2 + Y^4); \quad [(K+2)(2K-12) = (\alpha^2 + \beta^2)].$$

$$X = \tfrac{1}{2}(y-x), \quad Y = \tfrac{1}{2}(y+x), \quad X' = \tfrac{1}{2}(y'-x'), \quad Y' = \tfrac{1}{2}(y'+x').$$

$$F(X, Y) = \tfrac{1}{16}.f(x, y) = \tfrac{1}{16}(ax^4 - kx^2y^2 + ay^4); \quad [a = (K+2),\ k = (2K-12)].$$

$$F(X, Y) = F(X', Y') \quad \text{gives} \quad f(x, y) = f(x', y'); \qquad [ak = (\alpha^2 + \beta^2)].$$

$$x = x'; \quad y^2 + y'^2 = \frac{1}{a}.kx^2; \quad \text{solvable when} \quad \tau^2 - ak.v^2 = -1.$$

$$N = \tfrac{1}{16}aL.M; \qquad L = (x^2 - yy'), \quad M = (x^2 + yy').$$

Special Solutions (below). $y' = 1$; $y^2 - \frac{1}{a}.kx^2 = -1$.

a, L, M reduced to *odd* numbers by the factors κ, λ, μ; $[\kappa\lambda\mu = \tfrac{1}{16}]$.

K	a, k	x, y	X, Y; X′, Y′	κa; λL : μM
6	8, 0	x, y	X, Y; X′, Y′	N is a Dimorph Quartan (see p. 127)
7	9, 2	3, 1	1, 2; 1, 2	Identity
		15, 7	4, 11; 7, 8	9; 109:29;
		87, 41	23, 64; 43, 44	9; 941:5.761;
		507, 239	134, 373; 253, 254	9; 5.61.421:29.1109;
		2955, 1393	781, 2174; 1477, 1478	9; 1091329:4366709;
8	10, 4	5, 3	1, 4; 2, 3	5; 11:7;
		185, 117	34, 151; 92, 93	5; 8527:7.11.223;
		7025, 4443	1291, 5734; 3512, 3513	5; 24673091?†:7.1762681;
11	13, 10	65, 57	4, 61; 32, 33	13; 521:2141;
14	1, 1	x, y	X, Y; X′, Y′	N is a Dimorph Sextan (see pp. 190–194)
15	17, 18	. , .	. , . ; . , .	No solution with $x = x'$
16	18, 20	3, 3	0, 3; 0, 3	Identity.
		111, 117	3, 114; 55, 56	9; 27.113:9.691;
		4215, 4443	14, 4329; 2107, 2108	9; 3.2960297:3.193.7673;
23	25, 34	. , .	. , . ; . , .	No solution with $x = x'$
24	26, 36	. , .	. , . ; . , .	No solution with $y' = -1$

Dimorph Forms, $N = f(x, y) = f(x', y')$.

$f(x, y)$;	*Elements* (x, y, x', y'),	*Conditions and References.*
$(x^2 - y^2)$;	$x = a_1a_2 + b_1b_2,\quad y = a_1a_2 \sim b_1b_2$;	$x' = a_1b_2 + b_1a_2,\quad y' = a_1b_2 \sim b_1a_2$.
$(x^2 + y^2)$;	$x = a_1a_2 \sim b_1b_2,\quad y = a_1b_2 + b_1a_2$;	$x' = a_1a_2 + b_1b_2,\quad y' = a_1b_2 \sim b_1a_2$.
$(x^2 - qy^2)$;	$x = t_1t_2 - qu_1u_2,\quad y = t_1u_2 - u_1t_2$;	$x' = t_1t_2 + qu_1u_2,\quad y' = t_1u_2 + u_1t_2$.
$(x^2 + qy^2)$;	$x = t_1t_2 - qu_1u_2,\quad y = t_1u_2 + u_1t_2$;	$x' = t_1t_2 + qu_1u_2,\quad y' = t_1u_2 - u_1t_2$.
$(x^2 \mp xy + y^2)$;	$f(x, y) = A^2 + 3B^2 = A'^2 + 3B'^2 = f(x', y')$;	$q = 3$ in above.
$(x^2 \mp kxy + y^2)$;	$f(x, y) = \xi^2 + q\eta^2$; $\quad \xi = x \mp \frac{1}{2}ky,\quad \eta = y$;	$q = (1 - \frac{1}{4}k^2)$ in above.
$(x^3 \mp y^3)$;	see Tables, pages 221, 222.	
$(x^4 \mp y^4)$;	see Table, page 127.	
$(x^4 \mp 4y^4)$;	see Table, page 103.	
$(x^4 - x^2y^2 + y^4)$;	Dimorph Sextan: see Table, pages 190–194.	
$(x^4 - 3x^2y^2 + 9y^4)$;	Dimorph Trin-Aurifeuillian.	
$(a^2x^4 \mp 2abx^2y^2 + b^2y^4)$;	$N = (ax^2 \mp by^2)^2$.	
$(x^4 + 6x^2y^2 + y^4)$;	Dimorph Quartan; see Table, page 127.	
$(x^4 + 14x^2y^2 + y^4)$;	Dimorph Sextan; see Table, page 171.	
$(x^4 - kx^2y^2 + y^4)$;	$[k = \mathrm{a}^2 + \mathrm{b}^2,\ x = x']$; see Table, page 223.	
$(ax^4 - x^2y^2 + ay^4)$;	$[a = \alpha^2 + \beta^2,\ x = x']$; see Table, page 224.	
$(ax^4 - kx^2y^2 + ay^4)$;	$[ak = \alpha^2 + \beta^2,\ x = x']$; see Table, page 225.	
$(X^4 + KX^2Y^2 + Y^4)$;	$[(K + 2)(2K - 12) = \alpha^2 + \beta^2;\ x = x']$; see Table, page 225.	
$(x^4 \mp 2x^3y + 2x^2y^2 \mp 4xy^3 + 4y^4)$;	L, M of Bin-Aurifn. Sextan Chain; see Table, page 195.	
$(x^4 \mp 6x^3y + 18x^2y^2 \mp 18x^3y^3 + 9y^4)$;	L, M of Trin-Aurifn. Sextan Chain; see Table, page 197.	
$(ax^4 + bx^3y + cx^2y^2 + bxy^3 + ay^4)$;	$[(2a \mp 2b + c)(12a - 2k) = \mathrm{a}^2 + \mathrm{b}^2;\quad x \pm y = x' \pm y']$.	

Factorisation of Quartic Forms.

$$x^4 - y^4 = (x^2 - y^2)(x^2 + y^2) = (x - y)(x^3 + x^2y + xy^2 + y^3)$$
$$= (x + y)(x^3 - x^2y + xy^2 - y^3).$$

$$x^4 \sim 4y^4 = (x^2 \sim 2y^2)(x^2 + 2y^2).$$

$$a^2x^4 \sim b^2y^4 = (ax^2 \sim by^2)(ax^2 + by^2).$$

$$x^4 + 4y^4 = (x^2 - 2xy + 2y^2)(x^2 + 2xy + y^2).$$

$$x^4 + x^2y^2 + y^4 = (x^2 - xy + y^2)(x^2 + xy + y^2).$$

$$x^4 + 2x^2y^2 + y^4 = (x^2 + y^2)^2.$$

$$x^4 - 2x^2y^2 + y^4 = (x^2 \sim y^2)^2 = (x - y)(x^3 + x^2y - xy^2 - y^3)$$
$$= (x + y)(x^3 - x^2y - xy^2 + y^3).$$

$$x^4 - (k^2 - 2)x^2y^2 + y^4 = (x^2 - kxy + y^2)(x^2 + kxy + y^2).$$

$$x^4 \pm 4x^2y^2 + 4y^4 = (x^2 \pm 2y^2)^2.$$

$$x^4 - 8x^2y^2 + 4y^4 = (x^2 - 2xy - 2y^2)(x^2 + 2xy - 2y^2).$$

$$x^4 \pm 6x^2y^2 + 9y^4 = (x^2 \pm 3y^2)^2.$$

$$x^4 - 3x^2y^2 + 9y^4 = (x^2 - 3xy + 3y^2)(x^2 + 3xy + 3y^2).$$

$$x^4 - (k^2 \mp 2k)x^2y^2 + k^2y^4 = (x^2 - kxy \pm ky^2)(x^2 + kxy \pm ky^2).$$

$$acx^4 + (a^2 + c^2)x^2y^2 + acy^4 = (ax^2 + cy^2)(cx^2 + ay^2).$$

$$a^2x^4 - (b^2 - 2a^2)x^2y^2 + a^2y^4 = (ax^2 - bxy + ay^2)(ax^2 + bxy + ay^2).$$

$$a^2x^4 \mp 2abx^2y^2 + b^2y^4 = (ax^2 \mp by^2)^2.$$

$$a^2x^4 - (b^2 - 2ac)x^2y^2 + c^2y^4 = (ax^2 - bxy + cy^2)(ax^2 + bxy + cy^2).$$

$$aa'x^4 + (ac' + bb' + ca')x^2y^2 + cc'y^4 = (ax^2 - bxy + cy^2)(a'x^2 + b'xy + c'y^2)$$
$$[\text{where} \quad a : b : c = a' : b' : c'].$$

$$x^4 \pm x^3y \pm xy^3 + y^4 = (x \pm y)(x^3 \pm y^3) = (x \pm y)^2(x^2 \mp xy + y^2).$$

$$x^4 \pm x^3y \mp xy^3 - y^4 = (x \pm y)(x^3 \mp y^3) = (x^2 - y^2)(x^2 \pm xy + y^2).$$

$$a^2x^4 + a(b + b')(x^3y + xy^3) + a^2y^4 = (ax^2 + bxy + ay^2)(ax^2 + b'xy + ay^2);$$
$$[2a^2 + bb' = 0].$$

$$aa'x^4 + (ab' + a'b)x^3y + (ab + a'b')xy^3 + aa'y^4$$
$$= (ax^2 + bxy + a'y^2)(a'x^2 + b'xy + ay^2); \quad [a^2 + a'^2 + bb' = 0].$$

$$x^4 \pm 2x^3y + 2x^2y^2 \pm 2xy^2 + y^4 = (x \pm y)(x^3 \pm x^2y + xy^2 \pm y^3) = (x \pm y)^2(x^2 + y^2).$$

$$x^4 \pm 2x^3y + 3x^2y^2 \pm 2xy^3 + y^4 = (x^2 \pm xy + y^2)^2.$$

$$x^4 \pm 4x^3y + 6x^2y^2 \pm 4xy^3 + y^4 = (x \pm y)^4 = (x^2 \pm 2xy + y^2)^2$$
$$= (x \pm y)(x^3 \pm 3x^2y + 3xy^2 \pm y^3).$$

$$2x^4 \pm 6x^3y + 9x^2y^2 \pm 6xy^3 + 2y^4 = (2x^2 \pm 2xy + y^2)(x^2 \pm 2xy + 2y^2).$$

$$3x^4 \pm 12x^3y + 19x^2y^2 \pm 12xy^3 + 3y^4 = (3x^2 \pm 3xy + y^2)(x^2 \pm 3xy + 3y^2).$$

$$ax^4 \pm (a^2 + a)x^3y + (2a^2 + 1)x^2y^2 \pm (a^2 + a)xy^3 + ay^4$$
$$= (ax^2 \pm axy + y^2)(x^2 \pm axy + ay^2).$$

$$2x^4 \pm 2x^3y + x^2y^2 \mp 2xy^3 + 2y^4 = (2x^2 \mp 2xy + y^2)(x^2 \pm 2xy + 2y^2).$$

$$3x^4 \pm 6x^3y + x^2y^2 \mp 6xy^3 + 3y^4 = (3x^2 \mp 3xy + y^2)(x^2 \pm 3xy + 3y^2).$$

$$ax^4 \pm (a^2 - a)x^3y + x^2y^2 \mp (a^2 - a)xy^3 + ay^4 = (ax^2 \mp axy + y^2)(x^2 \pm axy + ay^2).$$

$$a^2x^4 + a(b + b')x^3y + (2a^2 + bb')x^2y^2 + a(b + b')xy^3 + a^2y^4$$
$$= (ax^2 + bxy + ay^2)(ax^2 + b'xy + ay^2).$$

$$aa'x^4 + b(a + a')x^3y + (a^2 + b^2 + a'^2)x^2y^2 + b(a + a')xy^3 + aa'y^4$$
$$= (ax^2 + bxy + a'y^2)(a'x^2 + bxy + ay^2).$$

$$aa'x^4 + b(a + a')x^3y + (ac' + b^2 + a'c)x^2y^2 + b(c + c')xy^3 + cc'y^4$$
$$= (ax^2 + bxy + cy^2)(a'x^2 + bxy + c'y^2).$$

Impossible Square Forms, $\mathrm{F}(x,y) \neq z^2$; $[xy > 1,\ x \neq y]$.

$(x^4 \mp y^4)$, $2(x^4 \mp y^4)$; $(x^4 \pm 2^m . y^4)$, [For m, see p. 231].

$(x^4 - kx^2y^2 + y^4)$; k = 0, 1, 3, 5, 6, 7, 8; 10, 12, 14, 17, 18, 19; 20, 21, 22, 23, 24, 29; 30, 31, 33, 34, 37, 38; 41, 45, 46, 48; 50, 52, 53, 54, 55, 56, 57, 58, 59; 60, 61, 62, 65, 66, 68, 69; 73, 75; 82, 83, 85, 88; 91, 93, 94, 95, 97, 98; 165, &c. *

$(x^4 + kx^2y^2 + y^4)$; k = 0, 1, 3, 4, 5, 6, 9; 10, 11, 15, 18, 19; 20, 21, 22, 25, 28, 29; 30, 32, 35, 37, 39; 40, 43, 45, 46; 50, 51, 53, 54, 58, 59; 65, 69; 70, 72, 74, 75, 76; 80, 81, 82, 85, 88; 91, 93, 97; 105, 111, 129, 165, 195; &c. *

* See *L'Interméd. d. Math.*, t. xv, 1908, pp. 30, 52, 159, 160, 282; t. xvi, 1909, p. 154.

Possible Square Forms, $\mathrm{F}(x,y) = z^2$.

$(x^2 - y^2)$;	$x = (t^2 + u^2)$, y or $z = (t^2 \sim u^2)$, z or $y = 2tu$.
$(x^2 + y^2)$;	$x = (t^2 \sim u^2)$, $y = 2tu$, $z = (t^2 + u^2)$.
$(x^2 - qy^2)$;	$x = (t^2 + qu^2)$, $y = 2tu$, $z = (t^2 \sim qu^2)$.
$(x^2 + qy^2)$;	$x = (t^2 \sim qu^2)$, $y = 2tu$, $z = (t^2 + qu^2)$.
$(x^2 \mp xy + y^2)$;	here $\mathrm{F}(x,y) = (A^2 + 3B^2)$, $q = 3$ in above.
$(x^2 \mp kxy + y^2)$;	here $\mathrm{F}(x,y) = (x \mp \frac{1}{2}ky)^2 + qy^2$, $q = (1 - \frac{1}{4}k^2)$ in above.
$(x^2 \mp bxy + cy^2)$;	here $\mathrm{F}(x,y) = (x \mp \frac{1}{2}by)^2 + qy^2$, $q = (c - \frac{1}{4}b^2)$ in above.
$(x^3 \mp y^3)$;	see p. 229.
$(x^3 \pm Cy^3)$;	see p. 234.
$\pm(x^4 \mp \mathrm{K}y^4)$;	$\mathrm{K} = \pm k$, see p. 230; $\mathrm{K} = \pm 2^m$ and $\pm k^2$, see p. 231.
$(x^4 \mp 2x^2y^2 + y^4)$;	$z = (x^2 \mp y^2)$.
$(1^4 - k^2y^2 + y^4)$;	$y = k$, $z = k^2$.
$[x^4 + (k^2 - 2)x^2y^2 + y^4]$;	$x = y$, $z = ky^2$.
$(x^4 \mp kx^2y^2 + y^4)$;	see pp. 232, 233.
$(x^4 \mp 2x^3y + 3x^2y^2 \mp 2xy^3 + y^4)$;	$z = (x^2 \mp xy + y^2)$.
$(x^4 \mp 4x^3y + 6x^2y^2 \mp 4xy^3 + y^4)$;	$z = (x \mp y)^2$.
$(x^4 \mp 4x^3y + 8x^2y^2 + \mp 8xy^3 + 4y^4)$;	$z = (x^2 \mp 2xy + 2y^2)$.
$(x^4 \mp 6x^3y + 15x^2y^2 \mp 18xy^3 + 9y^4)$;	$z = (x^2 \mp 3xy + 3y^2)$.
$x^4 \mp 2kx^3y + (k^2 + 2k)x^2y^2 \mp 2k^2xy^3 + k^2y^4)$;	$z = (x^2 \mp kxy + ky^2)$.
$(4x^4 \mp 4x^3y + 9x^2y^2 \mp 4xy^3 + 4y^4)$;	$z = (2x^2 \mp xy + 2y^2)$.
$(9x^4 \mp 6x^3y + 19x^2y^2 \mp 6xy^3 + 9y^4)$;	$z = (3x^2 \mp xy + 3y^2)$.
$\{k^2x^4 \mp 2kx^3y + (2k^2 + 1)x^2y^2 \mp 2kxy^3 + k^2y^4\}$;	$z = (kx^2 \mp xy + ky^2)$.

Solutions of $x^3 \sim y^3 = z^2 = (\lambda z_1 z_3)^2$.

$\lambda = 1$ or 3; $x \sim y = \lambda z_1^2$, $(x^3 \sim y^3) \div (x \sim y) = \lambda z_3^2$; [$y = \epsilon$ and may be $\pm$].

$z_3^2 = \alpha^2 + 3\beta^2$ [see Table on pp. 185–189 and 194]; $z_3 \not> 2{,}900$.

$\lambda = 1$; $x = \alpha \mp \beta$, $y = 2\beta$; $x \sim y = \alpha \mp 3\beta = z_1^2$.

$\lambda = 3$; $x = 3\beta \mp \alpha$, $y = 2\alpha$; $x \sim y = 3(\beta \mp \alpha) = 3z_1^2$.

$\lambda = 1$; $z = \omega$				$\lambda = 1$; $z = \epsilon$				$\lambda = 3$; $z = \omega$				$\lambda = 3$; $z = \epsilon$			
x	y	z_1	z_3	x	y	z_1	z_3	x	y	z_1	z_3	x	y	z_1	z_3
7	8	1	13	10	6	2	14	1	$\overline{2}$	1	1	22	$\overline{26}$	4	14
65	$\overline{56}$	11	61	114	110	2	194	47	74	3	61	2	$\overline{46}$	4	26
57	$\overline{112}$	13	97	450	$\overline{34}$	22	434	193	118	5	157	26	218	8	134
105	104	1	181	506	$\overline{170}$	26	446	433	$\overline{242}$	15	217	286	478	8	386
217	$\overline{312}$	23	277	522	38	22	542	431	506	5	469	262	694	12	494
111	280	13	349	946	270	26	1106	673	526	7	601	506	$\overline{1222}$	24	614
273	152	11	373	1786	$\overline{330}$	46	1646	1201	$\overline{122}$	21	661	626	1826	20	1274
305	$\overline{1064}$	37	949	1562	1558	2	2702	1969	$\overline{1706}$	35	1069	1534	$\overline{3266}$	40	1634
665	496	13	1009	2546	430	46	2786	1727	1874	7	1801	1726	2158	12	1946
1617	$\overline{592}$	47	1417					193	$\overline{3482}$	35	1957	1658	2426	16	2054
2345	$\overline{1376}$	61	2041									1894	$\overline{5018}$	48	2534
455	1824	37	2089												
1705	1176	23	2509												
1455	1456	1	2521												

If $x_r^4+K.y_r^4=\pm z_r^2$, then $x_{r+1}^4+K.y_{r+1}^4=+z_{r+1}^2$; [K = ±k]; where $x_{r+1}=x_r^4\sim K.y_r^4$, $y_{r+1}=2x_ry_rz_r$, $z_{r+1}=z_r^4+4K.x_r^4y_r^4$;

Quartic Squares, $x^4+Ky^4=\pm z^2$; [K = ±k].

[$z_1>10^6$ not printed].

$x^4+ky^4=z^2$		
k	x, y, z	x_1, y_1, z_1
1	None	None
2	None	None
*3	1, 1, 2	1, 2, 7; & v. inf.
4	None	None
5	1, 2, 9	79, 36, 6881
8	1, 1, 3	7, 6, 113
9	2, 1, 5	7, 20, 1201
13	3, 2, 17	127, 204, 150913
14	1, 2, 15	223, 60, 51521
15	1, 1, 4	7, 4, 79
16	None	None
18	1, 2, 17	287, 68, 84673
19	3, 1, 10	31, 30, 4039
21	5, 2, 31	289, 620, z_1
24	1, 1, 5	23, 10, 721
28	3, 2, 23	367, 276, 424993
29	5, 2, 33	161, 660, z_1
32	None	None
33	2, 1, 7	17, 28, 4513
	1, 2, 23	527, 92, 281953
	4, 1, 17	223, 136, 117313
34	3, 2, 25	463, 300, 566881
35	1, 1, 6	17, 6, 359
39	1, 2, 25	623, 100, 393121
	7, 2, 55	1777, 1540, z_1
40	3, 1, 11	41, 66, 27601
48	1, 1, 7	47, 14, 2593
51	5, 1, 26	287, 130, 146119
53	7, 2, 57	1553, 1596, z_1
55	3, 2, 31	799, 372, z_1
56	5, 2, 39	271, 780, z_1
60	1, 2, 31	959, 124, 927361
63	1, 1, 8	31, 8, 1087
	3, 2, 33	103, 44, z_1
	9, 2, 87	5553, 3132, z_1
64	None	None
65	2, 1, 9	49, 36, 10721
66	5, 2, 41	431, 820, z_1
68	1, 2, 43	1087, 172, z_1
73	6, 1, 37	1223, 444, z_1
80	1, 1, 9	79, 18, 6881
85	9, 2, 89	5201, 3204, z_1
88	3, 1, 13	7, 78, 57073
93	11, 2, 127	13153, 5588, z_1
95	1, 2, 39	1519, 156, z_1
99	1, 1, 10	49, 10, 2599
	5, 2, 47	959, 940, z_1
100	3, 2, 41	1519, 492, z_1
*3	(11, 3, 122), (47, 28, 2593) (7199, 8052, 12259565)	

$x^4-ky^4=\pm z^2$		
k	$x, y, z, \pm$	$x_1, y_1, z_1, \pm$
1	None	None
2	1, 1, 1, −	3, 2, 7, +
3		
4	None	None
†5	1, 1, 2, −	3, 2, 1, +; & v. inf.†
6	5, 2, 23, +	721, 460, 39841, +
7	2, 1, 3, +	23, 12, 367, +
8	None	None
10	1, 1, 3, −	11, 6, 41, +
12	7, 2, 47, +	2593, 1316, z_1, +
15	2, 1, 1, +	31, 4, 959, +
16	None	None
17	1, 1, 4, −	9, 4, 47, +
	2, 1, 1, −	33, 4, 1087, +
	3, 1, 8, +	49, 24, 353, +
20	9, 2, 79, +	6881, 2844, +
21	5, 2, 17, +	961, 340, 756479, +
25	2, 1, 3, −	41, 12, 1519, +
26	1, 1, 5, −	27, 10, 521, +
30	11, 2, 119, +	15121, 5236, +
31	4, 1, 15, +	287, 120, 18881, +
32	3, 1, 7, +	113, 42, 7967, +
34	5, 2, 9, +	81, 180, z_1, +
36	5, 2, 7, +	1201, 140, z_1, +
37	1, 1, 6, −	19, 6, 287, +
39	5, 2, 1, +	1249, 20, z_1, +
41	2, 1, 5, −	57, 20, 1999, +
42	13, 2, 167, +	29233, 8684, z_1, +
45	3, 1, 6, +	7, 2, 41, +
49	5, 1, 24, +	337, 120, 52319, +
50	1, 1, 7, −	51, 14, 2201, +
56	3, 1, 5, +	137, 30, 17519, +
	15, 2, 223, +	51521, 13380, +
64	None	None
65	1, 1, 8, −	33, 8, 959, +
	2, 1, 7, −	81, 28, 1759, +
	3, 1, 4, +	73, 12, 5201, +
71	6, 1, 35, +	1367, 420, z_1, +
72	17, 2, 287, +	84673, 19516, z_1, +
77	3, 1, 2, +	79, 6, 6233, +
	9, 2, 73, +	7792, 2628, z_1, +
80	3, 1, 1, +	161, 6, 25919, +
82	1, 1, 9, −	83, 18, 6233, +
	3, 1, 1, −	163, 6, 26567, +
	7, 2, 33, +	3713, 924, z_1, +
87	4, 1, 13, +	343, 104, 60527, +
90	7, 2, 31, +	2488, 868, z_1, +
	19, 2, 359, +	131761, 27284, z_1, +
95	9, 2, 71, +	8081, 2556, z_1, +
97	2, 1, 9, −	113, 36, 353, +
	3, 1, 2, −	89, 6, 7853, +
	7, 1, 48, +	2498, 672, z_1, +
†5	(161, 12, 25919, +),	

* See Ed. Lucas's *Recherches sur plusieurs ouvrages de Léonard de Pise*, Rome, 1877, Ch. iii, § 5.

I. *Square Quartic Forms*, $x^4 + \mathrm{K}.y^4 = \pm z^2$; $[\mathrm{K} = \pm 2^m]$.

$x^4 + 2^m.y^4 = z^2$ requires $m = 4\mu + 3$; $x^4 - 2^m.y^4 = \pm z^2$ requires $m = 4\mu + 1$.

i. *Solutions* of* $x_r^4 + 8y_r^4 = z_r^4$.

$$x_{r+1} = x_r^4 - 8y_r^4, \quad y_{r+1} = 2x_r y_r z_r, \quad z_{r+1} = z_r^4 + 32x_r^4 y_r^4.$$

$(x, y, z) = (1, 1, 3)$, $(7, 6, 113)$, $(7967, 9492, 262621633)$, (&c.).

ii. *Solutions* of* $x_{r+1}^4 - 2y_{r+1}^4 = +z_{r+1}^2$ *from* $x_r^2 - 2y_r^2 = \pm z_r^2$.

$$x_{r+1} = x_r^4 + 2y_r^4, \quad y_{r+1} = 2x_r y_r z_r, \quad z_{r+1} = z_r^4 - 8x_r^4 y_r^4.$$

x, y	3, 2	113, 84	57123, 6214	262621633, 151245528	&c.
z	7	7967	3262580153	6091245606 5182847	&c.

iii. *Solutions* of* $x_{r+1}^4 - 2y_{r+1}^4 = -z_{r+1}^2$ *from* $x_r^4 - 2y_r^4 = -z_r^2$.

Take $\mathrm{A} = x_r^2 + 2y_r^2, \quad \mathrm{B} = x_r y_r + z_r$;

then $z_{r+1} = (\mathrm{A}^2 x_r^2 + 2\mathrm{B}^2 y_r^2)^2 - 2(\mathrm{A}^2 y_r^2 - \mathrm{B}^2 x_r^2)^2$;

and $x_{r+1} = \mathrm{A}^2 x_r^2 - 2\mathrm{B}^2 y_r^2, \quad y_{r+1} = \mathrm{A}^2 y_r^2 + \mathrm{B}^2 x_r^2$.

$(x, y, z) = (1, 1, 1)$, $(1, 13, 239)$, $(1343, 1525, 2750257)$,
$(9788425919, 42422452969, z)$,
$(5705771236058721, 7658246457672229, z)$, (&c.).

II. $x^4 + \mathrm{K}.y^4 = \pm z^2$; $[\mathrm{K} = \pm k^2$; $\mathrm{c}^2 = \mathrm{a}^2 + \mathrm{b}^2$ gives Base-solutions].

i. *Solutions* of* $x_r^4 + k^2 y_r^4 = z_r^2$; [a or $b = \square$ gives Base-solutions].

$\mathrm{a} = \alpha^2$ gives $\alpha^4 + \mathrm{b}^2.1^4 = \mathrm{c}^2$; $\mathrm{b} = \beta^2$ gives $\beta^4 + \mathrm{a}^2.1^4 = \mathrm{c}^2$.

$$x_{r+1} = x_r^4 - k^2 y_r^4, \quad y_{r+1} = 2x_r y_r z_r, \quad z_{r+1} = z_r^4 + 4k^2 x_r^4 y_r^4.$$

ii. *Solutions* of* $x_r^4 - k^2 y_r^4 = \pm z_r^2$; [a, b, or $\mathrm{c} = \square$ gives Base-solutions].

$\mathrm{a} = \alpha^2$ gives $\alpha^4 - \mathrm{c}^2.1^4 = -\mathrm{b}^2$; $\mathrm{b} = \beta^2$ gives $\beta^4 - \mathrm{c}^2.1^4 = -\mathrm{a}^2$;

$\mathrm{c} = \gamma^2$ gives $\gamma^4 - \mathrm{a}^2.1^4 = +\mathrm{b}^2$, and $\gamma^4 - \mathrm{b}^2.1^4 = +\mathrm{c}^2$.

$$x_{r+1} = x_r^4 + k^2 y_r^4, \quad y_{r+1} = 2x_r y_r z_r, \quad z_{r+1} = z_r^4 - 4k^2 x_r^4 y_r^4.$$

[When two of a, b, c contain square factors, k may be reduced in II. i and II. ii.]

$x^4 + k^2y^4 = z^2$				$x^4 - k^2y^4 = \pm z^2$		
c, a, b	k	x_0, y_0, z_0	x_1, y_1, z_1	k	$x_0, y_0, z_0, \pm$	x_1, y_1, z_1
*5, 3, 4	3	2, 1, 5	7, 20, 1201	†5	2, 1, 3, −	41, 12, 1519
25, 7, 24	.	. . .	. . .	‡6	5, 2, 7, +	1201, 140, z_1
				7	5, 1, 24, +	674, 240, z_1
41, 9, 40	10	3, 2, 41	1519, 492, z_1	41	3, 1, 40, −	1762, 240, z_1
65, 63, 16	7	4, 3, 65	3713, 1560, z_1	65	4, 1, 63, −	4481, 504, z_1
85, 77, 36	77	6, 1, 85	4633, 1020, z_1	85	6, 1, 77, −	8521, 924, z_1
145, 17, 144	17	12, 1, 145	20447, 3480, z_1	145	12, 1, 17, −	41761, 408, z_1
169, 119, 120	.	. . .	. . .	119	13, 1, 120, +	42722, 3120, z_1
				30	13, 2, 119, +	42961, 6188, z_1
289, 161, 240	.	. . .	. . .	161	17, 1, 240, +	109442, 8160, z_1
				15	17, 4, 161, +	141121, 10948, z_1
*5, 3, 4	3	$x_2 = 1437599$, $y_2 = 336280$		†5	$x_2 = 3344161$, $y_2 = 1794696$	
				‡6	$x_2 = 2639802$, $y_2 = 7776485$	

*†‡ See Ed. Lucas's *Recherches sur plusieurs ouvrages de Léonard de Pise*, Rome, 1877, Chaps. ii, iii.

Quartic Squares, $x^4 - kx^2y^2 + y^4 = z^2$.

i. $k = k^2$; $x = 1$, $y = k$, $z = 1$.

ii. $x = 1$; $k = y^2 - \mathrm{K}$, where $z^2 - \mathrm{K}y^2 = 1$.

iii. $k = \lambda y^2 - 2C$, where $\lambda = (C^2 - 1) \div x^2$, $z = x^2 \sim Cy^2$; $[x, y$ arbitrary$]$.

Blanks in x, y, z columns mean that x, y, z are *possible*; see Euler's *Comment. Arithm.*, t. ii, p. 496, &c.

k	x, y	z	k	x, y	z	k	x, y	z	k	x, y	z
1	$x = y$	x^2	47	1, 8	33	104	4, 105	10159	169	1, 13	1
2	x, y	$(x^2 - y^2)$	49	1, 7	1	106	2, 33	851	179	24, 335	31999
4	1, 2	1		1, 8	31	107	1, 12	73		29, 780	527791
	4, 15	191		1, 15	199	109				55, 756	131111
9	1, 3	1		15, 209	37769	113	5, 63	2131	180	3, 70	4009
	3, 40	1559	51	3, 88	7511	116	4, 69	3719	182	1, 28	687
11	1, 4	9	64	1, 8	1	118	2, 55	2779	186	11, 204	28201
	24, 85	2599	67	4, 33	137	119	5, 56	711	187	12, 301	75953
13	1, 4	7	70	4, 35	359	121	1, 11	1	188		
15	24, 95	1951	72	3, 104	10487	123	1, 12	55	189		
16	1, 4	1	74	1, 10	51		24, 377	100655	190	1, 35	1126
	8, 255	64511	76			126	3, 35	334	191	1, 16	129
25	1, 5	1	77	7, 72	2705	131	39, 644	298951		1, 56	3039
26	1, 6	19	78	6, 55	811	132	56, 657	87457		8, 115	3639
27	4, 21	65	79	5, 48	871	134	12, 145	6031	193		
28	1, 6	17	81	1, 9	1	136	14, 209	27271	196	1, 14	1
32	1, 12	127	86	3, 28	89	142	1, 12	17		1, 70	4801
36	1, 6	1	89	1, 12	89	144	1, 12	1		56, 801	131521
	4, 39	1199	90			146	1, 14	99	197	5, 72	1159
	9, 70	3119	92	1, 20	351	148	1, 14	97		8, 399	152767
39	24, 155	6141	96	23, 408	138769	149	45, 592	130711		33, 464	12863
40	2, 15	121	100	1, 10	1	151	40, 689	332671	198		
42	2, 21	347	101	8, 287	79071	156	3, 152	22391	200	4, 145	19359
43	8, 55	911	102			166	2, 35	829			
44	3. 20	41	103	4, 55	2041	167	3, 56	2263			

Quartic Squares, $x^4 + kx^2y^2 + y^4 = z^2$; *up to* $k \not> 200$.

i. $k = \kappa^2 - 2$; $x = y$, $z = \kappa x^2$.

ii. $x = 1$; $k = \mathrm{K} - y^2$, where $z^2 - \mathrm{K}y^2 = +1$.

iii. $k = \lambda y^2 + 2C$, where $\lambda = (C^2-1) \div x^2$, $z = x^2 + Cy^2$; [x, y arbitrary].

k	x , y	z	k	x , y	z	k	x , y	z
2	x, y	x^2+y^2	78	1, 3	28		13, 70	11831
7	$x = y$	$3x^2$		10, 21	1909	141	7, 60	6151
8	1, 2	7	79	$x = y$	$9x^2$	142	$x = y$	$12x^2$
12	3, 2	23		1, 4	39	143	1, 3	37
13	3, 4	47		1, 5	51		1, 7	97
14	$x = y$	$4x^2$	83	1, 8	97		4, 5	241
16	1, 2	9	84	5, 12	569		4, 55	4009
17	1, 4	23	86	1, 2	19		17, 209	60937
23	$x = y$	$5x^2$	87	3, 8	233	151	1, 7	99
	1, 3	17	89	1, 4	41	152	1, 2	25
	1, 4	25		4, 5	191		3, 20	841
24	3, 2	31		7, 48	3919	153		
26	1, 2	11	90	6, 35	2339	155	104, 95	123809
27	3, 4	65	92	2, 7	143	156	2, 9	239
31	1, 3	19	94	2, 3	59		3, 4	151
33	8, 21	1063	95	7, 15	1049	159	3, 5	191
34	$x = y$	$6x^2$	96	3, 4	119	160	2, 5	129
36	1, 12	161		25, 408	194161	161	4, 7	359
38	1, 2	13	98	$x = y$	$10x^2$	162	2, 3	77
41	3, 4	79	99	312, 215	676081	166	5, 6	389
42	1, 6	53	100	4, 15	641	167	$x = y$	$13x^2$
44	1, 4	31	104	1, 6	71	168	2, 9	247
	2, 3	41		1, 20	449		6, 55	5239
	2, 5	71		2, 7	151	169	11, 56	8601
	11, 70	7079	106	1, 2	21	171		
	13, 198	42761		5, 28	1641	172	10, 21	2791
	16, 209	48991	107	4, 5	209	173	1, 4	55
47	$x = y$	$7x^2$	112	1, 6	73	174		
48	1, 6	55	118	3, 10	341	177	1, 12	215
49	9, 56	4721	119	$x = y$	$11x^2$	178	1, 2	27
52	1, 2	15	122	1, 4	47		5, 36	2729
	1, 4	33		1, 10	149	183	1, 12	217
	2, 21	535		2, 3	67	184	1, 6	89
55	1, 15	251		14, 55	9029	187	1, 4	57
56	2, 5	79	127	1, 3	35	188	1, 8	127
57	12, 55	5831		8, 21	1945	189	432, 65	428801
60	273, 10	77471	128	1, 2	23	191	3, 5	209
61	5, 4	159		1, 10	151	194	$x = y$	$14x^2$
62	$x = y$	$8x^2$		2, 55	3271		1, 5	74
63	3, 4	97		3, 4	137		1, 6	91
64	2, 3	49	131	1, 12	199	196	1, 8	129
	2, 15	329	132	3, 10	359		3, 4	169
	7, 12	689		7, 12	977		7, 20	2001
66	1, 3	26	133	4, 7	327	197	3, 8	343
67	4, 21	817	134	1, 4	49	198	2, 3	85
68	1, 2	17	135				13, 204	55897
71	1, 5	49	137	28, 377	188327	199	7, 15	1499
73	3, 8	215	140	2, 5	121	200		
77	1, 8	95		8, 15	1439			

☞ Blanks in x, y, z columns mean that x. y, z are possible; see Euler's *Comment. Arithm.*, t. ii, p. 494, &c.

$$x^3 \pm Cy^3 = z^2, \quad [\text{up to } C = 100].$$

Algebraic Solutions.—

$C = \zeta^2 - \xi^3;$	$C = \xi^3 - \zeta^2;$	$C = \xi^3 + \zeta^2;$
$\xi^3 + C.1^3 = \zeta^2.$	$\xi^3 - C.1^3 = \zeta^2.$	$\xi^3 - C.1^3 = -\zeta^2.$

$x^3 \pm Cy^3 = z^2$ with $C = k^3$ gives $(k^2x)^3 + C(ky)^3 = (Cz)^2$.

$x^3 + Cy^3 = z^2$		$x^3 - Cy^3 = z^2$		$x^3 - Cy^3 = -z^2$		$x^3 + Cy^3 = z^2$		$x^3 - Cy^3 = z^2$		$x^3 - Cy^3 = -z^2$	
C	x, y, z	C	x, y, z	C	x, y, z	C	x, y, z	C	x, y, z	C	x, y, z
1	1, 2, 3	1	8, 7, 13	1	7, 8, 13	23		19	24, 5, 107	24	23, 8, 11
2	17, 4, 71	2	3, 1, 5	2	1, 1, 1	24	1, 1, 5	20	6, 1, 14	25	15, 6, 45
3	1, 1, 2		7, 3, 17		15, 12, 9	25	1, 3, 26	21		26	1, 1, 5
	1, 2, 5	3	13, 3, 46	3	2, 2, 4	26		22			1, 5, 57
	10, 2, 32		16, 5, 61		14, 10, 16	27	9, 6, 81	23	3, 1, 2	27	
4	33, 12, 207		22, 6, 100		23, 16, 11	28	2, 1, 6	24	22, 3, 100	28	3, 1, 1
5	6, 2, 16	4	2, 1, 2	4	2, 3, 10		29, 5, 167	25	5, 1, 10	29	
	6, 5, 29		5, 1, 11		15, 10, 25	29	1, 3, 28		9, 2, 23	30	
	14, 10, 88		13, 9, 1		31, 20, 47		9, 2, 31	26	3, 1, 1	31	3, 1, 2
6	1, 2, 7		24, 15, 18	5	1, 1, 2	30	19, 5, 103		17, 4. 57	32	·8, 6, 80
	9, 6, 45	5	6, 3, 9	6	8, 6, 28	31			23, 7, 57	33	2, 1, 5
7	2, 2, 8		14, 2, 52	7	5, 3, 8	32	33, 6, 207	27		34	15, 6, 63
	15, 7, 76		19, 3, 82	8	7, 4, 13	33		28	4, 1, 6	35	
8	1, 1, 3	6	4, 2, 4	9	2, 1, 1	34			8, 1, 22	36	3, 1, 3
	2, 1, 4	7	2, 1, 1		2, 2, 8	35	1, 1, 6	29		37	1, 1, 6
9	3. 1, 6	8	10, 3, 28		2, 6, 44		16, 3, 71	30			1, 5, 68
	6, 10, 96	9	6, 2, 12	10	1, 1, 3	36	4, 1, 10	31	33, 6, 171		16, 5, 23
10	1, 2, 9		7, 3, 10		4, 2, 4		12, 1, 42	32		38	
	9, 4, 37		30, 14, 48	11	2, 3, 17	37	3, 1, 8	33	31, 3, 170	39	
	11, 7, 69		33, 8, 177		6, 3, 9		26, 5, 149	34	25, 6, 91	40	
11	4, 3, 19	10	11, 5, 9		7, 4, 19		41, 4, 267	35	11, 1, 36	41	
	34, 3, 199		16, 6, 44	12	2, 1, 2	38	11, 1, 37	36	9, 2, 21	42	
12	13, 1, 47	11	3, 1, 4		11, 5, 13	39			21, 5, 69	43	3, 1, 4
13	1, 6, 53		9, 4, 5	13	14, 6, 8	40	14, 5, 88	37	10, 3, 1		7, 2, 1
	22, 6, 116		14, 5, 37	14		41	2, 1, 7	38		44	2, 1, 6
14	1, 6, 55		15, 1, 58	15		42	9, 6, 99	39	4, 1, 5		7, 2, 3
	9, 2, 29	12	10, 3, 26	16	15, 6, 9	43	4, 3, 35		10, 1, 31		8, 3, 26
15	1, 1, 4	13	9, 2, 25	17	1, 1, 4		9, 4, 59	40	14, 1, 52	45	
	9, 6, 63		17, 1, 70		2, 1, 3	44	5, 1, 13	41		46	7, 2, 5
16	17, 2, 71	14		18	5, 3, 19	45	9, 2, 33	42			15, 6, 81
17	2, 1, 5	15	4, 1, 7		9, 5, 39	46	4, 6, 100	43		47	
	4, 1, 9	16	28, 6, 136	19	2, 6, 64	47	2, 7, 127	44	5, 1, 9	48	8, 3, 28
	8, 1, 23	17	19, 3, 80		8, 3, 1		17, 4, 89	45	21, 1, 96	49	7, 2, 7
18	7, 1, 19		18, 7, 1		15, 6, 27	48	1, 1, 7		34, 6, 172	50	1, 1, 7
	31, 5, 179		26, 10, 24	20	19, 7, 1	49		46	9, 2, 19		5, 3, 35
19	5, 1, 12		33, 12, 81	21		50		47	6, 1, 13	51	2, 3, 37
	17, 8, 121	18	3, 1, 3	22		51			12, 1, 41	52	3, 1, 5
20			57, 4, 429	23		52	17, 2, 73	48	4, 1, 4	53	7, 2, 9
21		19	7, 1, 18	24	2, 1, 4	53	24, 5, 143	49	21, 5, 56		11, 3, 10
22	3, 1, 7		17, 2, 69		14, 5, 16	54	3, 1, 9		22, 6, 8	54	15, 4, 9

$x^3 \pm Cy^3 = z^2$, [up to $C = 100$].

$x^3+Cy^3=z^2$		$x^3-Cy^3=z^2$		$x^3-Cy^3=-z^2$		$x^3-Cy^3=z^2$		$x^3-Cy^3=z^2$		$x^3-Cy^3=-z^2$	
C	x, y, z	C	x, y, z	C	x, y, z	C	x, y, z	C	x, y, z	C	x, y, z
55	9, 1, 28	49	25, 6, 71	55	19, 5, 4	80	9, 2, 37	74		82	1, 1, 9
56	2, 1, 8	50			24, 7, 71	81	9, 6, 135	75	34, 6, 152	83	5, 3, 46
57	4, 1, 11	51	19, 5, 22	56	20, 6, 64		10, 6, 136	76	5, 1, 7	84	26, 7, 106
	7, 1, 20	52		57	2, 1, 7	82			9, 2, 11	85	26, 6, 28
58		53	9, 1, 26	58	7, 2, 11	83			17, 4, 7	86	
59			17, 2, 67		23, 6, 19	84		77		87	
60	9, 6, 117		17, 4, 39	59	14, 6, 100	85		78		88	7, 2, 19
61	13, 3, 62	54	7, 1, 17	60	11, 3, 17	86		79	13, 3, 8		8, 6, 136
62	4, 6, 116	55	4, 1, 3	61	15, 4, 23	87			20, 1, 89	89	2, 1, 9
	9, 2, 35		9, 2, 17		15, 6, 99	88	16, 6, 152	80	16, 3. 44		4, 1, 5
	28, 6, 188	56	18, 1, 76	62		89	10, 1, 33	81	9, 2, 9	90	
63	1, 1, 8	57		63	3, 1, 6		17, 2, 75		13, 1, 46	91	3, 1, 8
	9, 4, 69	58	33, 6, 153	64	7, 2, 13		17, 4, 103	82	33, 6, 135	92	
	57, 4, 435	59	26, 5, 101	65	1, 1, 8	90		83	33, 4, 175	93	23, 6, 89
64	8, 1, 24	60	4, 1, 2		4, 1, 1	91		84		94	
65	14, 1, 53	61	5, 1, 8		9, 5, 86	92	2, 1, 10	85	9, 2, 7	95	
66		62		66		93		86	17, 2, 65	96	
67	10, 3, 53	63	4, 1, 1	67		94	3, 1, 11	87	7, 1, 16	97	
68			9, 2, 15	68	4, 1, 2	95		88	9, 2, 5	98	7, 2, 21
69			25, 3, 118	69		96		89	5, 1, 6	99	
70		64	40, 6, 224	70		97	11, 5, 116		36, 7, 127	100	4, 1, 6
71	5, 1, 14	65		71	7, 2, 15	98	7, 1, 21	90	9, 2, 3		11, 3, 37
	9, 5, 98	66	25, 6, 37	72	2, 1, 8	99	1, 1, 10	91	9, 2, 1		15, 4, 55
72	6, 5, 96	67	17, 4, 25		2, 3, 44		9, 2, 39	92			15, 6, 135
73	2, 1, 9	68	13, 3, 19	73	4, 1, 3	100	5. 1, 15	93			
	3, 1, 10	69		74				94			
74		70	9, 2, 13	75	14, 6, 116			95	6, 1, 11		
75	10, 3, 55	71	8, 1, 21	76	3, 1, 7			96	40, 6, 208		
76	4, 3, 46		25, 6, 17	77				97			
	25, 6, 179		29, 7, 6	78				98	28, 6, 28		
77		72	6, 1, 12	79	7, 2, 17			99			
78			28, 6, 80		15, 4, 41			100	5, 1, 5		
79			30, 7, 48		15, 6, 117				10, 1, 30		
80	1, 1, 9		33, 4, 177	80	4, 1, 4						
	4, 1, 12	73		81	27, 7, 90						

$$N = x^4 + Ky^4 = \pm z^2; \quad [K = \pm k].$$

Addition to Table on page 230.

	$x^4 + ky^4 = z^2$		$x^4 - ky^4 = \pm z^2$	
	k	x, y, z	k	$x, y, z, \pm$
Algebraic	ζ^2-1	$1, 1, \zeta$	ζ^2+1	$1, 1, \zeta, -$
	$\zeta^2-\xi^4$	$\xi, 1, \zeta$	$\xi^4-\zeta^2$	$\xi, 1, \zeta, +$
			$\xi^4+\zeta^2$	$\xi, 1, \zeta, -$
	$\eta^4\mp 2$	$1, \eta, \eta^4\mp 1$		
	$\eta^4-2\xi^2$	$\xi, \eta, \eta^4-\xi^2$	$2\xi^2-\eta^4$	$\xi, \eta, \eta^4\sim\xi^2, +$
	$\eta^4+2\xi^2$	$\xi, \eta, \xi^2+\eta^4$		
	$2\xi^2+1$	$\xi, 1, \xi^2+1$	$2\xi^2-1$	$\xi, 1, \xi^2-1, +$
	$4\xi^2+4$	$\xi, 1, \xi^2+2$	$4\xi^2-4$	$\xi, 1, \xi^2-2, +$
Numerical	14	11, 4, 135	12	2, 1, 2, +
	20	2, 1, 6	17	7, 3, 32, +
	31	5, 3, 56	23	18, 5, 301, +
	46	25, 6, 671	32	2, 1, 4, −
	46	45, 8, 2071	47	8, 3, 17, +
	47	17, 5, 336	52	2, 1, 6, −
	48	2, 1, 8	56	13, 3, 155, +
	49	4, 3, 65	60	4, 1, 14, +
	63	3, 1, 12	72	3, 1, 3, +
	63	19, 3, 368	80	2, 1, 8, −
	68	4, 1, 18	82	13, 4, 87, +
	73	2, 3, 77	82	71, 10, 4959, +
	79	1, 3, 80	85	3, 1, 2, −
	83	1, 3, 82	90	3, 1, 3, −
	84	2, 1, 10	96	5, 1, 23, +
	89	2, 3, 85	97	3, 1, 4, −
	89	34, 7, 1245	97	19, 5, 264, +
	94	9, 4, 175		
	99	7, 1, 50		

End of Tables of Dimorph and Square Forms.

Duan and Half-Duan Primes. All primes $p = (4\varpi + 1)$ are

$$= (x^2 + y^2) = \tfrac{1}{2}(x'^2 + y'^2).$$

Cuban and Trito-Cuban Primes.—All primes $p = (6\varpi + 1)$ are $= (A^2 + 3B^2)$.

$$p = A^2 + 3B^2 = \frac{x^3 \sim y^3}{x \sim y} = \frac{x'^3 + y'^3}{x' + y'} = \tfrac{1}{3}\frac{\xi^3 \sim \eta^3}{\xi \sim \eta} = \tfrac{1}{3}\frac{\xi'^3 + \eta'^3}{\xi' + \eta'}.$$

The Tables following (pp. 238–252) give the value of y yielding primes (p) of forms named below arising from all *Simple Duans* and *Simple Cubans* up to $y \ngtr 15{,}000$; and also the formulæ for the "2^{ic} parts" (a, b), (A, B) of their 2^{ic} partitions, as shown on the scheme below. The remaining pages (253–258) show *Quartan*, *Sextan*, *Octavan*, and *Duodeciman Primes*, together with the elements (x, y) which lead to them according to the scheme at foot of this page.

Source.	p	a or b ,	b or a	Limits of y,	p	Primes.	Pages.
Simple Duans	$(1+y^2)$	1 ,	y	15,000	225.10^6	1199	238, 239
	$\frac{1}{2}(1+y^2)$	$\frac{1}{2}(y-1)$,	$\frac{1}{2}(y+1)$		1125.10^5	1288	240, 241
	$\frac{1}{5}(1+y^2)$	$\frac{1}{5}(2y \mp 1)$,	$\frac{1}{5}(y \pm 2)$		45.10^6	744	242
	$\frac{1}{10}(1+y^2)$	$\frac{1}{10}(3y \mp 1)$,	$\frac{1}{10}(y \pm 3)$		225.10^5	763	243, 244
	$\frac{1}{13}(1+y^2)$	$\frac{1}{13}(3y \mp 2)$,	$\frac{1}{13}(2y \pm 3)$		173.10^5	251	244
	$\frac{1}{17}(1+y^2)$	$\frac{1}{17}(4y \mp 1)$,	$\frac{1}{17}(y \pm 4)$		132.10^5	185	244

Source.	p	A ,	B		y,	p	Primes.	Pages.
Simple Cubans	$\frac{y^3-1}{y-1}$	$(\frac{1}{2}y+1)$, $\frac{1}{2}(y-1)$,	$\frac{1}{2}y$ $\frac{1}{2}(y+1)$	$y = \epsilon$ $y = \omega$	15,000	225.10^6	1992	245–247
	$\frac{1}{3}\frac{y^3-1}{y-1}$	$\frac{1}{2}y$, $\frac{1}{2}(y+1)$,	$\frac{1}{3}(\frac{1}{2}y+1)$ $\frac{1}{6}(y-1)$	$y = \epsilon$ $y = \omega$		75.10^6	1061	248, 249
	$\frac{1}{7}\frac{y^3-1}{y-1}$	Express $(y^3-1)\div(y-1)$ in form (A^2+3B^2) as above. Reduce the fractions ($\frac{1}{7}$, $\frac{1}{13}$, $\frac{1}{19}$, $\frac{1}{21}$) by "conformal division" [see Text].				32.10^6	770	250, 251
	$\frac{1}{13}\frac{y^3-1}{y-1}$					173.10^5	399	251
	$\frac{1}{19}\frac{y^3-1}{y-1}$					117.10^5	269	252
	$\frac{1}{21}\frac{y^3-1}{y-1}$					107.10^5	393	252

Prime.	Form of p.	Limit of p.	x, y	Primes.	Pages.
Quartans	$x^4 + y^4$	$\ngtr 10^7$	ω, ϵ	240	253, 255
Half-Quartans	$\frac{1}{2}(x^4 + y^4)$	$\ngtr 10^7$	ω, ω	172	254
High Quartans	$x^4 + y^4$	$> 10^7$ to 32.10^8	ω, ϵ	21, [$x=1$]	255
High $\frac{1}{2}$-Quartans	$\frac{1}{2}(x^4 + y^4)$	$> 10^7$ to 25.10^8	ω, ω	18, [$x=1$]	255
Sextans	$(x^6 + y^6) \div (x^2 + y^2)$	$> 10^7$	ω, y	360	256, 257
High Simple do.	do.	$> 10^7$ to 32.10^8	1, y	30	257
Octavans	$x^8 + y^8$	$\ngtr 4.10^{12}$	ω, ϵ	4	258
Half-Octavans	$\frac{1}{2}(x^8 + y^8)$	$\ngtr 189.10$[illegible]	ω, ω	5	258
Duodecimans	$(x^{12} + y^{12}) \div (x^4 + y^4)$	$\ngtr 10^{10}$		20	258

Elements (y) *of Simple Duan Primes* $p = (y^2+1)$.

2	326	890	1 406	1 920	2 460	2 974	3 624	4 170	4 734	5 344	5 944	6 590
4	340	906	1 410	1 940	2 464	2 986	3 644	4 174	4 736	5 360	5 960	6 604
6	350	910	1 416	1 964	2 470	3 016	3 650	4 176	4 754	5 370	5 964	6 614
10	384	920	1 420	1 966	2 496	3 026	3 660	4 180	4 780	5 384	5 970	6 636
14	386	930	1 430	1 970	2 516	3 046	3 670	4 184	4 784	5 404	5 984	6 646
16	396	936	1 434	1 974	2 534	3 054	3 686	4 206	4 786	5 420	5 990	6 704
20	400	946	1 440	1 980	2 536	3 074	3 716	4 226	4 794	5 424	5 996	6 710
24	406	950	1 456	1 990	2 550	3 094	3 730	4 250	4 796	5 430	6 006	6 714
26	420	960	1 460	2 006	2 570	3 106	3 734	4 260	4 834	5 446	6 010	6 724
36	430	966	1 494	2 026	2 576	3 110	3 746	4 266	4 850	5 466	6 016	6 734
40	436	986	1 504	2 034	2 594	3 134	3 754	4 294	4 876	5 474	6 030	6 764
54	440	1 004	1 524	2 050	2 600	3 136	3 756	4 300	4 886	5 476	6 046	6 776
56	444	1 010	1 546	2 054	2 604	3 140	3 764	4 310	4 894	5 486	6 060	6 780
66	464	1 036	1 550	2 056	2 624	3 156	3 774	4 330	4 904	5 490	6 096	6 784
74	466	1 054	1 556	2 064	2 646	3 160	3 776	4 336	4 910	5 500	6 110	6 786
84	470	1 060	1 564	2 074	2 654	3 174	3 784	4 340	4 920	5 506	6 120	6 800
90	474	1 066	1 566	2 080	2 664	3 184	3 790	4 364	4 936	5 510	6 126	6 806
94	490	1 070	1 570	2 084	2 666	3 196	3 794	4 366	4 944	5 524	6 130	6 824
110	496	1 080	1 576	2 086	2 676	3 204	3 800	4 370	4 954	5 536	6 134	6 826
116	536	1 094	1 580	2 094	2 684	3 214	3 806	4 374	4 956	5 550	6 140	6 850
120	544	1 096	1 586	2 096	2 700	3 220	3 826	4 384	4 990	5 560	6 156	6 854
124	556	1 106	1 614	2 106	2 706	3 240	3 850	4 404	5 004	5 564	6 164	6 866
126	570	1 124	1 616	2 116	2 730	3 246	3 870	4 410	5 014	5 566	6 166	6 874
130	576	1 140	1 640	2 120	2 736	3 254	3 884	4 414	5 016	5 574	6 176	6 884
134	584	1 144	1 644	2 126	2 746	3 266	3 890	4 444	5 030	5 584	6 190	6 910
146	594	1 146	1 654	2 136	2 754	3 274	3 894	4 456	5 044	5 586	6 216	6 926
150	634	1 150	1 660	2 154	2 760	3 280	3 900	4 474	5 054	5 590	6 220	6 930
156	636	1 156	1 664	2 174	2 766	3 290	3 910	4 486	5 056	5 620	6 234	6 944
160	644	1 174	1 674	2 210	2 770	3 304	3 924	4 496	5 076	5 656	6 236	6 956
170	646	1 176	1 676	2 224	2 776	3 306	3 946	4 504	5 080	5 664	6 240	6 970
176	654	1 184	1 684	2 260	2 780	3 314	3 966	4 510	5 086	5 700	6 254	6 980
180	674	1 210	1 686	2 266	2 794	3 326	3 984	4 524	5 120	5 710	6 266	6 984
184	680	1 244	1 700	2 286	2 804	3 334	3 994	4 530	5 126	5 724	6 306	6 990
204	686	1 246	1 716	2 294	2 824	3 340	4 006	4 534	5 154	5 726	6 314	6 996
206	690	1 274	1 736	2 304	2 834	3 350	4 024	4 540	5 170	5 734	6 340	7 010
210	696	1 276	1 756	2 310	2 836	3 356	4 026	4 554	5 176	5 756	6 350	7 014
224	700	1 290	1 766	2 314	2 850	3 360	4 034	4 566	5 180	5 760	6 360	7 016
230	704	1 294	1 774	2 320	2 864	3 374	4 046	4 590	5 194	5 774	6 366	7 044
236	714	1 306	1 784	2 326	2 876	3 390	4 056	4 600	5 200	5 804	6 400	7 050
240	716	1 314	1 790	2 330	2 884	3 436	4 070	4 604	5 204	5 814	6 420	7 066
250	740	1 316	1 794	2 336	2 890	3 446	4 080	4 606	5 226	5 824	6 434	7 100
256	750	1 320	1 816	2 354	2 896	3 474	4 086	4 614	5 236	5 830	6 460	7 114
260	760	1 324	1 824	2 360	2 900	3 480	4 114	4 616	5 246	5 834	6 480	7 130
264	764	1 340	1 850	2 380	2 916	3 490	4 120	4 644	5 254	5 850	6 514	7 150
270	780	1 350	1 860	2 404	2 924	3 504	4 124	4 650	5 256	5 856	6 530	7 160
280	784	1 354	1 870	2 406	2 926	3 516	4 136	4 666	5 264	5 866	6 536	7 164
284	816	1 366	1 876	2 420	2 934	3 520	4 140	4 700	5 294	5 874	6 540	7 190
300	826	1 374	1 884	2 430	2 944	3 530	4 146	4 704	5 304	5 876	6 546	7 216
306	860	1 376	1 894	2 454	2 960	3 534	4 154	4 716	5 314	5 880	6 550	7 240
314	864	1 394	1 910	2 456	2 964	3 536	4 156	4 726	5 340	5 930	6 576	7 244

Elements (y) *of Simple Duan Primes* $p=(y^2+1)$.

7 260	8 014	8 784	9 460	10 126	10 790	11 456	12 154	12 816	13 550	14 226
7 286	8 030	8 786	9 474	10 130	10 796	11 480	12 184	12 820	13 576	14 264
7 304	8 034	8 790	9 476	10 150	10 804	11 514	12 194	12 844	13 620	14 270
7 316	8 064	8 816	9 486	10 160	10 814	11 520	12 214	12 854	13 650	14 290
7 326	8 080	8 846	9 494	10 166	10 836	11 560	12 224	12 874	13 656	14 294
7 364	8 100	8 854	9 520	10 216	10 840	11 566	12 234	12 876	13 660	14 330
7 384	8 114	8 876	9 530	10 240	10 844	11 586	12 256	12 880	13 666	14 356
7 404	8 116	8 880	9 546	10 246	10 846	11 596	12 276	12 896	13 674	14 374
7 410	8 174	8 894	9 554	10 256	10 854	11 600	12 294	12 910	13 680	14 380
7 414	8 176	8 940	9 564	10 270	10 866	11 610	12 300	12 920	13 686	14 406
7 420	8 180	8 964	9 596	10 276	10 890	11 626	12 314	12 926	13 724	14 410
7 434	8 184	8 974	9 600	10 284	10 894	11 650	12 334	12 964	13 754	14 414
7 456	8 194	8 976	9 630	10 294	10 896	11 674	12 336	12 970	13 756	14 426
7 460	8 196	8 996	9 650	10 324	10 914	11 680	12 344	12 986	13 786	14 466
7 466	8 206	9 000	9 666	10 326	10 936	11 720	12 354	13 056	13 806	14 476
7 474	8 210	9 010	9 670	10 350	10 960	11 750	12 356	13 064	13 820	14 484
7 490	8 226	9 016	9 696	10 360	10 966	11 766	12 360	13 066	13 830	14 486
7 504	8 230	9 020	9 714	10 376	10 970	11 790	12 386	13 076	13 846	14 494
7 516	8 254	9 024	9 724	10 384	10 984	11 800	12 390	13 090	13 854	14 496
7 520	8 270	9 046	9 744	10 414	11 010	11 804	12 396	13 100	13 870	14 504
7 524	8 290	9 054	9 760	10 416	11 024	11 810	12 404	13 106	13 880	14 506
7 536	8 296	9 120	9 770	10 424	11 026	11 814	12 416	13 110	13 886	14 544
7 550	8 304	9 124	9 786	10 426	11 034	11 816	12 434	13 130	13 900	14 550
7 596	8 324	9 126	9 804	10 430	11 056	11 830	12 450	13 136	13 924	14 560
7 604	8 350	9 154	9 806	10 490	11 074	11 866	12 454	13 180	13 940	14 566
7 624	8 376	9 164	9 826	10 504	11 076	11 886	12 460	13 224	13 964	14 576
7 656	8 386	9 180	9 844	10 506	11 096	11 894	12 484	13 246	13 984	14 580
7 674	8 420	9 204	9 860	10 516	11 116	11 910	12 486	13 254	13 994	14 606
7 716	8 424	9 214	9 874	10 520	11 130	11 924	12 490	13 266	14 000	14 634
7 720	8 434	9 240	9 876	10 530	11 154	11 934	12 506	13 274	14 010	14 636
7 734	8 454	9 246	9 880	10 550	11 170	11 946	12 546	13 284	14 016	14 660
7 744	8 500	9 260	9 894	10 556	11 200	11 970	12 564	13 286	14 020	14 674
7 754	8 540	9 266	9 896	10 560	11 204	11 980	12 570	13 310	14 026	14 694
7 770	8 550	9 270	9 900	10 580	11 236	11 990	12 590	13 336	14 034	14 714
7 774	8 554	9 276	9 904	10 594	11 244	11 996	12 614	13 344	14 036	14 716
7 780	8 576	9 280	9 956	10 614	11 246	12 000	12 620	13 350	14 040	14 746
7 796	8 584	9 294	9 970	10 634	11 256	12 014	12 624	13 360	14 086	14 774
7 804	8 610	9 310	9 980	10 640	11 270	12 016	12 630	13 376	14 104	14 790
7 806	8 626	9 314	9 986	10 654	11 286	12 024	12 634	13 390	14 120	14 814
7 810	8 634	9 324	9 990	10 666	11 320	12 036	12 636	13 394	14 144	14 824
7 820	8 656	9 336	10 006	10 674	11 330	12 060	12 684	13 420	14 156	14 826
7 836	8 670	9 340	10 014	10 690	11 336	12 064	12 694	13 430	14 166	14 850
7 854	8 680	9 356	10 016	10 700	11 346	12 084	12 710	13 436	14 180	14 886
7 856	8 684	9 374	10 024	10 726	11 350	12 090	12 724	13 466	14 186	14 914
7 864	8 694	9 386	10 050	10 734	11 364	12 094	12 730	13 490	14 190	14 926
7 906	8 706	9 406	10 056	10 744	11 374	12 096	12 744	13 506	14 194	14 936
7 910	8 720	9 424	10 074	10 764	11 400	12 120	12 756	13 516	14 196	14 940
7 944	8 750	9 426	10 084	10 770	11 404	12 126	12 764	13 520	14 200	14 950
7 946	8 760	9 434	10 086	10 780	11 416	12 140	12 766	13 534	14 210	14 996
7 956	8 774	9 436	10 116	10 784	11 436	12 144	12 776	13 546	14 220	

☞ *This Table gives the Elements* (y) *of all the Simple Duan Primes* $p=(y^2+1) \not> 225.10^6$.

Elements (y) *of Simple Half-Duan Primes* $p = \frac{1}{2}\mathrm{N} = \frac{1}{2}(y^2+1)$.

1	315	745	1 171	1 639	2 271	2 755	3 249	3 819	4 345	4 979	5 539	6 199	6 741
3	321	751	1 179	1 661	2 281	2 765	3 261	3 825	4 349	4 999	5 571	6 205	6 751
5	325	779	1 181	1 689	2 285	2 771	3 291	3 835	4 351	5 009	5 591	6 215	6 775
9	329	781	1 185	1 691	2 289	2 785	3 295	3 849	4 359	5 015	5 599	6 221	6 785
11	335	791	1 199	1 701	2 315	2 789	3 315	3 851	4 361	5 021	5 605	6 225	6 789
15	345	799	1 205	1 705	2 321	2 799	3 321	3 859	4 371	5 025	5 645	6 249	6 799
19	349	815	1 219	1 709	2 331	2 805	3 329	3 865	4 375	5 031	5 651	6 275	6 801
25	371	819	1 225	1 715	2 341	2 815	3 341	3 875	4 435	5 041	5 661	6 295	6 809
29	375	821	1 241	1 741	2 349	2 819	3 365	3 885	4 439	5 055	5 669	6 301	6 811
35	379	841	1 251	1 749	2 351	2 821	3 391	3 901	4 445	5 061	5 675	6 315	6 829
39	391	855	1 255	1 765	2 375	2 831	3 395	3 909	4 459	5 069	5 679	6 319	6 831
45	399	861	1 265	1 771	2 379	2 841	3 399	3 919	4 465	5 071	5 685	6 329	6 841
49	405	869	1 281	1 805	2 381	2 845	3 419	3 935	4 479	5 085	5 695	6 331	6 845
51	409	875	1 285	1 811	2 389	2 849	3 429	3 939	4 485	5 089	5 711	6 335	6 849
59	415	881	1 299	1 831	2 391	2 871	3 431	3 951	4 495	5 105	5 719	6 341	6 871
61	425	885	1 311	1 845	2 405	2 889	3 441	3 959	4 511	5 109	5 735	6 351	6 899
65	435	901	1 315	1 855	2 425	2 895	3 459	3 981	4 531	5 115	5 739	6 359	6 931
69	441	909	1 329	1 859	2 459	2 901	3 465	3 989	4 561	5 129	5 755	6 361	6 935
71	445	921	1 345	1 869	2 481	2 905	3 485	3 995	4 585	5 139	5 761	6 369	6 939
79	449	925	1 349	1 875	2 505	2 925	3 491	4 011	4 589	5 159	5 765	6 385	6 941
85	451	929	1 359	1 899	2 519	2 935	3 495	4 021	4 609	5 165	5 769	6 399	6 951
95	459	935	1 361	1 901	2 525	2 949	3 499	4 029	4 615	5 185	5 771	6 419	6 969
101	461	949	1 389	1 915	2 539	2 951	3 501	4 031	4 635	5 219	5 775	6 425	6 971
121	471	951	1 391	1 935	2 541	2 965	3 519	4 041	4 639	5 225	5 795	6 429	6 975
131	519	955	1 405	1 949	2 545	2 981	3 529	4 049	4 651	5 241	5 805	6 459	6 995
139	521	959	1 411	2 001	2 549	3 001	3 539	4 055	4 661	5 245	5 839	6 461	7 005
141	529	979	1 419	2 009	2 561	3 015	3 561	4 065	4 669	5 251	5 849	6 471	7 011
145	535	981	1 421	2 021	2 565	3 031	3 569	4 075	4 689	5 269	5 851	6 475	7 031
159	545	985	1 439	2 035	2 581	3 035	3 571	4 079	4 691	5 271	5 859	6 485	7 045
165	559	989	1 459	2 051	2 585	3 045	3 575	4 085	4 709	5 279	5 879	6 499	7 049
169	569	991	1 465	2 055	2 589	3 049	3 585	4 089	4 719	5 281	5 911	6 509	7 055
171	571	1 001	1 469	2 065	2 591	3 059	3 605	4 099	4 721	5 289	5 925	6 511	7 069
175	575	1 011	1 489	2 079	2 599	3 071	3 621	4 119	4 759	5 301	5 931	6 545	7 071
181	579	1 025	1 495	2 091	2 609	3 075	3 655	4 121	4 771	5 311	5 941	6 559	7 075
195	581	1 029	1 499	2 125	2 611	3 095	3 665	4 145	4 795	5 329	5 955	6 561	7 081
199	595	1 031	1 501	2 135	2 619	3 101	3 675	4 171	4 799	5 341	5 965	6 565	7 089
201	609	1 039	1 515	2 139	2 629	3 109	3 689	4 185	4 801	5 365	5 979	6 581	7 095
205	631	1 051	1 519	2 145	2 635	3 119	3 691	4 189	4 809	5 369	6 021	6 585	7 099
209	639	1 055	1 521	2 151	2 645	3 131	3 701	4 199	4 811	5 399	6 029	6 589	7 101
219	641	1 069	1 531	2 159	2 659	3 145	3 705	4 201	4 825	5 405	6 051	6 605	7 105
221	649	1 081	1 535	2 161	2 661	3 159	3 709	4 209	4 859	5 415	6 075	6 611	7 111
231	661	1 091	1 541	2 165	2 671	3 171	3 711	4 221	4 865	5 425	6 081	6 619	7 115
245	669	1 095	1 545	2 171	2 681	3 191	3 715	4 225	4 871	5 431	6 085	6 621	7 131
261	685	1 099	1 569	2 175	2 691	3 195	3 721	4 231	4 879	5 441	6 101	6 631	7 139
271	689	1 111	1 589	2 199	2 701	3 201	3 725	4 241	4 885	5 471	6 139	6 639	7 149
275	695	1 125	1 599	2 201	2 705	3 205	3 751	4 265	4 901	5 475	6 145	6 645	7 159
279	699	1 129	1 605	2 239	2 711	3 211	3 755	4 289	4 905	5 499	6 149	6 669	7 169
289	711	1 151	1 615	2 255	2 719	3 215	3 759	4 299	4 941	5 515	6 155	6 695	7 179
299	715	1 155	1 629	2 261	2 729	3 235	3 785	4 301	4 949	5 525	6 169	6 705	7 189
309	739	1 161	1 635	2 269	2 731	3 241	3 789	4 315	4 965	5 531	6 195	6 735	7 199

Elements (y) *of Simple Half-Duan Primes* $p = \frac{1}{2}N = \frac{1}{2}(y^2+1)$.

7 201	7 825	8 605	9 259	9 851	10 541	11 189	11 859	12 479	13 139	13 841	14 419
7 211	7 829	8 639	9 265	9 855	10 545	11 191	11 865	12 489	13 155	13 855	14 455
7 235	7 845	8 645	9 285	9 861	10 549	11 229	11 889	12 509	13 181	13 861	14 469
7 239	7 855	8 661	9 309	9 865	10 565	11 239	11 911	12 519	13 195	13 865	14 489
7 241	7 859	8 665	9 315	9 869	10 579	11 245	11 925	12 539	13 199	13 875	14 491
7 245	7 861	8 675	9 319	9 889	10 581	11 261	11 935	12 551	13 201	13 895	14 511
7 261	7 975	8 719	9 331	9 891	10 589	11 265	11 949	12 565	13 225	13 899	14 519
7 265	7 981	8 721	9 359	9 905	10 591	11 281	11 951	12 575	13 235	13 911	14 551
7 269	7 991	8 729	9 361	9 925	10 599	11 295	11 959	12 581	13 245	13 921	14 569
7 279	7 999	8 735	9 369	9 951	10 601	11 299	11 969	12 585	13 249	13 925	14 571
7 281	8 001	8 745	9 375	9 981	10 609	11 329	11 995	12 595	13 251	13 959	14 579
7 299	8 005	8 749	9 385	10 001	10 611	11 345	12 005	12 645	13 261	13 971	14 585
7 315	8 009	8 755	9 389	10 011	10 635	11 355	12 011	12 649	13 301	13 975	14 611
7 335	8 019	8 761	9 395	10 029	10 659	11 359	12 039	12 655	13 325	13 979	14 631
7 339	8 031	8 769	9 399	10 049	10 661	11 379	12 055	12 691	13 335	13 999	14 645
7 341	8 061	8 771	9 401	10 055	10 705	11 421	12 071	12 705	13 339	14 005	14 661
7 385	8 069	8 779	9 419	10 065	10 711	11 441	12 075	12 715	13 345	14 051	14 671
7 395	8 075	8 781	9 425	10 069	10 719	11 455	12 099	12 731	13 355	14 059	14 679
7 401	8 085	8 825	9 445	10 071	10 721	11 481	12 109	12 755	13 361	14 065	14 681
7 429	8 089	8 831	9 475	10 081	10 739	11 495	12 119	12 765	13 371	14 079	14 689
7 439	8 111	8 841	9 481	10 089	10 741	11 509	12 129	12 769	13 445	14 085	14 725
7 449	8 115	8 869	9 509	10 099	10 749	11 519	12 131	12 775	13 449	14 101	14 729
7 451	8 129	8 875	9 519	10 125	10 775	11 551	12 135	12 779	13 455	14 105	14 745
7 475	8 131	8 909	9 535	10 139	10 779	11 569	12 155	12 781	13 469	14 109	14 779
7 485	8 175	8 919	9 539	10 149	10 845	11 585	12 161	12 785	13 471	14 115	14 809
7 489	8 189	8 925	9 545	10 151	10 851	11 589	12 165	12 789	13 479	14 125	14 835
7 495	8 231	8 941	9 561	10 205	10 855	11 599	12 169	12 809	13 491	14 145	14 855
7 511	8 235	8 961	9 571	10 209	10 871	11 611	12 171	12 815	13 495	14 159	14 869
7 515	8 259	8 969	9 579	10 219	10 881	11 621	12 191	12 825	13 565	14 161	14 881
7 541	8 261	8 995	9 619	10 231	10 891	11 629	12 239	12 851	13 569	14 171	14 885
7 551	8 269	9 009	9 629	10 235	10 905	11 645	12 245	12 855	13 581	14 179	14 891
7 559	8 279	9 021	9 631	10 245	10 911	11 651	12 265	12 861	13 595	14 205	14 899
7 589	8 305	9 041	9 655	10 251	10 929	11 659	12 281	12 869	13 599	14 209	14 909
7 601	8 321	9 049	9 665	10 261	10 949	11 685	12 285	12 881	13 609	14 235	14 911
7 611	8 329	9 051	9 671	10 271	10 965	11 711	12 299	12 885	13 635	14 245	14 925
7 631	8 361	9 085	9 675	10 285	11 009	11 719	12 301	12 925	13 641	14 249	14 961
7 651	8 365	9 101	9 681	10 319	11 011	11 725	12 305	12 949	13 649	14 255	14 965
7 659	8 381	9 109	9 689	10 321	11 021	11 729	12 325	12 961	13 661	14 261	14 975
7 661	8 395	9 119	9 699	10 351	11 025	11 755	12 335	12 985	13 665	14 265	
7 681	8 399	9 135	9 701	10 355	11 039	11 761	12 361	12 989	13 731	14 275	
7 689	8 409	9 151	9 709	10 361	11 049	11 765	12 365	12 991	13 735	14 289	
7 695	8 415	9 155	9 715	10 371	11 065	11 769	12 369	12 999	13 739	14 291	
7 741	8 431	9 165	9 721	10 415	11 101	11 771	12 385	13 041	13 741	14 329	
7 749	8 439	9 169	9 751	10 419	11 109	11 779	12 409	13 045	13 765	14 349	
7 751	8 491	9 179	9 765	10 445	11 121	11 781	12 419	13 071	13 779	14 351	
7 759	8 511	9 185	9 789	10 461	11 141	11 795	12 421	13 081	13 789	14 365	
7 771	8 515	9 191	9 791	10 469	11 145	11 821	12 435	13 085	13 799	14 375	
7 785	8 545	9 195	9 795	10 475	11 155	11 829	12 441	13 095	13 805	14 381	
7 791	8 561	9 241	9 815	10 489	11 169	11 831	12 469	13 101	13 835	14 405	
7 809	8 579	9 255	9 821	10 529	11 171	11 841	12 471	13 119	13 839	14 411	

☞ *This Table gives the Elements* (y) *of all Simple Half-Duan Primes* $p = \frac{1}{2}(y^2+1) \ngtr \frac{1}{2}.225.10^6$.

Elements (y) *of Primes* $p = \frac{1}{5}N = \frac{1}{5}(y^2+1)$.

2	622	1 498	2 338	3 252	4 262	5 398	6 342	7 352	8 452	9 572	10 592	11 628	12 642	13 898
8	628	1 502	2 352	3 272	4 292	5 422	6 372	7 358	8 472	9 598	10 598	11 638	12 662	13 938
12	638	1 512	2 392	3 278	4 312	5 428	6 408	7 362	8 492	9 648	10 608	11 642	12 688	13 942
22	652	1 522	2 398	3 288	4 328	5 438	6 412	7 372	8 498	9 662	10 622	11 648	12 692	13 952
28	662	1 528	2 402	3 338	4 352	5 458	6 448	7 398	8 502	9 672	10 688	11 672	12 728	13 988
42	692	1 538	2 438	3 342	4 388	5 472	6 452	7 422	8 508	9 692	10 702	11 678	12 762	13 998
48	698	1 572	2 442	3 358	4 398	5 488	6 458	7 448	8 522	9 738	10 708	11 688	12 798	14 002
52	702	1 588	2 472	3 402	4 408	5 492	6 462	7 458	8 528	9 748	10 712	11 702	12 872	14 052
58	728	1 592	2 498	3 428	4 448	5 508	6 472	7 462	8 592	9 752	10 722	11 712	12 912	14 088
62	738	1 638	2 508	3 442	4 452	5 528	6 478	7 478	8 602	9 778	10 728	11 738	12 928	14 092
78	758	1 642	2 542	3 448	4 462	5 558	6 562	7 592	8 608	9 792	10 738	11 742	12 942	14 108
88	792	1 688	2 562	3 452	4 488	5 578	6 572	7 642	8 678	9 812	10 752	11 752	12 972	14 122
92	828	1 692	2 578	3 512	4 498	5 602	6 578	7 672	8 698	9 838	10 762	11 778	12 998	14 138
102	838	1 712	2 602	3 522	4 522	5 612	6 608	7 698	8 748	9 852	10 788	11 788	13 042	14 142
108	842	1 742	2 628	3 578	4 548	5 622	6 628	7 712	8 758	9 858	10 802	11 808	13 048	14 192
152	848	1 748	2 642	3 592	4 612	5 628	6 642	7 728	8 772	9 938	10 822	11 888	13 058	14 228
158	862	1 762	2 672	3 598	4 748	5 642	6 662	7 828	8 778	9 942	10 838	11 958	13 072	14 238
178	872	1 778	2 678	3 602	4 762	5 652	6 688	7 848	8 788	9 948	10 848	11 962	13 098	14 248
188	898	1 792	2 698	3 612	4 822	5 658	6 722	7 852	8 798	9 978	10 852	11 988	13 108	14 258
198	908	1 822	2 702	3 628	4 842	5 698	6 738	7 862	8 808	9 998	10 878	12 012	13 142	14 272
202	912	1 842	2 708	3 638	4 848	5 758	6 758	7 872	8 838	10 008	10 942	12 038	13 172	14 322
222	942	1 848	2 742	3 672	4 852	5 762	6 772	7 898	8 852	10 048	10 972	12 042	13 198	14 328
238	962	1 858	2 752	3 678	4 862	5 828	6 792	7 902	8 872	10 052	10 988	12 048	13 202	14 342
248	972	1 862	2 762	3 688	4 898	5 838	6 808	7 928	8 892	10 098	10 998	12 058	13 208	14 388
258	978	1 872	2 788	3 712	4 912	5 862	6 822	7 942	8 908	10 142	11 062	12 088	13 238	14 392
262	988	1 888	2 802	3 728	4 928	5 942	6 828	7 962	8 922	10 152	11 072	12 102	13 258	14 402
272	1 008	1 898	2 808	3 738	4 962	5 948	6 848	7 972	8 958	10 172	11 078	12 112	13 272	14 428
292	1 062	1 948	2 812	3 742	4 972	5 952	6 892	7 978	8 992	10 178	11 092	12 122	13 288	14 442
298	1 072	1 952	2 822	3 798	4 978	5 958	6 922	8 012	9 008	10 188	11 108	12 152	13 292	14 462
308	1 078	1 978	2 858	3 802	4 988	5 978	6 928	8 022	9 012	10 202	11 178	12 158	13 378	14 478
312	1 088	1 988	2 862	3 878	4 992	6 002	6 952	8 058	9 028	10 208	11 192	12 178	13 422	14 538
328	1 108	2 012	2 872	3 888	5 008	6 008	6 978	8 092	9 042	10 212	11 212	12 188	13 438	14 652
352	1 112	2 022	2 898	3 912	5 048	6 038	6 988	8 138	9 058	10 222	11 222	12 208	13 488	14 678
358	1 138	2 028	2 908	3 922	5 072	6 052	7 022	8 148	9 122	10 228	11 228	12 252	13 508	14 738
362	1 192	2 052	2 922	3 942	5 102	6 098	7 048	8 162	9 178	10 248	11 238	12 288	13 522	14 742
388	1 208	2 058	2 942	3 958	5 122	6 112	7 062	8 178	9 188	10 302	11 252	12 298	13 542	14 762
402	1 238	2 122	2 948	3 962	5 142	6 122	7 092	8 188	9 198	10 338	11 262	12 322	13 608	14 842
422	1 272	2 128	2 952	3 992	5 148	6 142	7 112	8 212	9 202	10 352	11 308	12 328	13 612	13 848
428	1 278	2 142	3 012	4 028	5 162	6 162	7 152	8 228	9 242	10 372	11 362	12 348	13 692	14 852
458	1 298	2 148	3 048	4 062	5 178	6 188	7 162	8 242	9 258	10 398	11 378	12 362	13 702	14 858
462	1 312	2 198	3 052	4 092	5 188	6 198	7 188	8 262	9 338	10 438	11 388	12 392	13 708	14 912
478	1 342	2 208	3 072	4 102	5 222	6 212	7 192	8 278	9 348	10 458	11 392	12 398	13 712	14 948
488	1 358	2 222	3 078	4 112	5 228	6 228	7 208	8 308	9 402	10 462	11 452	12 412	13 722	14 962
492	1 372	2 242	3 092	4 122	5 252	6 238	7 228	8 322	9 412	10 472	11 508	12 452	13 728	14 988
508	1 378	2 252	3 108	4 148	5 278	6 242	7 242	8 348	9 428	10 478	11 528	12 522	13 738	
522	1 402	2 258	3 142	4 172	5 328	6 262	7 278	8 362	9 438	10 488	11 542	12 548	13 748	
558	1 442	2 262	3 172	4 202	5 352	6 292	7 292	8 378	9 458	10 492	11 548	12 552	13 802	
572	1 452	2 272	3 198	4 208	5 372	6 298	7 302	8 392	9 512	10 552	11 592	12 572	13 812	
588	1 472	2 278	3 208	4 248	5 378	6 322	7 308	8 438	9 542	10 562	11 602	12 608	13 828	
602	1 488	2 298	3 212	4 258	5 388	6 338	7 322	8 448	9 548	10 572	11 622	12 612	13 888	

☞ *This Table gives the Elements* (y) *of all Primes* $p = \frac{1}{5}(y^2+1) \not> 45.10^6$.

Elements (y) *of Primes* $p = \frac{1}{10}N = \frac{1}{10}(y^2 + 1)$.

3	547	1 337	2 277	3 267	4 177	5 167	6 103	7 213	8 227	9 323	10 437	11 503	12 723	13 717
7	573	1 367	2 317	3 273	4 183	5 173	6 113	7 237	8 233	9 327	10 453	11 527	12 727	13 727
13	587	1 377	2 323	3 317	4 213	5 177	6 177	7 277	8 287	9 373	10 463	11 537	12 733	13 753
17	617	1 387	2 347	3 337	4 223	5 187	6 187	7 287	8 313	9 377	10 467	11 563	12 773	13 773
23	627	1 397	2 373	3 377	4 227	5 197	6 203	7 313	8 327	9 423	10 513	11 613	12 827	13 777
27	637	1 413	2 377	3 433	4 247	5 263	6 213	7 317	8 333	9 427	10 523	11 623	12 847	13 823
33	653	1 417	2 467	3 467	4 253	5 267	6 253	7 333	8 337	9 447	10 537	11 633	12 853	13 833
37	673	1 423	2 483	3 473	4 287	5 287	6 267	7 347	8 353	9 453	10 563	11 663	12 903	13 873
53	677	1 447	2 523	3 483	4 303	5 303	6 317	7 373	8 383	9 483	10 567	11 673	12 913	13 877
63	683	1 473	2 603	3 487	4 313	5 327	6 347	7 387	8 413	9 487	10 583	11 677	12 963	13 887
67	753	1 497	2 613	3 503	4 317	5 367	6 367	7 403	8 433	9 513	10 627	11 703	12 973	13 913
77	763	1 527	2 617	3 513	4 377	5 397	6 377	7 417	8 447	9 523	10 647	11 713	12 977	13 917
87	773	1 533	2 623	3 533	4 383	5 423	6 423	7 423	8 463	9 583	10 653	11 763	12 983	13 923
97	777	1 563	2 637	3 537	4 413	5 437	6 433	7 513	8 577	9 597	10 673	11 813	12 987	13 963
103	797	1 573	2 663	3 563	4 417	5 447	6 437	7 547	8 583	9 637	10 677	11 833	12 997	14 003
113	817	1 587	2 677	3 577	4 427	5 497	6 467	7 563	8 613	9 647	10 737	11 837	13 017	14 023
127	823	1 647	2 687	3 587	4 447	5 537	6 483	7 573	8 633	9 653	10 753	11 897	13 023	14 037
137	833	1 663	2 713	3 597	4 453	5 547	6 503	7 577	8 677	4 683	10 763	11 923	13 027	14 047
147	847	1 667	2 723	3 603	4 463	5 553	6 517	7 627	8 703	9 687	10 767	11 933	13 037	14 073
153	867	1 677	2 763	3 633	4 497	5 577	6 527	7 647	8 713	9 717	10 783	11 977	13 053	14 083
163	873	1 703	2 773	3 647	4 523	5 587	6 533	7 667	8 723	9 747	10 787	11 983	13 063	14 133
167	877	1 717	2 823	3 653	4 563	5 613	6 563	7 677	8 727	9 753	10 813	11 987	13 087	14 167
197	883	1 753	2 833	3 663	4 587	5 617	6 567	7 713	8 737	9 763	10 827	11 997	13 127	14 197
223	913	1 767	2 837	3 717	4 613	5 627	6 603	7 723	8 803	9 783	10 853	12 037	13 147	14 213
227	917	1 797	2 847	3 737	4 617	5 633	6 627	7 733	8 813	9 803	10 877	12 087	13 153	14 273
247	923	1 803	2 853	3 767	4 627	5 653	6 637	7 737	8 817	9 837	10 903	12 123	13 197	14 297
263	927	1 817	2 873	3 773	4 703	5 677	6 647	7 747	8 877	9 883	10 937	12 153	13 223	14 317
267	933	1 827	2 887	3 797	4 717	5 683	6 673	7 763	8 883	9 917	10 953	12 177	13 233	14 323
277	937	1 837	2 897	3 803	4 723	5 687	6 683	7 783	8 937	9 923	10 963	12 183	13 237	14 337
283	947	1 847	2 913	3 813	4 777	5 697	6 697	7 803	8 953	9 933	11 013	12 217	13 283	14 377
287	953	1 887	2 927	3 833	4 803	5 703	6 717	7 817	8 967	9 973	11 027	12 223	13 297	14 397
297	963	1 923	2 963	3 837	4 813	5 713	6 727	7 823	9 003	9 977	11 033	12 247	13 303	14 413
303	997	1 927	2 983	3 847	4 823	5 737	6 737	7 837	9 013	9 997	11 073	12 263	13 327	14 447
323	1 047	1 933	3 017	3 867	4 833	5 753	6 747	7 903	9 037	10 003	11 083	12 317	13 337	14 467
347	1 053	1 947	3 023	3 877	4 837	5 763	6 823	7 913	9 063	10 053	11 103	12 347	13 353	14 527
363	1 063	1 973	3 027	3 883	4 863	5 787	6 853	7 917	9 073	10 073	11 117	12 403	13 367	14 533
367	1 073	1 987	3 053	3 887	4 877	5 823	6 873	7 923	9 077	10 087	11 163	12 447	13 377	14 563
373	1 103	2 003	3 067	3 913	4 887	5 833	6 883	7 933	9 083	10 097	11 183	12 467	13 397	14 567
383	1 117	2 067	3 087	3 917	4 897	5 863	6 903	7 947	9 087	10 113	11 223	12 497	13 427	14 577
397	1 137	2 073	3 103	3 937	4 903	5 887	6 927	7 953	9 097	10 123	11 277	12 503	13 433	14 583
417	1 147	2 083	3 117	3 967	4 923	5 917	6 933	7 967	9 103	10 173	11 283	12 513	13 453	14 603
427	1 163	2 113	3 123	3 977	4 963	5 947	6 953	7 997	9 113	10 203	11 287	12 523	13 487	14 613
433	1 167	2 117	3 127	3 997	4 973	5 953	6 967	8 053	9 127	10 217	11 313	12 547	13 503	14 623
453	1 173	2 133	3 133	4 003	5 003	5 973	7 003	8 067	9 137	10 253	11 317	12 573	13 523	14 653
503	1 187	2 167	3 147	4 013	5 067	5 983	7 027	8 083	9 163	10 297	11 333	12 597	13 527	14 697
513	1 197	2 173	3 153	4 023	5 073	5 987	7 097	8 087	9 177	10 313	11 347	12 647	13 533	14 703
517	1 213	2 227	3 163	4 133	5 077	6 023	7 133	8 123	9 187	10 333	11 363	12 653	13 617	14 723
527	1 233	2 237	3 187	4 153	5 103	6 047	7 137	8 153	9 213	10 337	11 387	12 663	13 623	14 727
533	1 247	2 247	3 233	4 163	5 123	6 083	7 163	8 177	9 223	10 377	11 397	12 673	13 627	14 753
537	1 273	2 273	3 247	4 167	5 133	6 097	7 183	8 217	9 253	10 433	11 453	12 713	13 683	14 767

☞ *Continued on page* 244,

Elements (y) *of Primes* $p = \frac{1}{10}\mathrm{N} = \frac{1}{10}(y^2+1)$.

(*Continued from page* 243.)

14 783	14 797	14 847	14 883	14 953	14 967	14 987
14 787	14 813	14 853	14 927	14 963	14 977	

Elements (y) *of Primes* $p = \frac{1}{13}\mathrm{N} = \frac{1}{13}(y^2+1)$.

8	694	1 516	2 764	3 856	5 416	6 820	8 390	9 914	11 136	12 446	13 840
34	720	1 604	2 790	3 866	5 650	6 934	8 484	9 940	11 146	12 524	13 866
44	736	1 630	2 800	3 934	5 686	6 960	8 546	9 950	11 214	12 654	13 954
60	814	1 646	2 816	3 960	5 764	6 986	8 624	9 976	11 354	12 680	13 996
86	840	1 734	2 904	3 996	5 884	7 090	8 650	10 044	11 370	12 696	14 100
96	866	1 750	3 930	4 090	5 894	7 236	8 744	10 096	11 474	12 706	14 204
164	876	1 776	3 050	4 100	5 910	7 376	8 884	10 106	11 484	12 810	14 230
190	954	1 890	3 154	4 116	6 024	7 444	8 900	10 184	11 500	12 836	14 240
226	980	1 916	3 164	4 204	6 040	7 454	8 926	10 226	11 656	12 930	14 360
304	1 006	1 994	3 180	4 230	6 066	7 574	8 936	10 236	11 770	12 940	14 474
320	1 074	2 046	3 190	4 490	6 144	7 584	9 004	10 330	11 864	13 034	14 490
330	1 100	2 114	3 206	4 646	6 170	7 730	9 066	10 444	11 874	13 044	14 516
346	1 110	2 254	3 216	4 766	6 326	7 766	9 144	10 590	11 890	13 304	14 594
356	1 204	2 270	3 440	4 776	6 414	7 834	9 186	10 600	11 916	13 346	14 620
424	1 240	2 400	3 544	4 854	6 466	7 860	9 300	10 616	12 046	13 424	14 656
434	1 256	2 504	3 570	4 880	6 570	7 886	9 420	10 694	12 124	13 476	14 724
486	1 266	2 530	3 580	4 896	6 586	7 896	9 430	10 730	12 254	13 606	14 734
554	1 334	2 556	3 596	5 114	6 596	7 964	9 586	10 954	12 264	13 694	14 906
564	1 386	2 644	3 606	5 166	6 664	7 974	9 680	10 964	12 306	13 710	14 984
590	1 396	2 660	3 674	5 234	6 700	8 286	9 784	11 006	12 410	13 720	14 994
671	1 490	2 686	3 814	5 400	6 794	8 354	9 820	11 110	12 420	13 814	

Elements (y) *of Primes* $p = \frac{1}{17}\mathrm{N} = \frac{1}{17}(y^2+1)$.

4	574	1 466	2 520	3 880	5 674	7 306	8 564	9 720	11 394	13 264	14 480
30	676	1 704	2 546	3 906	5 716	7 374	8 606	9 796	11 420	13 290	14 514
64	744	1 730	2 580	3 914	5 750	7 416	8 674	10 000	11 700	13 596	14 616
106	786	1 764	2 690	4 016	5 844	7 646	8 700	10 034	11 896	13 604	14 624
140	846	1 806	2 724	4 186	5 980	7 654	8 776	10 060	11 930	13 630	14 650
166	854	1 840	2 784	4 390	6 056	7 790	8 810	10 204	12 210	13 706	14 684
234	880	1 866	2 860	4 450	6 116	7 824	8 836	10 306	12 236	13 766	14 820
276	990	1 900	2 886	4 526	6 124	7 884	8 904	10 374	12 270	13 774	14 930
310	1 024	1 934	2 996	4 696	6 260	7 926	9 074	10 476	12 346	13 936	14 964
344	1 050	1 976	3 064	4 934	6 626	7 960	9 116	10 604	12 380	14 004	
370	1 160	2 180	3 124	4 960	6 634	8 266	9 184	10 680	12 406	14 046	
404	1 220	2 240	3 226	4 994	6 694	8 326	9 320	10 910	12 440	14 114	
446	1 296	2 316	3 396	5 206	6 770	8 334	9 346	10 986	12 474	14 216	
480	1 330	2 376	3 464	5 274	6 906	8 436	9 354	11 156	12 890	14 250	
506	1 356	2 444	3 566	5 300	7 144	8 470	9 414	11 326	13 196	14 284	
514	1 364	2 486	3 676	5 640	7 280	8 530	9 694	11 386	13 230	14 310	

☞ *These Tables give the Elements* (y) *of all Primes* $p = \frac{1}{\mu}(y^2+1)$, $[\mu = 10, 13, 17]$, *up to* $y = 15{,}000$.

Elements (y) *of Simple Cuban Primes* $p = \mathrm{N} = (y^3-1)\div(y-1)$.

1	168	453	773	1 067	1 365	1 665	2 033	2 373	2 673	3 024	3 407	3 720	4 068
2	173	455	782	1 070	1 373	1 676	2 043	2 385	2 682	3 027	3 417	3 732	4 074
3	176	456	785	1 074	1 380	1 685	2 049	2 387	2 684	3 030	3 419	3 737	4 082
5	188	476	792	1 077	1 386	1 692	2 054	2 390	2 688	3 039	3 423	3 741	4 089
6	189	489	798	1 091	1 392	1 695	2 058	2 397	2 696	3 041	3 437	3 744	4 103
8	192	495	801	1 092	1 401	1 700	2 064	2 402	2 703	3 048	3 438	3 746	4 107
12	194	500	812	1 097	1 415	1 701	2 073	2 409	2 708	3 050	3 443	3 762	4 110
14	203	512	818	1 098	1 421	1 707	2 075	2 411	2 714	3 053	3 444	3 771	4 119
15	206	518	819	1 107	1 422	1 709	2 079	2 429	2 721	3 069	3 449	3 785	4 136
17	209	525	825	1 116	1 427	1 716	2 085	2 430	2 723	3 093	3 458	3 794	4 140
20	215	530	827	1 118	1 434	1 718	2 091	2 435	2 729	3 099	3 459	3 801	4 142
21	218	531	836	1 125	1 440	1 746	2 105	2 442	2 730	3 101	3 464	3 828	4 151
24	231	533	839	1 130	1 442	1 748	2 112	2 450	2 735	3 107	3 470	3 834	4 157
27	236	537	846	1 133	1 443	1 757	2 117	2 451	2 736	3 114	3 473	3 836	4 158
33	245	540	855	1 142	1 448	1 767	2 129	2 456	2 742	3 128	3 479	3 839	4 175
38	246	551	857	1 146	1 449	1 770	2 136	2 463	2 771	3 135	3 482	3 842	4 178
41	266	554	860	1 148	1 454	1 788	2 138	2 465	2 787	3 137	3 485	3 843	4 184
50	272	560	864	1 149	1 473	1 800	2 142	2 471	2 789	3 143	3 491	3 846	4 193
54	278	566	875	1 163	1 475	1 806	2 145	2 478	2 793	3 144	3 492	3 855	4 196
57	279	567	878	1 181	1 484	1 809	2 157	2 483	2 814	3 153	3 501	3 872	4 199
59	287	572	890	1 182	1 487	1 811	2 159	2 484	2 828	3 170	3 510	3 879	4 203
62	288	579	894	1 191	1 494	1 814	2 162	2 493	2 840	3 174	3 515	3 881	4 217
66	290	582	897	1 193	1 505	1 818	2 163	2 495	2 841	3 195	3 534	3 884	4 224
69	293	584	899	1 196	1 515	1 826	2 168	2 498	2 849	3 197	3 540	3 885	4 235
71	309	603	911	1 200	1 520	1 832	2 171	2 511	2 862	3 200	3 542	3 888	4 236
75	314	605	915	1 202	1 526	1 841	2 183	2 513	2 864	3 204	3 543	3 891	4 238
77	329	609	918	1 209	1 529	1 847	2 201	2 523	2 868	3 212	3 549	3 899	4 245
78	332	612	920	1 217	1 536	1 848	2 204	2 535	2 870	3 213	3 554	3 905	4 257
80	336	621	927	1 221	1 539	1 853	2 205	2 540	2 871	3 221	3 557	3 914	4 259
89	342	624	950	1 230	1 547	1 863	2 238	2 541	2 873	3 234	3 561	3 918	4 268
90	344	626	959	1 254	1 548	1 886	2 241	2 555	2 883	3 242	3 563	3 920	4 275
99	348	635	960	1 256	1 559	1 895	2 247	2 556	2 885	3 251	3 573	3 927	4 284
101	351	642	969	1 260	1 560	1 902	2 255	2 558	2 891	3 254	3 576	3 939	4 287
105	357	644	974	1 263	1 562	1 923	2 259	2 561	2 894	3 255	3 587	3 954	4 290
110	369	668	981	1 272	1 566	1 931	2 262	2 562	2 898	3 270	3 594	3 965	4 304
111	378	671	987	1 274	1 568	1 932	2 264	2 568	2 906	3 281	3 603	3 975	4 308
117	381	677	990	1 275	1 581	1 952	2 273	2 574	2 922	3 288	3 606	3 986	4 311
119	383	686	992	1 281	1 583	1 956	2 276	2 589	2 925	3 296	3 608	3 989	4 313
131	392	696	993	1 295	1 590	1 970	2 280	2 597	2 936	3 297	3 611	3 995	4 326
138	395	701	1 001	1 308	1 592	1 973	2 301	2 604	2 943	3 302	3 626	3 996	4 340
141	398	720	1 002	1 314	1 596	1 974	2 303	2 607	2 948	3 314	3 629	3 998	4 350
143	402	726	1 007	1 317	1 611	1 977	2 309	2 618	2 955	3 330	3 638	4 002	4 352
147	404	728	1 011	1 323	1 616	1 982	2 313	2 625	2 957	3 338	3 639	4 017	4 359
150	405	735	1 016	1 331	1 620	1 986	2 318	2 628	2 966	3 339	3 645	4 025	4 361
153	414	743	1 020	1 340	1 632	1 994	2 324	2 639	2 969	3 345	3 654	4 028	4 367
155	416	747	1 022	1 343	1 644	2 000	2 339	2 646	2 982	3 380	3 671	4 031	4 374
161	426	755	1 029	1 349	1 646	2 010	2 345	2 649	2 996	3 381	3 687	4 047	4 380
162	434	761	1 041	1 352	1 650	2 015	2 360	2 651	2 997	3 395	3 692	4 056	4 385
164	435	762	1 058	1 359	1 658	2 016	2 364	2 658	3 011	3 398	3 711	4 061	4 389
167	447	768	1 065	1 364	1 659	2 022	2 372	2 663	3 020	3 405	3 713	4 067	4 394

Elements (y) *of Simple Cuban Primes* $p = \mathrm{N} = (y^3 - 1) \div (y - 1)$.

4 395	4 772	5 165	5 570	5 993	6 378	6 761	7 164	7 619	8 036	8 400	8 762	9 156	9 596
4 404	4 781	5 178	5 571	6 005	6 389	6 767	7 181	7 652	8 042	8 408	8 774	9 159	9 600
4 406	4 791	5 181	5 594	6 006	6 396	6 774	7 182	7 661	8 049	8 435	8 777	9 164	9 605
4 409	4 796	5 199	5 601	6 014	6 408	6 788	7 190	7 670	8 051	8 463	8 802	9 171	9 611
4 418	4 802	5 208	5 603	6 018	6 417	6 791	7 196	7 677	8 057	8 469	8 807	9 176	9 612
4 424	4 803	5 211	5 610	6 023	6 420	6 798	7 199	7 694	8 060	8 475	8 813	9 177	9 626
4 430	4 808	5 220	5 621	6 033	6 426	6 807	7 203	7 700	8 064	8 478	8 825	9 180	9 645
4 431	4 812	5 223	5 627	6 042	6 434	6 810	7 220	7 701	8 070	8 480	8 826	9 183	9 647
4 437	4 815	5 228	5 640	6 047	6 440	6 812	7 223	7 707	8 072	8 484	8 835	9 191	9 653
4 443	4 821	5 234	5 642	6 051	6 441	6 816	7 229	7 728	8 078	8 496	8 883	9 198	9 654
4 446	4 844	5 241	5 643	6 063	6 446	6 833	7 238	7 736	8 079	8 501	8 886	9 204	9 656
4 448	4 847	5 246	5 649	6 065	6 447	6 837	7 253	7 745	8 084	8 510	8 900	9 218	9 660
4 467	4 863	5 249	5 663	6 077	6 450	6 842	7 271	7 748	8 088	8 517	8 910	9 231	9 672
4 473	4 877	5 250	5 670	6 090	6 455	6 875	7 274	7 752	8 093	8 522	8 925	9 240	9 680
4 506	4 880	5 253	5 691	6 095	6 459	6 882	7 286	7 754	8 106	8 525	8 928	9 248	9 684
4 511	4 886	5 256	5 694	6 096	6 462	6 884	7 295	7 761	8 114	8 540	8 930	9 255	9 686
4 515	4 889	5 265	5 696	6 102	6 482	6 888	7 301	7 769	8 118	8 541	8 942	9 261	9 689
4 530	4 896	5 267	5 715	6 107	6 489	6 905	7 311	7 785	8 130	8 543	8 945	9 288	9 695
4 535	4 898	5 276	5 718	6 111	6 495	6 909	7 314	7 787	8 132	8 552	8 946	9 306	9 702
4 536	4 905	5 283	5 727	6 114	6 497	6 914	7 334	7 791	8 142	8 559	8 954	9 308	9 705
4 539	4 914	5 285	5 738	6 117	6 510	6 915	7 343	7 812	8 148	8 562	8 963	9 309	9 719
4 550	4 940	5 292	5 748	6 128	6 537	6 917	7 344	7 845	8 153	8 564	8 972	9 318	9 722
4 556	4 941	5 304	5 753	6 137	6 539	6 921	7 355	7 847	8 168	8 568	8 981	9 320	9 723
4 557	4 952	5 334	5 754	6 156	6 551	6 944	7 356	7 850	8 174	8 601	8 988	9 323	9 743
4 560	4 964	5 340	5 759	6 170	6 552	6 954	7 358	7 857	8 195	8 603	8 991	9 344	9 747
4 574	4 973	5 351	5 766	6 173	6 567	6 956	7 365	7 859	8 198	8 624	8 996	9 351	9 749
4 581	4 976	5 355	5 774	6 177	6 569	6 963	7 388	7 860	8 202	8 625	9 000	9 357	9 750
4 584	4 980	5 358	5 778	6 200	6 573	6 968	7 392	7 869	8 210	8 627	9 002	9 360	9 756
4 604	4 983	5 367	5 780	6 207	6 576	6 986	7 397	7 871	8 217	8 645	9 003	9 362	9 768
4 619	5 003	5 382	5 792	6 216	6 579	6 987	7 425	7 883	8 226	8 646	9 008	9 366	9 780
4 625	5 010	5 390	5 804	6 219	6 590	6 993	7 427	7 889	8 231	8 651	9 030	9 399	9 789
4 626	5 012	5 400	5 816	6 222	6 593	7 013	7 430	7 890	8 237	8 658	9 033	9 401	9 800
4 632	5 018	5 405	5 817	6 231	6 599	7 014	7 448	7 895	8 247	8 664	9 035	9 404	9 803
4 640	5 025	5 409	5 834	6 237	6 602	7 026	7 460	7 901	8 259	8 669	9 042	9 443	9 828
4 653	5 039	5 432	5 838	6 251	6 614	7 034	7 467	7 908	8 267	8 673	9 050	9 462	9 842
4 658	5 043	5 447	5 852	6 272	6 627	7 047	7 479	7 922	8 268	8 681	9 059	9 465	9 845
4 661	5 055	5 453	5 865	6 285	6 632	7 059	7 482	7 925	8 279	8 688	9 063	9 495	9 848
4 682	5 064	5 459	5 867	6 303	6 636	7 080	7 502	7 929	8 288	8 702	9 068	9 504	9 861
4 695	5 082	5 484	5 871	6 305	6 644	7 083	7 503	7 932	8 300	8 715	9 071	9 516	9 864
4 698	5 085	5 487	5 876	6 306	6 653	7 091	7 505	7 938	8 315	8 718	9 072	9 521	9 866
4 707	5 087	5 501	5 894	6 324	6 669	7 106	7 509	7 950	8 321	8 720	9 086	9 533	9 875
4 712	5 094	5 514	5 906	6 326	6 681	7 110	7 514	7 967	8 322	8 727	9 099	9 540	9 882
4 721	5 097	5 516	5 907	6 341	6 692	7 112	7 524	7 971	8 324	8 730	9 105	9 555	9 885
4 724	5 115	5 522	5 913	6 348	6 702	7 115	7 526	7 973	8 366	8 741	9 107	9 560	9 897
4 731	5 117	5 526	5 936	6 350	6 719	7 118	7 539	7 976	8 373	8 744	9 113	9 561	9 899
4 733	5 130	5 529	5 942	6 354	6 720	7 122	7 545	7 983	8 379	8 748	9 114	9 570	9 908
4 745	5 136	5 538	5 951	6 363	6 725	7 143	7 547	7 986	8 384	8 753	9 117	9 575	9 912
4 749	5 139	5 549	5 969	6 368	6 734	7 152	7 572	8 000	8 385	8 756	9 126	9 576	9 918
4 751	5 159	5 552	5 979	6 371	6 740	7 157	7 580	8 016	8 387	8 757	9 134	9 579	9 924
4 770	5 162	5 561	5 984	6 377	6 749	7 160	7 605	8 027	8 393	8 760	9 143	9 581	9 947

Elements (y) *of Simple Cuban Primes* $p = \mathrm{N} = (y^3-1)\div(y-1)$.

9 950	10 319	10 755	11 114	11 549	11 921	12 423	12 909	13 359	13 886	14 217	14 666
9 957	10 323	10 766	11 121	11 555	11 922	12 435	12 915	13 371	13 902	14 222	14 684
9 960	10 332	10 769	11 129	11 562	11 927	12 438	12 920	13 377	13 905	14 246	14 696
9 962	10 358	10 781	11 138	11 565	11 933	12 449	12 948	13 392	13 908	14 253	14 703
9 966	10 368	10 788	11 166	11 567	11 934	12 459	12 956	13 398	13 910	14 267	14 705
9 971	10 388	10 790	11 171	11 574	11 948	12 461	12 972	13 410	13 914	14 273	14 724
9 975	10 398	10 794	11 172	11 577	11 954	12 468	12 974	13 415	13 916	14 279	14 726
9 989	10 401	10 800	11 178	11 600	11 975	12 473	12 986	13 431	13 923	14 286	14 727
9 996	10 415	10 808	11 187	11 609	11 985	12 486	12 995	13 434	13 928	14 288	14 733
9 999	10 428	10 815	11 199	11 613	11 987	12 488	13 005	13 454	13 931	14 297	14 735
10 008	10 431	10 821	11 207	11 621	12 006	12 501	13 020	13 460	13 937	14 304	14 762
10 011	10 436	10 823	11 213	11 627	12 017	12 512	13 032	13 461	13 947	14 306	14 766
10 029	10 437	10 827	11 214	11 640	12 018	12 524	13 044	13 469	13 949	14 307	14 768
10 034	10 443	10 835	11 219	11 642	12 026	12 530	13 046	13 473	13 970	14 313	14 799
10 059	10 445	10 848	11 220	11 649	12 029	12 542	13 070	13 476	13 979	14 328	14 804
10 064	10 466	10 850	11 234	11 655	12 066	12 545	13 077	13 487	13 992	14 330	14 817
10 086	10 473	10 869	11 243	11 667	12 075	12 557	13 086	13 499	13 994	14 339	14 820
10 092	10 479	10 874	11 256	11 676	12 083	12 582	13 089	13 502	14 001	14 346	14 822
10 107	10 482	10 881	11 264	11 684	12 087	12 594	13 091	13 517	14 013	14 364	14 834
10 109	10 491	10 893	11 276	11 691	12 104	12 599	13 095	13 527	14 019	14 369	14 840
10 116	10 493	10 895	11 282	11 693	12 111	12 615	13 097	13 545	14 021	14 376	14 852
10 121	10 496	10 911	11 303	11 700	12 113	12 617	13 109	13 548	14 027	14 379	14 853
10 122	10 508	10 914	11 304	11 705	12 123	12 638	13 112	13 553	14 028	14 385	14 861
10 125	10 517	10 920	11 324	11 714	12 131	12 641	13 142	13 565	14 031	14 393	14 867
10 128	10 538	10 934	11 325	11 717	12 134	12 647	13 158	13 574	14 034	14 397	14 880
10 151	10 542	10 935	11 336	11 724	12 143	12 657	13 163	13 593	14 042	14 412	14 882
10 158	10 547	10 944	11 343	11 738	12 159	12 666	13 167	13 595	14 055	14 421	14 889
10 160	10 550	10 947	11 361	11 756	12 165	12 683	13 173	13 613	14 066	14 435	14 892
10 199	10 554	10 953	11 367	11 760	12 173	12 696	13 203	13 632	14 078	14 460	14 895
10 202	10 583	10 961	11 375	11 763	12 176	12 705	13 209	13 635	14 084	14 474	14 904
10 205	10 589	10 967	11 403	11 766	12 185	12 711	13 214	13 655	14 087	14 495	14 910
10 206	10 605	10 976	11 409	11 777	12 201	12 720	13 229	13 683	14 091	14 505	14 918
10 212	10 620	10 982	11 415	11 780	12 204	12 726	13 238	13 686	14 096	14 507	14 934
10 223	10 622	10 983	11 424	11 784	12 206	12 731	13 245	13 704	14 112	14 510	14 939
10 226	10 631	10 991	11 429	11 789	12 227	12 740	13 257	13 713	14 118	14 516	14 943
10 239	10 640	10 995	11 438	11 796	12 230	12 747	13 266	13 728	14 129	14 519	14 951
10 242	10 646	10 997	11 441	11 802	12 237	12 752	13 275	13 730	14 132	14 549	14 952
10 244	10 652	10 998	11 450	11 814	12 251	12 753	13 286	13 739	14 133	14 558	14 976
10 251	10 671	11 009	11 457	11 819	12 264	12 771	13 296	13 754	14 136	14 574	14 987
10 263	10 680	11 016	11 465	11 838	12 269	12 780	13 298	13 784	14 145	14 586	14 990
10 265	10 692	11 019	11 466	11 843	12 276	12 785	13 299	13 793	14 148	14 594	14 993
10 272	10 694	11 021	11 472	11 849	12 293	12 803	13 310	13 812	14 154	14 598	14 997
10 289	10 697	11 024	11 499	11 864	12 311	12 824	13 314	13 814	14 169	14 600	
10 298	10 706	11 028	11 511	11 868	12 339	12 825	13 317	13 818	14 175	14 601	
10 302	10 716	11 039	11 520	11 870	12 344	12 848	13 319	13 821	14 178	14 607	
10 304	10 718	11 060	11 522	11 879	12 360	12 852	13 322	13 842	14 187	14 616	
10 307	10 730	11 075	11 523	11 880	12 375	12 878	13 329	13 844	14 189	14 631	
10 311	10 737	11 094	11 541	11 892	12 381	12 887	13 331	13 845	14 190	14 642	
10 314	10 748	11 102	11 543	11 900	12 393	12 888	13 338	13 856	14 208	14 649	
10 316	10 752	11 112	11 546	11 910	12 402	12 897	13 356	13 866	14 213	14 652	

☞ *This Table gives the Elements* (y) *of all Simple Cuban Primes* $p = \dfrac{y^3-1}{y-1} \ngtr 225.10^6$.

Elements (y) *of Simple Trito-Cuban Primes,* $p = \frac{1}{3}N = \frac{1}{3}(y^3-1) \div (y-1)$.

1	409	922	1 522	2 176	2 821	3 592	4 243	4 843	5 668	6 397	7 264
4	421	925	1 534	2 191	2 833	3 613	4 249	4 870	5 680	6 445	7 267
7	427	943	1 540	2 194	2 866	3 619	4 255	4 924	5 689	6 481	7 294
10	442	946	1 561	2 197	2 884	3 664	4 261	4 942	5 701	6 502	7 327
13	460	964	1 573	2 203	2 890	3 667	4 264	4 957	5 746	6 511	7 330
19	469	994	1 585	2 227	2 905	3 676	4 291	4 999	5 767	6 520	7 339
28	472	1 000	1 594	2 233	2 920	3 685	4 294	5 017	5 773	6 541	7 342
31	475	1 006	1 609	2 236	2 929	3 703	4 297	5 020	5 797	6 544	7 357
34	493	1 021	1 624	2 260	2 932	3 706	4 333	5 026	5 803	6 565	7 378
40	496	1 027	1 630	2 275	2 953	3 709	4 348	5 038	5 818	6 583	7 390
43	511	1 039	1 639	2 290	2 962	3 718	4 357	5 113	5 830	6 592	7 405
52	514	1 051	1 651	2 296	2 974	3 724	4 360	5 134	5 836	6 622	7 411
70	517	1 057	1 657	2 353	2 983	3 748	4 369	5 143	5 839	6 643	7 435
73	526	1 060	1 666	2 359	3 022	3 778	4 375	5 167	5 845	6 655	7 441
76	538	1 063	1 669	2 380	3 034	3 802	4 417	5 173	5 851	6 664	7 477
82	553	1 072	1 672	2 422	3 073	3 808	4 432	5 197	5 860	6 667	7 483
85	556	1 078	1 678	2 437	3 079	3 814	4 441	5 215	5 902	6 679	7 498
91	559	1 093	1 690	2 446	3 088	3 823	4 471	5 218	5 923	6 697	7 519
97	574	1 105	1 708	2 455	3 106	3 832	4 474	5 230	5 932	6 700	7 525
103	580	1 126	1 711	2 464	3 121	3 862	4 486	5 272	5 941	6 727	7 540
112	586	1 144	1 720	2 470	3 151	3 886	4 492	5 278	5 956	6 739	7 546
115	589	1 165	1 723	2 476	3 160	3 904	4 504	5 284	5 962	6 748	7 552
124	616	1 174	1 729	2 491	3 169	3 907	4 513	5 302	5 971	6 811	7 570
127	622	1 195	1 750	2 497	3 202	3 913	4 516	5 305	5 995	6 826	7 579
136	628	1 198	1 753	2 509	3 205	3 928	4 522	5 311	5 998	6 865	7 591
145	637	1 216	1 762	2 527	3 211	3 967	4 537	5 323	6 016	6 898	7 594
148	649	1 252	1 795	2 533	3 226	3 976	4 543	5 341	6 046	6 901	7 603
157	658	1 258	1 804	2 539	3 232	3 991	4 549	5 344	6 082	6 910	7 612
166	661	1 288	1 825	2 560	3 235	4 009	4 570	5 347	6 103	6 931	7 633
175	673	1 312	1 828	2 569	3 247	4 018	4 597	5 356	6 109	6 937	7 642
187	682	1 324	1 837	2 581	3 277	4 021	4 600	5 362	6 118	6 979	7 663
190	700	1 330	1 861	2 584	3 283	4 042	4 606	5 377	6 121	6 991	7 684
199	712	1 333	1 879	2 605	3 286	4 045	4 612	5 383	6 124	6 994	7 720
202	715	1 336	1 882	2 611	3 310	4 051	4 633	5 416	6 130	7 024	7 735
223	727	1 345	1 909	2 623	3 328	4 060	4 639	5 425	6 154	7 048	7 747
241	736	1 351	1 939	2 632	3 346	4 075	4 642	5 446	6 160	7 063	7 750
244	754	1 354	1 960	2 644	3 382	4 087	4 654	5 458	6 166	7 066	7 756
259	775	1 363	1 963	2 665	3 391	4 105	4 690	5 470	6 187	7 084	7 768
265	778	1 372	1 993	2 677	3 400	4 108	4 711	5 491	6 214	7 087	7 798
271	799	1 396	1 996	2 695	3 457	4 126	4 726	5 509	6 244	7 090	7 810
274	832	1 405	2 029	2 698	3 472	4 129	4 747	5 524	6 247	7 099	7 813
280	838	1 414	2 056	2 710	3 478	4 138	4 756	5 530	6 259	7 105	7 825
286	850	1 438	2 065	2 716	3 496	4 144	4 759	5 533	6 310	7 108	7 834
316	859	1 441	2 068	2 719	3 514	4 147	4 765	5 545	6 328	7 138	7 840
325	868	1 447	2 080	2 761	3 520	4 180	4 777	5 551	6 331	7 171	7 843
358	883	1 456	2 086	2 773	3 547	4 186	4 789	5 557	6 352	7 201	7 846
370	889	1 483	2 107	2 794	3 550	4 192	4 801	5 563	6 355	7 210	7 885
376	895	1 492	2 110	2 803	3 556	4 201	4 816	5 572	6 361	7 216	7 888
385	910	1 510	2 140	2 806	3 577	4 213	4 828	5 587	6 364	7 222	7 906
388	916	1 519	2 170	2 812	3 589	4 240	4 840	5 641	6 382	7 243	7 924

Elements (y) *of Simple Trito-Cuban Primes* (*continued*).

7 927	8 632	9 289	10 093	11 047	11 740	12 568	13 348	14 035	14 824
7 936	8 650	9 292	10 099	11 056	11 758	12 571	13 363	14 041	14 833
7 951	8 653	9 295	10 102	11 074	11 812	12 577	13 375	14 047	14 857
7 957	8 659	9 322	10 129	11 095	11 815	12 586	13 378	14 050	14 866
7 960	8 662	9 331	10 165	11 128	11 842	12 601	13 390	14 059	14 899
7 966	8 683	9 358	10 198	11 140	11 851	12 607	13 396	14 089	14 908
7 993	8 686	9 379	10 216	11 152	11 854	12 610	13 408	14 092	14 917
8 008	8 701	9 388	10 219	11 158	11 893	12 628	13 411	14 098	14 923
8 053	8 704	9 400	10 225	11 182	11 899	12 631	13 426	14 113	14 932
8 062	8 707	9 406	10 276	11 191	11 941	12 649	13 468	14 125	14 962
8 071	8 734	9 430	10 282	11 203	11 983	12 685	13 522	14 155	14 965
8 074	8 737	9 436	10 321	11 212	12 010	12 706	13 534	14 176	
8 083	8 746	9 439	10 330	11 224	12 025	12 727	13 543	14 182	
8 107	8 764	9 442	10 345	11 245	12 040	12 733	13 585	14 209	
8 119	8 776	9 451	10 354	11 266	12 043	12 739	13 597	14 227	
8 137	8 788	9 472	10 360	11 269	12 052	12 748	13 606	14 236	
8 140	8 800	9 490	10 363	11 284	12 055	12 781	13 609	14 248	
8 146	8 812	9 520	10 375	11 296	12 061	12 787	13 636	14 278	
8 155	8 833	9 547	10 384	11 299	12 082	12 829	13 639	14 281	
8 161	8 854	9 574	10 393	11 308	12 085	12 838	13 648	14 344	
8 170	8 872	9 598	10 471	11 317	12 103	12 844	13 651	14 365	
8 179	8 893	9 607	10 492	11 338	12 136	12 871	13 669	14 383	
8 191	8 896	9 619	10 501	11 374	12 169	12 880	13 678	14 386	
8 197	8 911	9 670	10 510	11 383	12 181	12 895	13 681	14 395	
8 230	8 923	9 673	10 549	11 389	12 187	12 913	13 684	14 416	
8 242	8 926	9 736	10 573	11 395	12 193	12 955	13 693	14 437	
8 260	8 935	9 745	10 633	11 416	12 202	12 967	13 699	14 449	
8 263	8 965	9 763	10 654	11 458	12 232	12 991	13 702	14 461	
8 266	8 974	9 775	10 675	11 467	12 235	13 000	13 714	14 479	
8 275	9 010	9 793	10 678	11 476	12 244	13 012	13 726	14 503	
8 293	9 016	9 808	10 681	11 497	12 256	13 021	13 747	14 539	
8 317	9 022	9 814	10 717	11 500	12 265	13 051	13 756	14 554	
8 326	9 028	9 817	10 738	11 518	12 277	13 054	13 762	14 566	
8 356	9 049	9 820	10 744	11 536	12 316	13 063	13 768	14 572	
8 380	9 085	9 859	10 750	11 548	12 355	13 069	13 777	14 581	
8 392	9 094	9 877	10 774	11 551	12 367	13 084	13 795	14 587	
8 398	9 115	9 880	10 792	11 593	12 382	13 093	13 819	14 593	
8 410	9 145	9 892	10 804	11 602	12 400	13 123	13 840	14 617	
8 422	9 154	9 913	10 807	11 611	12 403	13 153	13 846	14 623	
8 473	9 157	9 940	10 870	11 614	12 409	13 177	13 879	14 629	
8 491	9 199	9 943	10 900	11 623	12 439	13 186	13 891	14 659	
8 494	9 205	9 946	10 909	11 626	12 445	13 195	13 909	14 665	
8 527	9 211	9 973	10 918	11 641	12 466	13 210	13 912	14 692	
8 548	9 217	9 985	10 927	11 656	12 481	13 222	13 930	14 701	
8 554	9 229	10 027	10 969	11 674	12 484	13 249	13 963	14 722	
8 566	9 247	10 036	10 972	11 704	12 493	13 258	13 999	14 734	
8 569	9 253	10 048	10 984	11 710	12 517	13 279	14 005	14 740	
8 590	9 268	10 057	11 026	11 719	12 523	13 291	14 008	14 782	
8 611	9 271	10 066	11 032	11 728	12 526	13 300	14 020	14 806	
8 620	9 280	10 072	11 038	11 737	12 538	13 312	14 026	14 815	

☞ *This Table gives the Elements of all Primes* $p = \frac{1}{3}(y^3-1)\div(y-1) \not> 75.10^6$.

Elements (y) *of Primes* $p = \frac{1}{7}\mathrm{N} = \frac{1}{7}(y^3-1) \div (y-1)$.

2	585	1 311	2 048	3 026	4 062	5 063	6 029	6 941	7 940	8 969	10 194	11 232
4	597	1 313	2 067	3 035	4 064	5 070	6 036	6 953	7 947	8 976	10 203	11 244
9	599	1 320	2 069	3 054	4 071	5 135	6 038	6 960	7 982	9 048	10 236	11 246
11	611	1 325	2 090	3 075	4 076	5 142	6 066	6 962	8 033	9 053	10 257	11 253
23	641	1 332	2 111	3 096	4 118	5 147	6 071	6 981	8 040	9 062	10 334	11 265
32	660	1 334	2 130	3 126	4 125	5 154	6 092	6 983	8 061	9 081	10 341	11 279
39	662	1 346	2 132	3 131	4 139	5 189	6 120	6 995	8 075	9 104	10 371	11 286
44	669	1 376	2 160	3 140	4 155	5 219	6 141	7 053	8 087	9 123	10 376	11 321
51	683	1 388	2 165	3 159	4 160	5 231	6 150	7 065	8 138	9 125	10 392	11 328
53	690	1 397	2 214	3 173	4 181	5 238	6 162	7 095	8 145	9 209	10 397	11 330
60	702	1 404	2 244	3 182	4 209	5 240	6 164	7 109	8 229	9 228	10 425	11 372
65	723	1 409	2 256	3 273	4 223	5 303	6 176	7 128	8 283	9 249	10 434	11 400
72	725	1 418	2 277	3 287	4 239	5 315	6 183	7 158	8 285	9 284	10 446	11 414
86	732	1 437	2 286	3 299	4 274	5 336	6 206	7 200	8 304	9 293	10 469	11 442
93	746	1 458	2 300	3 308	4 316	5 343	6 218	7 226	8 327	9 314	10 488	11 463
95	758	1 460	2 391	3 327	4 370	5 345	6 227	7 242	8 334	9 333	10 502	11 498
114	788	1 467	2 396	3 329	4 386	5 364	6 234	7 247	8 339	9 354	10 523	11 519
123	795	1 479	2 403	3 348	4 398	5 366	6 246	7 254	8 369	9 368	10 544	11 538
156	807	1 481	2 433	3 369	4 412	5 373	6 248	7 268	8 376	9 384	10 551	11 580
170	816	1 502	2 445	3 371	4 421	5 387	6 255	7 275	8 402	9 396	10 574	11 622
179	830	1 509	2 475	3 413	4 454	5 436	6 311	7 305	8 411	9 440	10 595	11 652
186	849	1 514	2 501	3 434	4 463	5 462	6 332	7 338	8 418	9 452	10 616	11 678
200	872	1 523	2 550	3 453	4 482	5 499	6 365	7 424	8 430	9 489	10 677	11 694
207	879	1 535	2 571	3 483	4 512	5 511	6 381	7 443	8 432	9 510	10 679	11 706
212	891	1 577	2 580	3 516	4 545	5 525	6 407	7 464	8 439	9 515	10 691	11 720
219	905	1 593	2 585	3 530	4 554	5 532	6 416	7 485	8 444	9 531	10 698	11 727
228	921	1 605	2 592	3 537	4 580	5 637	6 423	7 487	8 451	9 566	10 733	11 736
233	933	1 619	2 594	3 567	4 601	5 660	6 458	7 520	8 486	9 594	10 763	11 757
240	956	1 649	2 606	3 593	4 617	5 693	6 465	7 578	8 514	9 599	10 791	11 771
249	963	1 656	2 657	3 614	4 638	5 735	6 507	7 590	8 558	9 650	10 796	11 799
261	968	1 661	2 678	3 635	4 671	5 744	6 519	7 599	8 577	9 657	10 805	11 811
270	977	1 703	2 706	3 677	4 694	5 765	6 521	7 641	8 579	9 669	10 817	11 832
303	1 019	1 724	2 732	3 684	4 743	5 772	6 549	7 646	8 586	9 699	10 826	11 853
317	1 061	1 731	2 760	3 686	4 755	5 777	6 554	7 655	8 591	9 713	10 866	11 862
333	1 080	1 754	2 769	3 719	4 764	5 786	6 570	7 676	8 598	9 755	10 880	11 916
338	1 115	1 766	2 783	3 749	4 769	5 793	6 582	7 688	8 619	9 774	10 901	11 946
345	1 122	1 775	2 790	3 770	4 778	5 805	6 591	7 695	8 670	9 795	10 931	11 951
375	1 131	1 787	2 802	3 810	4 790	5 828	6 617	7 704	8 682	9 809	10 943	11 967
389	1 143	1 796	2 816	3 824	4 811	5 868	6 647	7 709	8 696	9 881	10 985	11 972
401	1 164	1 808	2 825	3 861	4 827	5 870	6 654	7 716	8 712	9 888	11 013	11 979
443	1 178	1 815	2 858	3 866	4 860	5 891	6 666	7 739	8 717	9 893	11 048	12 009
452	1 187	1 817	2 865	3 875	4 862	5 912	6 687	7 767	8 733	9 900	11 076	12 056
473	1 215	1 857	2 900	3 882	4 883	5 919	6 708	7 779	8 738	9 923	11 078	12 098
480	1 229	1 866	2 942	3 945	4 916	5 945	6 722	7 793	8 754	10 028	11 097	12 105
492	1 236	1 887	2 951	3 957	4 925	5 966	6 759	7 823	8 759	10 061	11 106	12 140
515	1 241	1 908	2 963	3 971	4 944	5 975	6 773	7 851	8 780	10 103	11 132	12 149
534	1 248	1 922	2 991	3 980	4 946	5 982	6 806	7 863	8 787	10 133	11 148	12 161
548	1 269	1 955	3 005	4 029	4 958	6 003	6 848	7 865	8 852	10 145	11 204	12 168
564	1 271	1 997	3 014	4 041	5 009	6 017	6 864	7 886	8 913	10 152	11 211	12 191
578	1 278	2 004	3 021	4 050	5 030	6 024	6 869	7 914	8 955	10 166	11 225	12 240

Elements (y) *of Primes* $p = \frac{1}{7}N = \frac{1}{7}(y^3-1) \div (y-1)$.

12 254	12 485	12 744	13 017	13 206	13 500	13 778	13 941	14 291	14 555	14 814
12 282	12 497	12 833	13 031	13 274	13 512	13 785	13 974	14 298	14 597	14 835
12 296	12 504	12 882	13 038	13 281	13 533	13 794	14 018	14 310	14 627	14 856
12 324	12 539	12 891	13 073	13 290	13 547	13 806	14 058	14 319	14 655	14 858
12 336	12 555	12 903	13 085	13 337	13 556	13 808	14 072	14 408	14 676	14 912
12 371	12 602	12 924	13 092	13 344	13 610	13 829	14 156	14 415	14 697	14 919
12 387	12 630	12 933	13 115	13 353	13 661	13 850	14 184	14 438	14 711	14 928
12 408	12 644	12 959	13 134	13 400	13 682	13 883	14 193	14 492	14 744	14 942
12 422	12 681	12 975	13 157	13 430	13 694	13 892	14 207	14 499	14 786	14 949
12 443	12 702	12 980	13 176	13 449	13 703	13 904	14 247	14 529	14 795	14 984
12 462	12 714	13 010	13 190	13 472	13 743	13 920	14 256	14 541	14 807	

Elements (y) *of Primes* $p = \frac{1}{13}N = \frac{1}{13}(y^3-1) \div (y-1)$.

3	705	1 608	2 954	4 488	5 697	7 049	8 349	9 785	11 157	12 398	13 776
9	731	1 634	3 045	4 494	5 703	7 055	8 394	9 870	11 235	12 411	13 802
29	770	1 706	3 090	4 553	5 762	7 068	8 420	9 915	11 261	12 470	13 887
35	776	1 725	3 129	4 676	5 820	7 127	8 433	9 980	11 280	12 554	13 958
42	783	1 784	3 149	4 689	5 846	7 146	8 505	10 071	11 319	12 587	14 108
48	789	1 790	3 162	4 728	5 885	7 224	8 531	10 097	11 339	12 626	14 225
113	815	1 862	3 227	4 787	5 918	7 302	8 583	10 130	11 378	12 743	14 231
120	848	1 875	3 240	4 800	5 963	7 328	8 589	10 136	11 423	12 801	14 244
126	854	1 914	3 272	4 884	6 035	7 374	8 609	10 188	11 436	12 827	14 283
152	887	1 940	3 324	4 917	6 054	7 406	8 615	10 247	11 462	12 860	14 309
185	906	1 959	3 363	4 949	6 249	7 419	8 687	10 338	11 501	12 873	14 381
204	932	2 037	3 396	4 956	6 308	7 491	8 693	10 344	11 534	12 879	14 426
224	945	2 115	3 428	4 962	6 321	7 679	8 771	10 403	11 612	12 944	14 517
237	965	2 141	3 480	5 001	6 392	7 712	8 849	10 416	11 618	12 951	14 537
243	978	2 187	3 500	5 034	6 405	7 731	8 856	10 487	11 670	12 977	14 582
276	1 004	2 213	3 506	5 040	6 425	7 757	8 882	10 526	11 690	13 055	14 589
302	1 043	2 252	3 552	5 073	6 431	7 764	8 966	10 611	11 709	13 074	14 615
308	1 088	2 369	3 584	5 144	6 438	7 770	8 973	10 617	11 735	13 133	14 621
321	1 095	2 408	3 597	5 229	6 483	7 803	9 012	10 643	11 742	13 146	14 628
341	1 127	2 421	3 786	5 235	6 516	7 829	9 135	10 650	11 781	13 172	14 693
386	1 140	2 427	3 851	5 300	6 548	7 881	9 161	10 676	11 859	13 191	14 706
399	1 212	2 525	3 870	5 333	6 587	7 946	9 168	10 695	11 865	13 217	14 738
419	1 238	2 577	3 981	5 339	6 594	7 952	9 194	10 734	11 885	13 263	14 771
432	1 277	2 603	4 007	5 372	6 620	8 030	9 233	10 760	11 982	13 289	14 829
477	1 316	2 687	4 046	5 417	6 665	8 037	9 252	10 877	12 008	13 334	14 972
503	1 335	2 700	4 098	5 489	6 672	8 043	9 311	10 890	12 054	13 347	
510	1 361	2 759	4 208	5 528	6 678	8 102	9 330	10 923	12 060	13 464	
516	1 368	2 772	4 215	5 547	6 776	8 141	9 369	10 988	12 080	13 523	
542	1 407	2 778	4 254	5 573	6 789	8 154	9 428	11 033	12 138	13 529	
549	1 433	2 798	4 299	5 580	6 795	8 225	9 434	11 040	12 158	13 536	
588	1 452	2 817	4 319	5 606	6 860	8 232	9 506	11 066	12 197	13 562	
633	1 478	2 856	4 338	5 645	6 867	8 316	9 623	11 079	12 249	13 607	
659	1 595	2 889	4 397	5 684	6 893	8 336	9 642	11 105	12 320	13 685	
666	1 602	2 934	4 403	5 690	7 029	8 342	9 714	11 144	12 333	13 692	

☞ *These Tables give the Elements* (y) *of all Primes* $p = \frac{1}{\mu}(y^3-1) \div (y-1)$, $[\mu = 7, 13]$, *up to* $y \not> 15{,}000$.

Elements (y) *of Primes* $p = \frac{1}{19}\mathrm{N} = \frac{1}{19}(y^3-1)\div(y-1)$.

7	524	1 835	2 705	4 058	4 856	5 958	7 307	8 139	9 416	10 632	11 882	13 140	14 432
11	539	1 869	2 747	4 115	4 913	6 011	7 379	8 177	9 435	10 647	11 886	13 193	14 447
26	558	1 911	2 766	4 149	4 970	6 053	7 440	8 196	9 450	10 685	12 057	13 212	14 546
45	600	1 949	2 861	4 229	4 989	6 144	7 493	8 405	9 530	10 704	12 129	13 292	14 622
83	752	1 968	2 990	4 248	5 061	6 182	7 535	8 424	9 602	10 818	12 167	13 349	14 675
125	771	2 021	2 994	4 301	5 141	6 186	7 554	8 447	9 621	10 841	12 209	13 364	14 694
140	843	2 078	3 066	4 305	5 270	6 224	7 626	8 504	9 663	10 913	12 228	13 368	14 808
182	881	2 120	3 108	4 343	5 369	6 410	7 664	8 618	9 716	10 970	12 570	13 520	14 846
197	885	2 192	3 237	4 415	5 426	6 524	7 721	8 652	9 777	11 145	12 585	13 691	14 922
201	923	2 211	3 275	4 434	5 445	6 543	7 740	8 846	9 887	11 217	12 722	13 706	
216	980	2 253	3 408	4 472	5 460	6 642	7 763	8 865	9 891	11 297	12 794	13 725	
239	999	2 420	3 450	4 529	5 498	6 695	7 782	8 975	10 043	11 312	12 836	13 748	
258	1 071	2 439	3 465	4 571	5 559	6 699	7 839	8 994	10 062	11 331	12 851	13 767	
311	1 113	2 477	3 503	4 586	5 631	6 809	7 896	9 089	10 115	11 373	12 908	13 782	
330	1 170	2 481	3 507	4 628	5 654	6 866	7 934	9 203	10 191	11 411	12 965	13 881	
353	1 223	2 519	3 564	4 662	5 726	6 942	7 953	9 222	10 233	11 502	12 984	13 896	
444	1 337	2 553	3 659	4 700	5 783	7 098	8 006	9 264	10 248	11 597	13 007	14 052	
467	1 356	2 576	3 750	4 704	5 787	7 208	8 025	9 302	10 290	11 654	13 026	14 276	
482	1 470	2 610	3 921	4 742	5 825	7 227	8 048	9 374	10 457	11 711	13 079	14 295	
486	1 664	2 633	3 963	4 833	5 939	7 269	8 105	9 393	10 613	11 730	13 083	14 318	

Elements (y) *of Primes* $p = \frac{1}{21}\mathrm{N} = \frac{1}{21}(y^3-1)\div(y-1)$.

4	550	1 306	2 368	3 397	4 300	5 455	6 598	7 954	9 076	10 231	11 470	12 595	14 002
16	604	1 411	2 377	3 406	4 309	5 485	6 640	7 963	9 130	10 264	11 491	12 679	14 053
25	613	1 423	2 398	3 418	4 363	5 548	6 799	7 984	9 151	10 285	11 512	12 688	14 065
37	688	1 495	2 482	3 502	4 393	5 569	6 820	8 059	9 160	10 294	11 524	12 763	14 137
46	697	1 507	2 545	3 511	4 456	5 623	6 883	8 122	9 172	10 315	11 575	12 784	14 158
58	709	1 528	2 587	3 523	4 498	5 644	6 934	8 131	9 214	10 336	11 587	12 793	14 305
88	730	1 558	2 620	3 544	4 519	5 674	7 009	8 185	9 235	10 411	11 608	12 835	14 317
109	739	1 579	2 641	3 574	4 603	5 728	7 051	8 194	9 256	10 432	11 713	12 898	14 422
130	751	1 612	2 755	3 616	4 708	5 737	7 156	8 269	9 361	10 453	11 764	12 910	14 494
142	760	1 642	2 767	3 628	4 720	5 875	7 207	8 320	9 370	10 558	11 797	12 919	14 515
151	793	1 705	2 797	3 637	4 792	5 896	7 240	8 332	9 391	10 579	11 932	12 961	14 548
184	844	1 747	2 818	3 670	4 834	5 938	7 312	8 341	9 433	10 621	11 953	12 982	14 557
193	865	1 768	2 839	3 679	4 876	5 968	7 375	8 353	9 517	10 714	12 007	13 129	14 578
205	886	1 780	2 881	3 700	4 888	6 001	7 408	8 416	9 559	10 735	12 037	13 171	14 590
247	907	1 801	2 893	3 733	4 939	6 010	7 438	8 467	9 580	10 789	12 049	13 192	14 674
268	970	1 852	2 923	3 796	4 993	6 031	7 450	8 509	9 613	10 831	12 058	13 318	14 683
298	1 012	1 864	2 935	3 817	5 002	6 043	7 480	8 551	9 643	10 840	12 070	13 435	14 695
310	1 045	1 915	2 965	3 859	5 035	6 085	7 492	8 572	9 676	10 882	12 079	13 486	14 746
319	1 054	1 948	2 998	3 910	5 044	6 136	7 501	8 584	9 706	10 924	12 154	13 498	14 851
331	1 066	1 957	3 040	3 943	5 056	6 169	7 513	8 626	9 718	10 987	12 217	13 519	14 935
340	1 117	1 969	3 049	3 952	5 098	6 190	7 522	8 677	9 748	11 071	12 226	13 570	14 968
382	1 138	2 032	3 061	3 964	5 107	6 199	7 534	8 689	9 907	11 113	12 238	13 675	14 977
394	1 150	2 041	3 112	4 099	5 119	6 211	7 585	8 710	9 958	11 125	12 289	13 717	14 989
403	1 171	2 062	3 217	4 132	5 149	6 304	7 618	8 731	9 970	11 155	12 352	13 759	
415	1 180	2 074	3 238	4 141	5 161	6 325	7 681	8 761	9 991	11 230	12 364	13 771	
424	1 201	2 104	3 250	4 162	5 245	6 346	7 732	8 773	10 033	11 260	12 373	13 855	
457	1 213	2 179	3 271	4 174	5 296	6 430	7 753	8 794	10 054	11 281	12 469	13 918	
478	1 222	2 221	3 334	4 204	5 380	6·493	7 774	8 803	10 105	11 344	12 478	13 948	
487	1 285	2 251	3 343	4 237	5 392	6 514	7 807	8 857	10 147	11 356	12 511	13 969	
541	1 297	2 263	3 385	4 288	5 413	6 589	7 942	8 929	10 180	11 377	12 520	13 990	

☞ *These Tables give the Elements* (y) *of all Primes* $p = \frac{1}{\mu}(y^3-1)\div(y-1)$, [$\mu = 19, 21$], *up to* $y \ngtr 15{,}000$.

Quartan Primes, $p = x^4 + y^4$ [x odd, y even].

p	x, y	p	x, y	p	x, y	p	x, y
17	1, 2	280 097	23, 4	1 338 737	7, 34	3 649 777	39, 34
97	3, 2	283 937	23, 8	1 342 897	9, 34	3 653 057	43, 22
257	1, 4	284 881	15, 22	1 345 921	33, 20	3 750 577	43, 24
337	3, 4	289 841	23, 10	1 350 977	11, 34	3 818 977	29, 42
641	5, 2	317 777	17, 22	1 364 897	13, 34	3 874 337	41, 32
881	5, 4	331 777	1, 24	1 466 657	19, 34	3 942 577	21, 44
1 297	1, 6	334 177	7, 24	1 501 921	35, 6	3 959 297	37, 38
2 417	7, 2	346 417	11, 24	1 521 361	35, 12	4 035 217	31, 42
2 657	7, 4	360 337	13, 24	1 682 017	7, 36	4 100 641	45, 2
3 697	7, 6	384 817	23, 18	1 763 137	17, 36	4 100 881	45, 4
4 177	3, 8	391 921	25, 6	1 800 577	33, 28	4 104 721	45, 8
4 721	5, 8	394 721	25, 8	1 809 937	19, 36	4 162 097	41, 34
6 577	9, 2	411 361	25, 12	1 874 177	37, 2	4 279 537	27, 44
10 657	9, 8	457 057	3, 26	1 874 417	37, 4	4 398 577	39, 38
12 401	7, 10	459 377	7, 26	1 878 257	37, 8	4 467 377	43, 32
14 657	11, 2	462 097	19, 24	1 912 577	37, 14	4 477 457	1, 46
14 897	11, 4	463 537	9, 26	1 959 457	23, 36	4 477 537	3, 46
15 937	11, 6	471 617	11, 26	1 972 097	31, 32	4 478 081	5, 46
16 561	9, 10	531 457	27, 2	2 034 161	37, 20	4 505 377	41, 36
28 817	13, 4	587 297	19, 26	2 043 617	29, 34	4 506 017	13, 46
38 561	13, 10	596 977	27, 16	2 070 241	25, 36	4 560 977	17, 46
39 041	5, 14	614 657	1, 28	2 085 217	3, 38	4 607 777	19, 46
49 297	13, 12	621 217	9, 28	2 168 657	17, 38	4 671 937	21, 46
54 721	15, 8	643 217	13, 28	2 279 617	21, 38	4 715 281	45, 28
65 537	1, 16	728 017	29, 12	2 351 857	39, 14	4 755 137	43, 34
65 617	3, 16	736 817	23, 26	2 378 977	39, 16	4 879 937	47, 4
66 161	5, 16	744 977	19, 28	2 473 441	39, 20	4 880 977	47, 6
66 977	13, 14	745 697	29, 14	2 522 257	33, 34	4 910 897	41, 38
80 177	11, 16	812 257	29, 18	2 566 561	9, 40	4 918 097	47, 14
83 537	17, 2	812 401	7, 30	2 616 577	27, 38	5 039 681	47, 20
83 777	17, 4	824 641	11, 30	2 684 161	37, 30	5 211 457	47, 24
89 041	15, 14	838 561	13, 30	2 690 321	19, 40	5 308 417	1, 48
105 601	5, 18	847 601	25, 26	2 754 481	21, 40	5 309 041	5, 48
107 377	7, 18	867 281	29, 20	2 825 777	41, 2	5 385 761	41, 40
119 617	11, 18	893 521	17, 30	2 836 961	35, 34	5 391 937	17, 48
121 937	17, 14	941 537	29, 22	2 839 841	23, 40	5 436 961	45, 34
130 337	19, 2	944 257	31, 12	2 922 737	37, 32	5 663 377	33, 46
131 617	19, 6	961 937	31, 14	2 930 737	41, 18	5 764 817	49, 2
134 417	19, 8	988 417	27, 26	3 112 321	5, 42	5 768 897	49, 8
140 321	19, 10	1 049 201	5, 32	3 157 537	41, 24	5 785 537	49, 12
149 057	17, 16	1 050 977	7, 32	3 195 217	17, 42	5 978 801	43, 40
151 057	19, 12	1 055 137	9, 32	3 242 017	19, 42	6 015 697	29, 48
160 001	1, 20	1 089 841	23, 30	3 362 017	39, 32	6 185 761	45, 38
160 081	3, 20	1 146 097	27, 28	3 391 537	23, 42	6 252 401	7, 50
166 561	9, 20	1 178 897	19, 32	3 428 801	43, 10	6 278 561	13, 50
168 737	19, 14	1 224 337	33, 14	3 439 537	43, 12	6 333 521	17, 50
204 481	21, 10	1 328 417	23, 32	3 457 217	43, 14	6 444 481	21, 50
243 521	17, 20	1 336 337	1, 34	3 553 777	37, 36	6 765 217	51, 2
260 017	21, 16	1 336 417	3, 34	3 578 801	43, 20	6 769 297	51, 8
279 857	23, 2	1 336 961	5, 34	3 635 761	41, 30	6 775 201	51, 10

☞ *Continued on page* 255.

Half-Quartan Primes, $p = \frac{1}{2}(x^4 + y^4)$, [*x and y odd*].

v	x, y	p	x, y	p	x, y	p	x, y
1	1, 1	353 681	29, 3	2 057 633	45, 11	4 715 233	55, 23
41	3, 1	378 953	29, 15	2 092 073	45, 17	4 795 481	51, 41
313	5, 1	405 641	27, 23	2 093 801	39, 37	4 928 953	55, 29
353	5, 3	450 881	29, 21	2 163 193	41, 35	4 932 713	49, 45
1 201	7, 1	461 801	31, 3	2 171 161	43, 31	5 101 961	53, 39
3 593	9, 5	462 073	31, 5	2 190 233	45, 23	5 278 001	57, 1
4 481	9, 7	465 041	31, 9	2 439 881	47, 3	5 319 761	57, 17
7 321	11, 1	476 041	31, 13	2 440 153	47, 5	5 473 313	57, 25
8 521	11, 7	487 073	31, 15	2 441 041	47, 7	5 654 641	53, 43
10 601	11, 9	548 953	29, 25	2 447 161	47, 11	5 822 441	51, 47
14 281	13, 1	559 001	31, 21	2 454 121	47, 13	5 988 193	55, 41
14 321	13, 3	593 273	33, 5	2 481 601	47, 17	6 028 313	57, 35
14 593	13, 5	594 161	33, 7	2 537 081	47, 21	6 058 993	59, 5
21 601	13, 11	750 313	35, 1	2 705 561	47, 27	6 083 993	59, 15
26 513	15, 7	750 353	35, 3	2 793 481	47, 29	6 123 841	59, 19
32 633	15, 11	757 633	35, 11	2 866 121	43, 39	6 198 601	59, 23
41 761	17, 1	764 593	35, 13	2 882 441	49, 3	6 253 993	59, 25
41 801	17, 3	792 073	35, 17	2 901 601	47, 31	6 265 001	51, 49
42 073	17, 5	815 401	31, 29	2 907 713	49, 15	6 324 401	59, 27
42 961	17, 7	937 121	37, 3	2 947 561	49, 19	6 412 321	59, 29
49 081	17, 11	940 361	37, 9	3 032 801	47, 33	6 520 441	59, 31
56 041	17, 13	951 361	37, 13	3 122 281	43, 41	6 690 881	57, 41
66 361	19, 7	1 002 241	37, 19	3 148 121	49, 27	6 922 921	61, 1
67 073	17, 15	1 016 033	35, 27	3 190 153	47, 35	6 930 241	61, 11
72 481	19, 11	1 054 721	33, 31	3 236 041	49, 29	6 948 233	61, 15
90 473	19, 15	1 132 393	37, 25	3 344 161	49, 31	6 995 761	59, 37
97 241	21, 1	1 156 721	39, 1	3 383 801	51, 7	7 020 161	61, 21
97 553	21, 5	1 157 033	39, 5	3 522 521	51, 23	7 215 401	59, 39
104 561	21, 11	1 198 481	39, 17	3 577 913	51, 25	7 471 561	59, 41
106 921	19, 17	1 398 841	37, 31	3 736 241	51, 29	7 768 081	59, 43
111 521	21, 13	1 414 081	41, 7	3 759 713	45, 43	7 941 641	63, 19
139 921	23, 1	1 416 161	41, 9	3 819 481	49, 37	8 160 401	57, 49
141 121	23, 7	1 420 201	41, 11	3 948 521	53, 9	8 230 121	63, 29
165 233	23, 15	1 510 121	41, 21	3 952 561	53, 11	8 338 241	63, 31
195 353	25, 3	1 510 361	39, 29	3 987 001	53, 17	8 925 313	65, 1
198 593	25, 9	1 618 481	39, 31	4 132 913	51, 35	8 928 593	65, 9
205 081	23, 19	1 678 601	41, 27	4 295 281	49, 41	8 967 073	65, 17
237 073	25, 17	1 687 393	37, 35	4 298 881	53, 29	9 223 241	57, 53
237 161	23, 21	1 709 713	43, 5	4 319 681	51, 37	9 278 953	65, 29
266 921	27, 7	1 710 601	43, 7	4 589 593	55, 13	9 441 281	59, 51
280 001	27, 13	1 734 713	43, 15	4 591 801	49, 43	9 585 881	63, 43
307 481	27, 17	1 975 121	43, 27	4 617 073	55, 17	9 853 313	57, 55
353 641	29, 1	2 005 841	41, 33	4 672 553	55, 21	9 862 393	65, 37

☞ *This Table gives all Half-Quartan Primes,* $p = \frac{1}{2}(x^4 + y^4) \ngtr 10^7$.

Quartan Primes
(*Continued from page* 253).

p	x, y
6 790 897	39, 46
6 925 201	51, 20
6 964 817	47, 38
6 999 457	51, 22
7 101 137	49, 34
7 166 897	43, 44
7 222 177	51, 26
7 326 257	11, 52
7 435 921	33, 50
7 439 681	47, 40
7 506 097	21, 52
7 591 457	23, 52
7 813 777	51, 32
7 843 057	27, 52
7 891 777	53, 6
7 894 577	53, 8
7 900 481	53, 10
7 911 217	53, 12
7 928 897	53, 14
8 050 481	53, 20
8 124 161	37, 50
8 222 257	53, 24
8 324 801	49, 40
8 503 057	1, 54
8 503 681	5, 54
8 505 137	53, 28
8 531 617	13, 54
8 586 577	17, 54
8 627 777	47, 44
8 633 377	19, 54
8 812 241	35, 52
8 939 057	53, 32
9 075 761	41, 50
9 189 041	55, 14
9 216 161	55, 16
9 226 817	53, 34
9 255 601	55, 18
9 426 577	31, 54
9 834 497	1, 56
9 918 017	17, 50

☞ *This Table gives all Quartan Primes,* $p = (x^4+y^4) \ngtr 10^7$.

High Quartan Primes.
$p = (x^4+y^4)$,
[*x odd, y even*].

	p	x, y
	29 986 577	1, 74
B	40 960 001	1, 80
	45 212 177	1, 82
	59 969 537	1, 88
B	65 610 001	1, 90
Da	100 000 081	3, 100
	100 006 561	9, 100
	126 247 697	1, 106
	193 877 777	1, 118
	303 595 777	1, 132
	384 160 001	1, 140
	406 586 897	1, 142
	562 448 657	1, 154
	655 360 001	1, 160
	723 394 817	1, 164
	916 636 177	1, 174
	1 049 760 001	1, 180
	1 416 468 497	1, 194
	1 536 953 617	1, 198
	1 731 891 457	1, 204
	1 944 810 001	1, 210
	2 342 560 001	1, 220
	2 702 336 257	1, 228

☞ *This List is complete* (*with* $x=1$) *up to* $y \ngtr 236$.

High Half-Quartan Primes,
$p = \frac{1}{2}(1+y)^4$, [*y odd*].

	p	x, y
B	12 705 841	1, 71
B	14 199 121	1, 73
BJ	21 523 361	1, 81
	56 275 441	1, 103
	60 775 313	1, 105
	81 523 681	1, 113
	87 450 313	1, 115
	100 266 961	1, 119
	107 182 721	9, 121
	138 461 441	1, 129
	273 990 641	1, 153
	370 600 313	1, 165
	407 865 361	1, 169
	427 518 041	1, 171
	784 119 601	1, 199
	849 090 841	1, 203
	883 050 313	1, 205
	1 984 563 001	1, 251
	2 249 930 281	1, 259

☞ *This List is complete* (*with* $x=1$) *up to* $y \ngtr 265$.

Sextan Primes, $p = (x^6 + y^6) \div (x^2 + y^2)$.

p	x, y	p	x, y	p	x, y	p	x, y
1	1, 1	79 153	17, 4	479 761	23, 28	1 352 521	35, 33
13	1, 2	81 001	17, 3	495 613	27, 26	1 388 593	17, 36
61	3, 2	83 233	1, 17	513 841	27, 5	1 405 693	37, 26
73	1, 3	97 501	5, 18	530 713	1, 27	1 417 393	37, 24
193	3, 4	99 721	19, 15	547 753	29, 17	1 429 801	37, 23
241	1, 4	101 281	19, 11	554 641	29, 24	1 457 821	35, 6
541	5, 2	107 641	19, 9	557 521	29, 16	1 481 281	35, 4
601	1, 5	118 621	19, 6	572 281	29, 25	1 486 561	37, 20
1 021	5, 6	121 921	19, 5	595 741	29, 26	1 489 153	13, 36
1 801	7, 5	126 241	11, 20	606 913	29, 12	1 510 273	37, 19
1 873	7, 4	127 921	17, 20	607 681	3, 28	1 535 581	37, 18
1 933	7, 6	134 161	9, 20	613 741	23, 30	1 537 441	11, 36
2 221	7, 2	148 513	21, 13	620 161	29, 11	1 563 901	27, 38
3 121	5, 8	165 601	21, 19	694 081	29, 4	1 569 241	37, 33
3 361	7, 8	170 353	21, 8	694 201	31, 21	1 620 973	31, 38
4 993	9, 7	184 081	21. 5	699 793	29, 3	1 642 813	21, 38
5 521	9, 4	189 853	19, 22	701 761	31, 24	1 678 321	1, 36
6 481	1, 9	209 953	23, 16	706 921	31, 19	1 753 441	39, 25
8 461	9, 10	210 481	23, 17	717 133	31, 18	1 775 281	39, 31
9 181	3, 10	211 441	23, 15	768 301	7, 30	1 788 673	39, 23
9 901	1, 10	219 001	23, 13	784 753	31, 28	1 790 641	37, 8
10 993	11, 8	224 401	23, 12	789 673	31, 13	1 804 513	39, 32
11 113	11, 7	229 981	3, 22	791 473	21, 32	1 809 481	37, 7
12 241	11, 5	243 553	23, 9	805 873	31, 12	1 811 533	39, 22
12 541	11, 10	254 161	19, 24	809 101	1, 30	1 816 861	35, 38
13 633	11, 3	258 061	23, 22	836 161	17, 32	1 826 173	37, 6
14 173	11, 2	275 161	23, 3	868 801	15, 32	1 840 561	37, 5
17 761	5, 12	276 721	11, 24	878 833	31, 7	1 861 921	37, 3
20 593	1, 12	277 741	23, 2	890 221	31, 6	1 868 701	37, 2
21 433	13, 9	306 541	25, 14	900 121	31, 5	1 891 501	39, 34
21 661	13, 10	306 913	23, 24	919 693	31, 2	1 921 681	29, 40
21 841	13, 8	309 481	25, 21	922 561	1, 31	1 925 041	27, 40
23 773	13, 6	313 561	25, 13	946 801	33, 28	1 993 441	23, 40
26 113	13, 4	318 001	5, 24	988 033	31, 32	2 083 693	1, 38
27 901	13, 2	339 841	25, 23	1 004 461	25, 34	2 122 513	41, 28
28 393	1, 13	343 261	19, 26	1 023 601	5, 32	2 144 041	39, 11
29 101	9, 14	345 133	17, 26	1 030 441	33, 13	2 171 341	39, 10
34 141	5, 14	346 561	25, 9	1 062 913	33, 31	2 181 073	41, 33
41 161	15, 13	353 341	21, 26	1 120 321	33, 8	2 189 281	41, 24
49 201	11, 16	355 501	15, 26	1 126 861	15, 34	2 202 253	39, 38
49 741	15, 2	380 881	25, 4	1 129 501	35, 26	2 218 861	41, 34
50 833	13, 16	385 081	25, 3	1 134 961	33, 7	2 220 193	39, 8
51 361	9, 16	390 001	1, 25	1 139 041	35, 27	2 241 313	39, 7
63 241	17, 13	410 353	27, 16	1 148 941	31, 34	2 276 041	39, 5
63 313	3, 16	425 101	25, 26	1 177 681	35, 29	2 307 373	39, 2
64 621	17, 10	425 641	27, 23	1 181 581	33, 2	2 311 921	1, 39
65 293	17, 14	426 253	7, 26	1 188 721	35, 19	2 398 633	41, 37
71 761	17, 7	426 973	27, 14	1 263 373	33, 34	2 399 821	25, 42
74 413	17, 6	436 801	27, 13	1 282 093	7, 34	2 460 961	41, 16
78 781	13, 18	466 441	27, 25	1 322 161	35, 13	2 545 681	3, 40

☞ *Continued on page* 257.

Sextan Primes (*Continued from page* 256).

p	x, y	p	x, y	p	x, y	p	x, y
2 564 701	43, 30	4 433 281	47, 15	6 909 841	55, 36	9 138 541	55, 2
2 570 233	41, 13	4 452 841	49, 29	6 941 293	47, 54	9 147 601	1, 55
2 570 941	37, 42	4 483 201	49, 40	6 999 073	11, 52	9 168 961	59, 45
2 571 073	43, 29	4 545 913	49, 27	7 043 713	51, 52	9 226 033	59, 37
2 582 401	41, 39	4 554 481	49, 41	7 085 341	53, 18	9 307 513	59, 47
2 637 001	41, 11	4 582 321	47, 12	7 240 333	23, 54	9 405 553	59, 48
2 653 801	43, 25	4 598 701	49, 26	7 287 361	3, 52	9 587 041	9, 56
2 654 401	43, 35	4 654 801	49, 25	7 308 913	1, 52	9 734 161	41, 60
2 702 113	43, 36	4 701 661	37, 50	7 313 881	55, 29	9 831 361	1, 56
2 722 273	41, 8	4 726 081	17, 48	7 349 473	53, 51	9 957 613	57, 14
2 758 141	43, 22	4 760 941	39, 50	7 378 081	39, 56		
2 810 713	41, 3	4 771 021	31, 50	7 378 333	53, 14		
2 819 053	41, 2	4 773 841	47, 7	7 393 681	55, 28		
2 839 201	43, 20	4 774 513	49, 23	7 476 841	55, 27		
2 912 893	11, 42	4 854 781	29, 50	7 562 701	55, 26		
2 919 913	43, 39	4 870 861	47, 2	7 580 701	19, 54		
2 971 873	37, 44	4 947 601	13, 48	7 606 561	53, 52		
3 020 401	43, 40	5 189 161	51, 31	7 619 581	53, 10		
3 075 601	45, 32	5 207 341	23, 50	7 652 401	55, 49		
3 088 801	21, 44	5 306 113	1, 48	7 714 801	53, 8		
3 094 813	43, 14	5 387 593	49, 13	7 740 001	55, 24		
3 096 061	45, 34	5 439 793	49, 12	7 820 881	53, 5		
3 127 681	45, 28	5 477 821	19, 50	7 845 793	53, 4		
3 134 881	43, 13	5 488 921	49, 11	7 865 281	53, 3		
3 188 701	45, 26	5 499 841	35, 52	7 917 601	57, 40		
3 268 861	43, 42	5 530 201	51, 25	7 920 193	57, 41		
3 353 533	43 6	5 576 881	49, 9	8 014 033	57, 44		
3 354 781	45, 22	5 636 593	31, 52	8 047 801	55, 51		
3 402 241	43, 3	5 730 721	43, 52	8 164 861	11, 54		
3 416 953	1, 43	5 744 833	29, 52	8 258 641	57, 47		
3 454 813	37, 46	5 871 841	27, 52	8 265 121	25, 56		
3 499 921	45, 19	5 899 273	51, 47	8 277 601	57, 32		
3 598 921	45, 17	5 929 633	53, 36	8 357 233	57, 31		
3 655 633	7, 44	5 979 613	53, 34	8 359 921	55, 17		
3 666 241	47, 32	6 115 441	53, 43	8 362 573	7, 54		
3 729 721	47, 37	6 161 041	23, 52	8 441 761	55, 16		
3 738 781	21, 46	6 172 381	53, 30	8 487 373	41, 58		
3 775 201	45, 43	6 200 353	53, 44	8 543 881	55, 53		
3 800 761	47, 27	6 218 161	47, 52	8 667 961	55, 13		
3 833 233	47, 39	6 293 821	51, 14	8 735 761	55, 12		
3 949 453	17, 46	6 385 213	37, 54	8 765 101	47, 58		
3 983 773	43, 46	6 448 573	53, 26	8 816 653	57, 26		
3 986 641	35, 48	6 602 833	51, 8	8 832 721	19, 56		
4 148 413	13, 46	6 684 361	53, 23	8 839 021	33, 58		
4 156 081	47, 20	6 726 961	53, 48	8 915 953	53, 56		
4 236 061	11, 46	6 753 841	15, 52	9 007 213	31, 58		
4 332 721	49, 36	6 757 981	29, 54	9 091 561	59, 41		
4 376 173	7, 46	6 765 181	53, 22	9 099 793	59, 48		
4 382 893	49, 38	6 846 193	53, 21	9 123 481	55, 3		
4 425 181	5, 46	6 883 561	55, 37	9 136 201	59, 39		

☞ *This Table gives all Sextan Primes,* $p = (x^6 + y^6) \div (x^2 + y^2) \not> 10^7$.

High Simple Sextan Primes,
$p = (1^6 + y^6) \div (1^2 + y^2)$.

	p	x,	y
BC	13 842 121	1,	61
BC	14 772 493	1,	62
BC	17 846 401	1,	65
	47 451 433	1,	83
	71 630 833	1,	92
	78 066 061	1,	94
	96 049 801	1,	99
Lo	99 990 001	1,	100
	116 975 041	1,	104
	121 539 601	1,	105
	141 146 281	1,	109
	168 883 021	1,	114
	193 863 853	1,	118
	252 031 501	1,	126
	294 482 761	1,	131
	759 305 581	1,	166
	796 565 953	1,	168
	815 702 161	1,	169
	875 183 473	1,	172
	1 121 479 633	1,	183
	1 171 316 401	1,	185
	1 303 173 901	1,	190
	1 416 430 861	1,	194
	1 475 750 641	1,	196
	1 536 914 413	1,	198
	1 907 986 081	1,	209
	2 517 580 801	1,	224
	2 562 840 001	1,	225
	2 750 006 041	1,	229
	2 847 342 961	1,	231
Lo	999 999 000 001	1,	1000

☞ *This Table is complete* (*for* $x = 1$) *up to* $y \not> 238$.

Octavan Primes.

	$p = x^8 + y^8$	x, y
$p < 10^7$	257	1, 2
	65 537	1, 4
	2 070 241	5, 6
	Complete to $p \not> 10^7$	
$p > 10^7$	100 006 561	3, 10
	None $> 10^7$, up to 4.10^{12}	1, y

Half-Octavan Primes.

	$p = \frac{1}{2}(x^8 + y^8)$	x, y
$p < 10^7$	1	1, 1
	198 593	3, 5
	Complete to $p \not> 10^7$	
$p > 10^7$	BJ 21 523 361	1, 9
	107 182 721	3, 4
	407 865 361	1, 13
	No more $< 189.10^8$	1, y

Duodeciman Primes.

	$p = (x^{12} + y^{12}) \div (x^4 + y^4)$	x, y
$p < 10^7$	1	1, 1
	241	1, 2
	5 521	3, 2
	6 481	1, 3
	51 361	3, 4
	346 561	3, 5
	380 881	5, 2
	390 001	1, 5
	1 678 321	1, 6
	4 332 721	7, 6
	4 654 801	7, 5
	5 576 881	7, 3
	Complete to $p \not> 10^7$	

	$p = (x^{12} + y^{12}) \div (x^4 + y^4)$	x, y
$p > 10^7$	12 707 521	7, 8
	39 336 721	9, 5
	41 432 641	9, 4
	42 942 001	9, 2
	99 990 001	1, 10
	815 702 161	1, 13
	1 475 750 641	1, 14
	2 562 840 001	1, 15
	Complete to $p \not> 10^{10}$	1, y

Sextodeciman Primes.

$$65\,537 = (1^{16} + 2^{16}).$$
$$1 = \tfrac{1}{2}(1^{16} + 1^{16}).$$
$$21\,523\,361 = \tfrac{1}{2}(1^{16} + 3^{16}).$$

24-man Primes.

$$1 = (1^{24} + 1^{24}) \div (1^8 + 1^8).$$
$$41\,432\,641 = (3^{24} + 2^{24}) \div (3^8 + 2^8).$$

Cuban Primes $p = (x^3-y^3)\div(x-y)$, *up to* $p \not> 10^6$, $[x-y=1]$.

p	x	p	x	p	x	p	x	p	x	p	x
1	1	10 267	59	81 181	165	200 467	259	383 419	358	698 419	483
7	2	11 719	63	82 171	166	202 021	260	387 721	360	707 131	486
19	3	12 097	64	87 211	171	213 067	267	398 581	365	733 591	495
37	4	13 267	67	88 237	172	231 019	278	407 377	369	742 519	498
61	5	13 669	68	89 269	173	234 361	280	423 001	376	760 537	504
127	7	16 651	75	92 401	176	241 117	284	436 627	382	769 627	507
271	10	19 441	81	96 661	180	246 247	287	452 797	389	772 669	508
331	11	19 927	82	102 121	185	251 431	290	459 817	392	784 897	512
397	12	22 447	87	103 231	186	260 191	295	476 407	399	791 047	514
547	14	23 497	89	104 347	187	263 737	297	478 801	400	812 761	521
631	15	24 571	91	110 017	192	267 307	299	493 291	406	825 301	525
919	18	25 117	92	112 327	194	276 337	304	522 919	418	837 937	529
1 657	24	26 227	94	114 661	196	279 991	306	527 941	420	847 477	532
1 801	25	27 361	96	115 837	197	283 669	308	553 411	430	863 497	537
1 951	26	33 391	106	126 691	206	285 517	309	574 219	438	879 667	542
2 269	28	35 317	109	129 169	208	292 969	313	584 767	442	886 177	544
2 437	29	42 841	120	131 671	210	296 731	315	590 077	444	895 987	547
2 791	31	45 757	124	135 469	213	298 621	316	592 741	445	909 151	551
3 169	33	47 251	126	140 617	217	310 087	322	595 411	446	915 769	553
3 571	35	49 537	129	144 541	220	329 677	332	603 457	449	925 741	556
4 219	38	50 311	130	145 861	221	333 667	334	608 851	451	929 077	557
4 447	39	55 897	137	151 201	225	337 681	336	611 557	452	932 419	558
5 167	42	59 221	141	155 269	228	347 821	341	619 711	455	939 121	560
5 419	43	60 919	143	163 567	234	351 919	343	627 919	458	952 597	564
6 211	46	65 209	148	169 219	238	360 187	347	650 071	466	972 991	570
7 057	49	70 687	154	170 647	239	368 551	351	658 477	469	976 411	571
7 351	50	73 477	157	176 419	243	372 769	353	666 937	472	986 707	574
8 269	53	74 419	158	180 811	246	374 887	354	689 761	480	990 151	575
9 241	56	75 367	159	189 757	252	377 011	355	692 641	481	997 057	577

Cuban Primes $p = (x^3-y^3)\div(x-y)$, *up to* $p \not> 10^6$, $[x-y=2]$.

p	x	p	x	p	x	p	x	p	x	p	x
1	1	13 873	69	76 801	161	193 549	255	355 009	345	618 349	455
13	3	18 253	79	84 673	169	209 089	265	363 313	349	640 333	463
109	7	20 173	83	106 033	189	221 953	273	367 501	351	645 889	465
193	9	21 169	85	108 301	191	238 573	283	397 489	365	685 453	479
433	13	22 189	87	112 909	195	245 389	287	410 701	371	720 301	491
769	17	28 813	99	115 249	197	259 309	295	415 153	373	762 049	505
1 201	21	37 633	113	129 793	209	270 001	301	424 129	377	786 433	513
1 453	23	43 201	121	139 969	217	273 613	303	433 201	381	823 729	525
2 029	27	47 629	127	142 573	219	280 909	307	442 369	385	842 701	531
3 469	35	60 493	143	147 853	223	284 593	309	534 253	423	940 801	561
3 889	37	63 949	147	169 933	239	299 569	317	544 429	427	961 069	567
4 801	41	65 713	149	172 801	241	307 201	321	549 553	429	967 873	569
10 093	59	69 313	153	178 609	245	326 701	331	565 069	435		
12 289	65	73 009	157	181 549	247	342 733	339	596 749	447		

Trito-Cuban Primes, $p = \frac{1}{3}(x^3-y^3)\div(x-y)$, *up to* $p \not> 10^6$, $[x-y = 3]$.

p	x	p	x	p	x	p	x	p	x	p	x
1	1, 2	9 901	101	55 933	238	173 473	418	375 157	614	681 451	827
7	4	10 303	103	60 271	247	181 903	428	386 263	623	684 757	829
13	5	11 131	107	60 763	248	188 791	436	390 001	626	699 733	838
31	7	12 211	112	71 023	268	189 661	437	392 503	628	704 761	841
43	8	12 433	113	74 257	274	200 257	449	403 861	637	716 563	848
73	10	13 807	119	77 563	280	205 663	455	412 807	644	731 881	857
157	14	14 281	121	78 121	281	207 481	457	415 381	646	735 307	859
211	16	17 293	133	82 657	289	208 393	458	446 893	670	740 461	862
241	17	19 183	140	83 233	290	227 053	478	450 913	673	747 361	866
307	19	20 023	143	84 391	292	239 611	491	459 007	679	766 501	877
343	20	20 593	145	86 143	295	245 521	497	471 283	688	771 763	880
421	22	21 757	149	95 791	311	250 501	502	485 113	698	792 991	892
463	23	22 651	152	98 911	316	262 657	514	492 103	703	800 131	896
601	26	23 563	155	108 571	331	268 843	520	519 121	722	805 507	899
757	29	24 181	157	110 557	334	276 151	527	527 803	728	809 101	901
1 123	35	26 083	163	113 233	338	281 431	532	530 713	730	830 833	913
1 483	40	26 407	164	117 307	344	282 493	533	540 961	737	838 141	917
1 723	43	27 061	166	118 681	346	284 623	535	552 793	745	843 643	920
2 551	52	28 057	169	121 453	350	288 907	539	558 757	749	847 321	922
2 971	56	28 393	170	123 553	353	292 141	542	570 781	757	860 257	929
3 307	59	30 103	175	127 807	359	304 153	553	579 883	763	903 451	952
3 541	61	31 153	178	136 531	371	307 471	556	581 407	764	920 641	961
3 907	64	35 533	190	143 263	380	314 161	562	590 593	770	922 561	962
4 423	68	35 911	191	145 543	383	320 923	568	598 303	775	939 931	971
4 831	71	37 057	194	147 073	385	322 057	569	612 307	784	949 651	976
5 113	73	37 831	196	154 057	394	327 757	574	617 011	787	963 343	983
5 701	77	41 413	205	156 421	397	335 821	581	628 057	794	975 157	989
6 007	79	42 643	208	158 803	400	339 307	584	637 603	800	981 091	992
6 163	80	43 891	211	162 007	404	341 641	586	642 403	803	985 057	994
6 481	82	46 441	217	163 621	406	364 213	605	660 157	814	987 043	995
8 011	91	47 743	220	164 431	407	366 631	607	669 943	820		
8 191	92	53 593	233	171 811	416	371 491	611	671 581	821		

Prime Aurifeuillian Factors p = L *or* M *of Sextans* (B, B′, D′, D″).

B, B′, D′, D″ are all of form $N_{vi} = (x^6 + y^6) \div (x^2 + y^2) = L.M.$

B has $x = \xi^2,\ y = 2\eta^2$; B′ has $x' = \xi'^2,\ y' = 2\eta'^2$.

B = LM, B′ = L′M′; see Table, pages 172–179.

D′ has $x' = t'^2 + u'^2,\ y' = t'^2 \sim u'^2$; D″ has $x'' = \frac{1}{2}(t''^2 + u''^2),\ y'' = t''u''$.

D′ = L′M′, D″ = L″M″; see Table, pages 190–194.

Then B, B′, D′, D″ have a common factor (L or M) if

$$\xi = 2\eta' - \xi',\quad \eta = \eta' \sim \xi';\qquad \xi' = \xi \pm 2\eta,\quad \eta' = \xi \pm \eta.$$

$$t' = \eta',\qquad u' = \eta;\qquad t'' = t' + u',\ u'' = t' \sim u'.$$

p	B ξ, η	B′ ξ', η'	D′ t', u'	D″ t'', u''
1	L, 1, 1			
13	M, 1, 1	L, 3, 2	L, 2, 1	L, 3, 1
37	L, 3, 1	L, 1, 2	M, 2, 1	M, 3, 1
109	M, 1, 2	L, 5, 3	L, 3, 2	L, 5, 1
229	L, 5, 2	L, 1, 3	M, 3, 2	M, 5, 1
409	L, 5, 1	L, 3, 4	M, 4, 1	M, 5, 3
421	M, 3, 2	L, 7, 5	L, 5, 2	L, 7, 3
457	M, 1, 3	L, 7, 4	L, 4, 3	L, 7, 1
1 321	M, 1, 4	L, 9, 5	L, 5, 4	L, 9, 1
1 549	M, 5, 2	L, 9, 7	L, 7, 2	L, 9, 5
1 789	L, 7, 1	L, 5, 6	M, 6, 1	M, 7, 5
2 377	M, 3, 4	L, 11, 7	L, 7, 4	L, 11, 3
2 689	M, 5, 3	L, 11, 8	L, 8, 3	L, 11, 5
3 061	M, 1, 5	L, 11, 6	L, 6, 5	L, 11, 1
3 217	M, 7, 1	L, 9, 8	L, 8, 1	L, 9, 7
4 729	M, 5, 4	L, 13, 9	L, 9, 4	L, 13, 5
4 801	M, 3, 5	L, 13, 8	L, 8, 5	L, 13, 3
5 233	L, 9, 1	L, 7, 8	M, 8, 1	M, 9, 7
6 073	L, 11, 4	L, 3, 7	M, 7, 4	M, 11, 3
6 133	M, 1, 6	L, 13, 7	L, 7, 6	L, 13, 1
6 421	M, 7, 3	L, 13, 10	L, 10, 3	L, 13, 7
8 221	M, 9, 1	L, 11, 10	L, 10, 1	L, 11, 9
8 317	L, 13, 6	L, 1, 7	M, 7, 6	M, 13, 1
10 477	M, 9, 2	L, 13, 11	L, 11, 2	L, 13, 9
14 281	M, 7, 5	L, 17, 12	L, 12, 5	L, 17, 7
14 449	L, 15, 7	L, 1, 8	M, 8, 7	M, 15, 1
15 061	M, 3, 7	L, 17, 10	L, 10, 7	L, 17, 3
17 341	L, 13, 3	L, 7, 10	M, 10, 3	M, 13, 7
18 313	M, 9, 4	L, 17, 13	L, 13, 4	L, 17, 9
20 773	L, 13, 2	L, 9, 11	M, 11, 2	M, 13, 9
21 277	M, 7, 6	L, 19, 13	L, 13, 6	L, 19, 7
23 473	L, 17, 8	L, 1, 9	M, 9, 8	M, 17, 1
26 317	M, 11, 3	L, 17, 14	L, 14, 3	L, 17, 11
33 349	M, 13, 1	L, 15, 14	L, 14, 1	L, 15, 13
35 869	L, 17, 6	L, 5, 11	M, 11, 6	M, 17, 5
39 181	M, 13, 2	L, 17, 15	L, 15, 2	L, 17, 13
42 841	L, 17, 5	L, 7, 12	M, 12, 5	M, 17, 7
44 221	M, 1, 10	L, 21, 11	L, 11, 10	L, 21, 1
44 257	L, 19, 8	L, 3, 11	M, 11, 8	M, 19, 3
44 269	L, 15, 1	L, 13, 14	M, 14, 1	M, 15, 13
54 013	M, 11, 6	L, 23, 17	L, 17, 6	L, 23, 11
54 421	M, 3, 10	L, 23, 13	L, 13, 10	L, 23, 3
55 897	M, 13, 4	L, 21, 17	L, 17, 4	L, 21, 13
62 773	L, 19, 6	L, 7, 13	M, 13, 6	M, 19, 7
63 409	M, 9, 8	L, 25, 17	L, 17, 8	L, 25, 9
64 153	M, 1, 11	L, 23, 12	L, 12, 11	L, 23, 1
65 701	L, 17, 2	L, 13, 15	M, 15, 2	M, 17, 13
69 829	M, 11, 7	L, 25, 18	L, 18, 7	L, 25, 11
74 209	L, 17, 1	L, 15, 16	M, 16, 1	M, 17, 15
76 129	L, 21, 8	L, 5, 13	M, 13, 8	M, 21, 5
89 689	M, 15, 4	L, 23, 19	L, 19, 4	L, 23, 15
93 937	L, 19, 3	L, 13, 16	M, 16, 3	M, 19, 13
93 997	M, 17, 1	L, 19, 18	L, 18, 1	L, 19, 17
105 229	L, 19, 2	L, 15, 17	M, 17, 2	M, 19, 15
105 769	L, 25, 12	L, 1, 13	M, 13, 12	M, 25, 1
113 341	M, 9, 10	L, 29, 19	L, 19, 10	L, 29, 9
123 397	M, 1, 13	L, 27, 14	L, 14, 13	L, 27, 1
139 393	L, 23, 7	L, 9, 16	M, 16, 7	M, 23, 9
143 053	L, 27, 13	L, 1, 14	M, 14, 13	M, 27, 1

Prime Aurifeuillian Factors $p =$ L *or* M *of Sextans* (B, B′, D′, D″).

(*Continued from page* 261.)

p	B ξ, η	B′ ξ', η'	D′ t', u'	D″ t'', u''
160 357	L, 21, 2	L, 17, 19	M, 19, 2	M, 21, 17
170 509	M, 5, 13	L, 31, 18	L, 18, 13	L, 31, 5
175 621	L, 23, 5	L, 13, 18	M, 18, 5	M, 23, 13
189 421	L, 29, 14	L, 1, 15	M, 15, 14	M, 29, 1
194 569	L, 23, 4	L, 15, 19	M, 19, 4	M, 23, 15
198 301	M, 13, 10	L, 33, 23	L, 23, 10	L, 33, 13
203 641	M, 7, 13	L, 33, 20	L, 20, 13	L, 33, 7
213 553	M, 17, 7	L, 31, 24	L, 24, 7	L, 31, 17
213 973	M, 21, 1	L, 23, 22	L, 22, 1	L, 23, 21
216 481	M, 1, 15	L, 31, 16	L, 16, 15	L, 31, 1
228 961	M, 19, 5	L, 29, 24	L, 24, 5	L, 29, 19
231 709	L, 25, 6	L, 13, 19	M, 19, 6	M, 25, 13
234 733	L, 23, 2	L, 19, 21	M, 21, 2	M, 23, 19
235 789	M, 21, 2	L, 25, 23	L, 23, 2	L, 25, 21
246 241	L, 31, 15	L, 1, 16	M, 16, 15	M, 31, 1
270 913	L, 27, 8	L, 11, 19	M, 19, 8	M, 27, 11
278 413	L, 29, 11	L, 7, 18	M, 18, 11	M, 29, 7
279 073	M, 1, 16	L, 33, 17	L, 17, 16	L, 33, 1
280 249	L, 25, 4	L, 17, 21	M, 21, 4	M, 25, 17
297 397	M, 19, 7	L, 33, 26	L, 26, 7	L, 33, 19
299 881	L, 27, 7	L, 13, 20	M, 20, 7	M, 27, 13
301 057	M, 11, 13	L, 37, 24	L, 24, 13	L, 37, 11
305 329	M, 23, 1	L, 25, 24	L, 24, 1	L, 25, 23
305 749	L, 25, 3	L, 19, 22	M, 22, 3	M, 25, 19
311 173	M, 9, 14	L, 37, 23	L, 23, 14	L, 37, 9
315 589	L, 31, 13	L, 5, 18	M, 18, 13	M, 31, 5
341 569	M, 19, 8	L, 35, 27	L, 27, 8	L, 35, 19
346 201	L, 29, 9	L, 11, 20	M, 20, 9	M, 29, 11
365 173	M, 23, 3	L, 29, 26	L, 26, 3	L, 29, 23
381 697	L, 29, 8	L, 13, 21	M, 21, 8	M, 29, 13
394 201	L, 31, 11	L, 9, 20	M, 20, 11	M, 31, 9
398 029	L, 33, 14	L, 5, 19	M, 19, 14	M, 33, 5
399 241	M, 3, 17	L, 37, 20	L, 20, 17	L, 37, 3
401 017	M, 23, 4	L, 31, 27	L, 27, 4	L, 31, 23
410 617	M, 17, 11	L, 39, 28	L, 28, 11	L, 39, 17
418 069	L, 29, 7	L, 15, 22	M, 22, 7	M, 29, 15
423 229	M, 25, 1	L, 27, 26	L, 26, 1	L, 27, 25
443 881	L, 33, 13	L, 7, 20	M, 20, 13	M, 33, 7
443 917	M, 1, 18	L, 37, 19	L, 19, 18	L, 37, 1
444 529	L, 35, 16	L, 3, 19	M, 19, 16	M, 35, 3
451 669	M, 5, 17	L, 39, 22	L, 22, 17	L, 39, 5
455 701	M, 19, 10	L, 39, 29	L, 29, 10	L, 39, 19
478 813	L, 31, 9	L, 13, 22	M, 22, 9	M, 31, 13
485 113	M, 17, 12	L, 41, 29	L, 29, 12	L, 41, 17
548 521	M, 13, 15	L, 43, 28	L, 28, 15	L, 43, 13
549 481	M, 1, 19	L, 39, 20	L, 20, 19	L, 39, 1
561 553	M, 11, 16	L, 43, 27	L, 27, 16	L, 43, 11
572 581	M, 17, 13	L, 43, 30	L, 30, 13	L, 43, 17
573 277	L, 29, 3	L, 23, 26	M, 26, 3	M, 29, 23
591 901	M, 21, 10	L, 41, 31	L, 31, 10	L, 41, 21
606 589	L, 35, 13	L, 9, 22	M, 22, 13	M, 35, 9
613 177	M, 19, 12	L, 43, 31	L, 31, 12	L, 43, 19
616 933	M, 27, 2	L, 31, 29	L, 29, 2	L, 31, 27
629 701	M, 7, 18	L, 43, 25	L, 25, 18	L, 43, 7
660 073	L, 29, 1	L, 27, 28	M, 28, 1	M, 29, 27
660 529	M, 13, 16	L, 45, 29	L, 29, 16	L, 45, 13
660 661	L, 31, 5	L, 21, 26	M, 26, 5	M, 31, 21
675 313	M, 21, 11	L, 43, 32	L, 32, 11	L, 43, 21
677 857	M, 23, 9	L, 41, 32	L, 32, 9	L, 41, 23
711 889	M, 19, 13	L, 45, 32	L, 32, 13	L, 45, 19
714 529	M, 25, 7	L, 39, 32	L, 32, 7	L, 39, 25
741 721	L, 41, 20	L, 1, 21	M, 21, 20	M, 41, 1
764 149	M, 7, 19	L, 45, 26	L, 26, 19	L, 45, 7
780 721	M, 27, 5	L, 37, 32	L, 32, 5	L, 37, 27
788 209	M, 25, 8	L, 41, 33	L, 33, 8	L, 41, 25
790 501	M, 13, 17	L, 47, 30	L, 30, 17	L, 47, 13
808 993	L, 37, 13	L, 11, 24	M, 24, 13	M, 37, 11
817 153	L, 39, 16	L, 7, 23	M, 23, 16	M, 39, 7
826 093	M, 19, 14	L, 47, 33	L, 33, 14	L, 47, 19
856 549	M, 23, 11	L, 45, 34	L, 34, 11	L, 45, 23
865 741	L, 31, 1	L, 29, 30	M, 30, 1	M, 31, 29
867 001	L, 33, 5	L, 23, 28	M, 28, 5	M, 33, 23
872 269	M, 25, 9	L, 43, 34	L, 34, 9	L, 43, 25
879 961	L, 37, 12	L, 13, 25	M, 25, 12	M, 37, 13
883 117	M, 21. 13	L, 47, 34	L, 34, 13	L, 47, 21
895 357	L, 43. 21	L, 1, 22	M, 22, 21	M, 43, 1
897 349	L, 41, 18	L, 5, 23	M, 23, 18	M, 41, 5
916 129	L, 35, 8	L, 19, 27	M, 27, 8	M, 35, 19
925 849	L, 33, 4	L, 25, 29	M, 29, 4	M, 33, 25
952 669	L, 37, 11	L, 15, 26	M, 26, 11	M, 37, 15
975 661	M, 11, 19	L, 49, 30	L, 30, 19	L, 49, 11
991 069	M, 5, 21	L, 47, 26	L, 26, 21	L, 47, 5

☞ *This Table gives all primes* L, M *of* B, B′, D′, D″ *up to* M $\not>$ 9.10^6.

Prime Aurifeuillian Factors p = L *or* M *of Sextans* (S′, S″, T′, T″).

S′, S″, T′, T″ are all of form $N_{vi} = (x^6+y^6) \div (x^2+y^2) = L.M.$

S′ has $x = \xi^2$, $y = 6\eta^2$; S″ has $x'' = 3\xi''^2$, $y = 2\eta''^2$.

S′ = L′M′, S″ = L″M″; see Tables, pages 179–182, and 183, 184, 194.

T′ has $\begin{cases} x' = t'^2 \sim 3x'^2, \\ y' = t'^2 + 3u'^2; \end{cases}$ T″ has $\begin{cases} x'' = \frac{1}{2}(t''^2 \sim 3u''^2), \\ y'' = \frac{1}{2}(t''^2 + 3u''^2). \end{cases}$

T′ = L′M′, T″ = L″M″; see Table, pages 185–189, 194.

Then S′, S″, T′, T″ have a common factor (L or M), if

$$\xi' = 3\xi'' \pm 2\eta'', \quad \eta' = \xi'' \pm \eta''; \quad \xi'' = \xi' \mp 2\eta', \quad \eta'' = \xi' \mp 3\eta'.$$

$$t' = \eta'', \quad u' = \eta'; \quad t'' = \xi', \quad u'' = \xi''.$$

p	S′ ξ', η'	S″ ξ'', η''	T′ t', u'	T″ t'', u''
1		L, 1, 1		L, 1, 1
13	L, 1, 1	L, 1, 2	L, 2, 1	M, 1, 1
61	L, 5, 2	M, 1, 1	L, 1, 2	L, 5, 1
97	M, 1, 1	L, 3, 4	L, 4, 1	L, 1, 3
181	L, 5, 1	L, 3, 2	M, 2, 1	L, 5, 3
277	L, 7, 3	M, 1, 2	L, 2, 3	L, 7, 1
349	L, 1, 2	L, 3, 5	L, 5, 2	M, 1, 3
373	L, 7, 2	L, 3, 1	M, 1, 2	L, 7, 3
1 009	L, 7, 1	L, 5, 4	M, 4, 1	L, 7, 5
2 089	L, 1, 3	L, 5, 8	L, 8, 3	M, 1, 5
2 161	L, 11, 5	M, 1, 4	L, 4, 5	L, 11, 1
2 521	L, 5, 4	L, 3, 7	L, 7, 4	M, 5, 3
3 769	L, 13, 4	L, 5, 1	M, 1, 4	L, 13, 5
4 549	L, 13, 6	M, 1, 5	L, 5, 6	L, 13, 1
4 789	L, 11, 2	L, 7, 5	M, 5, 2	L, 11, 7
6 673	L, 13, 3	L, 7, 4	M, 4, 3	L, 13, 7
7 177	L, 1, 4	L, 7, 11	L, 11, 4	M, 1, 7
8 389	L, 17, 6	M, 5, 1	L, 1, 6	L, 17, 5
11 197	L, 13, 2	L, 9, 7	M, 7, 2	L, 13, 9
11 257	L, 13, 7	L, 1, 8	L, 8, 7	M, 13, 1
11 833	M, 1, 4	L, 9, 13	L, 13, 4	L, 1, 9
12 637	M, 7, 2	L, 11, 13	L, 13, 2	L, 7, 11
14 029	L, 7, 6	L, 5, 11	L, 11, 6	M, 7, 5
14 737	L, 17, 8	M, 1, 7	L, 7, 8	L, 17, 1
16 069	L, 11, 7	L, 3, 10	L, 10, 7	M, 11, 3
17 989	L, 13, 1	L, 11, 10	M, 10, 1	L, 13, 11
20 101	L, 5, 6	L, 7, 13	L, 13, 6	M, 5, 7
23 833	L, 19, 9	M, 1, 8	L, 8, 9	L, 19, 1

p	S′ ξ', η'	S″ ξ'', η''	T′ t', u'	T″ t'', u''
24 001	L, 19, 5	L, 9, 4	M, 4, 5	L, 19, 9
25 237	M, 11, 1	L, 13, 14	L, 14, 1	L, 11, 13
25 633	L, 13, 8	L, 3, 11	L, 11, 8	M, 13, 3
27 481	M, 1, 5	L, 11, 16	L, 16, 5	L, 1, 11
28 297	L, 17, 3	L, 11, 8	M, 8, 3	L, 17, 11
28 753	L, 23, 8	M, 7, 1	L, 1, 8	L, 23, 7
29 629	L, 17, 9	L, 1, 10	L, 10, 9	M, 17, 1
34 849	L, 11, 8	L, 5, 13	L, 13, 8	M, 11, 5
45 289	M, 13, 1	L, 15, 16	L, 16, 1	L, 13, 15
49 789	L, 19, 3	L, 13, 10	M, 10, 3	L, 19, 13
49 801	L, 25, 11	M, 3, 8	L, 8, 11	L, 25, 3
51 349	L, 13, 9	L, 5, 14	L, 14, 9	M, 13, 5
55 117	M, 1, 6	L, 13, 19	L, 19, 6	L, 1, 13
56 941	L, 17, 10	L, 3, 13	L, 13, 10	M, 17, 3
63 361	L, 25, 7	L, 11, 4	M, 4, 7	L, 25, 11
66 697	L, 11, 9	L, 7, 16	L, 16, 9	M, 11, 7
69 109	L, 19, 2	L, 15, 13	M, 13, 2	L, 19, 15
71 413	M, 13, 2	L, 17, 19	L, 19, 2	L, 13, 17
71 809	M, 11, 3	L, 17, 20	L, 20, 3	L, 11, 17
71 881	L, 23, 5	L, 13, 8	M, 8, 5	L, 28, 13
73 609	L, 29, 12	M, 5, 7	L, 7, 12	L, 29, 5
74 929	L, 1, 7	L, 13, 20	L, 20, 7	M, 1, 13
78 721	L, 5, 8	L, 11, 19	L, 19, 8	M, 5, 11
80 557	L, 19, 11	L, 3, 14	L, 14, 11	M, 19, 3
85 381	L, 25, 6	L, 13, 7	M, 7, 6	L, 25, 13
88 741	M, 7, 5	L, 17, 22	L, 22, 5	L, 7, 17
90 697	L, 23, 12	L, 1, 13	L, 13, 12	M, 23, 1
91 573	L, 29, 9	L, 11, 2	M, 2, 9	L, 29, 11
92 941	L, 13, 10	L, 7, 17	L, 17, 10	M, 13, 7
97 789	L, 29, 13	M, 3, 10	L, 10, 13	L, 29, 3

Prime Aurifeuillian Factors p = L *or* M *of Sextans* (S′, S″, T′, T″).

(*Continued from page* 263.)

p	S′ ξ', η'	S″ ξ'', η''	T′ t', u'	T″ t'', u''
99 529	L, 31, 13	M, 5, 8	L, 8, 13	L, 31, 5
99 709	M, 1, 7	L, 15, 22	L, 22, 7	L, 1, 15
101 209	L, 17, 11	L, 5, 16	L, 16, 11	M, 17, 5
106 861	M, 5, 6	L, 17, 23	L, 23, 6	L, 5, 17
118 369	L, 29, 8	L, 13, 5	M, 5, 8	L, 29, 13
124 021	L, 25, 13	L, 1, 14	L, 14, 13	M, 25, 1
132 157	L, 31, 14	M, 3, 11	L, 11, 14	L, 31, 3
135 301	L, 5, 9	L, 13, 22	L, 22, 9	M, 5, 13
138 577	L, 31, 9	L, 13, 4	M, 4, 9	L, 31, 13
149 113	L, 23, 13	L, 3, 16	L, 16, 13	M, 23, 3
155 809	L, 13, 11	L, 9, 20	L, 20, 11	M, 13, 9
165 877	L, 23, 2	L, 19, 17	M, 17, 2	L, 23, 19
169 129	M, 13, 4	L, 21, 25	L, 25, 4	L, 13, 21
177 109	L, 37, 14	M, 9, 5	L, 5, 14	L, 37, 9
178 021	L, 7, 10	L, 13, 23	L, 23, 10	M, 7, 13
178 693	M, 19, 1	L, 21, 22	L, 22, 1	L, 19, 21
186 841	L, 37, 15	M, 7, 8	L, 8, 15	L, 37, 7
188 941	L, 35, 11	L, 13, 2	M, 2, 11	L, 35, 13
189 517	L, 37, 13	M, 11, 2	L, 2 13	L, 37, 11
196 501	L, 25, 14	L, 3, 17	L, 17, 14	M, 25, 3
223 549	L, 31, 7	L, 17, 10	M, 10, 7	L, 31, 17
235 069	L, 23, 14	L, 5, 19	L, 19, 14	M, 23, 5
238 213	M, 17, 3	L, 23, 26	L, 26, 3	L, 17, 23
244 669	M, 19, 2	L, 23, 25	L, 25, 2	L, 19, 23
246 217	L, 13, 12	L, 11, 23	L, 23, 12	M, 13, 11
263 077	L, 17, 13	L, 9, 22	L, 22, 13	M, 17, 9
263 953	M, 1, 9	L, 19, 28	L, 28, 9	L, 1, 19
268 993	L, 41, 16	M, 9, 7	L, 7, 16	L, 41, 9
273 157	M, 11, 6	L, 23, 29	L, 29, 6	L, 11, 23
278 149	L, 7, 11	L, 15, 26	L, 26, 11	M, 7, 15
294 649	L, 11, 12	L, 13, 25	L, 25, 12	M, 11, 13
307 261	L, 25, 1	L, 23, 22	M, 22, 1	L, 25, 23
316 201	L, 35, 17	M, 1, 16	L, 16, 17	L, 35, 1
325 009	L, 43, 16	M, 11, 5	L, 5, 16	L, 43, 11
333 049	M, 17, 4	L, 25, 29	L, 29, 4	L, 17, 25
348 949	L, 41, 13	L, 15, 2	M, 2, 13	L, 41, 15
355 261	L, 23, 15	L, 7, 22	L, 22, 15	M, 23, 7
356 989	L, 41, 18	M, 5, 13	L, 13, 18	L, 41, 5
364 909	M, 13, 6	L, 25, 31	L, 31, 6	L, 13, 25
398 557	L, 37, 18	M, 1, 17	L, 17, 18	L, 37, 1
400 069	L, 37, 9	L, 19, 10	M, 10, 9	L, 37, 19
411 241	M, 5, 9	L, 23, 32	L, 32, 9	L, 5, 23
423 097	L, 31, 4	L, 23, 19	M, 19, 4	L, 31, 23
443 629	L, 43, 19	M, 5, 14	L, 14, 19	L, 43, 5
445 141	L, 35, 18	L, 1, 19	L, 19, 18	M, 35, 1
451 897	M, 19, 4	L, 27, 31	L, 31, 4	L, 19, 27
459 961	M, 17, 5	L, 27, 32	L, 32, 5	L, 17, 27
469 153	L, 43, 13	L, 17, 4	M, 4, 13	L, 43, 17
489 061	L, 19, 15	L, 11, 26	L, 26, 15	M, 19, 11
500 713	L, 41, 11	L, 19, 8	M, 8, 11	L, 41, 19
512 269	M, 7, 9	L, 25, 34	L, 34, 9	L, 7, 25
517 249	L, 23, 16	L, 9, 25	L, 25, 16	M, 23, 9
520 609	L, 47, 16	M, 15, 1	L, 1, 16	L, 47, 15
558 457	L, 43, 12	L, 19, 7	M, 7, 12	L, 43, 19
575 077	L, 29, 1	L, 27, 26	M, 26, 1	L, 29, 27
583 753	L, 37, 7	L, 23, 16	M, 16, 7	L, 37, 23
594 829	L, 49, 17	M, 15, 2	L, 2, 17	L, 49, 15
630 901	M, 25, 2	L, 29, 31	L, 31, 2	L, 25, 29
668 509	L, 29, 18	L, 7, 25	L, 25, 18	M, 29, 7
715 777	M, 13, 8	L, 29, 37	L, 37, 8	L, 13, 29
723 181	M, 7, 10	L, 27, 37	L, 37, 10	L, 7, 27
729 457	L, 23, 17	L, 11, 28	L, 28, 17	M, 23, 11
746 041	L, 53, 20	M, 13, 7	L, 7, 20	L, 53, 13
753 001	L, 35, 4	L, 27, 23	M, 23, 4	L, 35, 27
756 601	L, 37, 20	L, 3, 23	L, 23, 20	M, 37, 3
760 993	L, 31, 1	L, 29, 28	M, 28, 1	L, 31, 29
805 573	M, 19, 6	L, 31, 37	L, 37, 6	L, 19, 31
808 837	L, 31, 19	L, 7, 26	L, 26, 19	M, 31, 7
812 137	L, 47, 13	L, 21, 8	M, 8, 13	L, 47, 21
826 621	L, 37, 5	L, 27, 22	M, 22, 5	L, 37, 27
845 809	L, 41, 8	L, 25, 17	M, 17, 8	L, 25, 41
864 361	L, 55, 21	M, 13, 8	L, 8, 21	L, 55, 13
870 901	L, 11, 15	L, 19, 34	L, 34, 15	M, 11, 19
922 513	L, 29, 19	L, 9, 28	L, 28, 19	M, 29, 9
951 061	L, 55, 19	M, 17, 2	L, 2, 19	L, 55, 17
952 873	L, 47, 12	L, 23, 11	M, 11, 12	L, 47, 23
970 969	M, 13, 9	L, 31, 40	L, 40, 9	L, 13, 31
978 541	L, 55, 23	M, 9, 14	L, 14, 23	L, 55, 9
994 249	M, 7, 11	L, 29, 40	L, 40, 11	L, 7, 29

☞ *This Table gives all primes* L, M *of* S′, S″, T′, T″ *up to* M $\not>$ 9.10[6].

Product Cubic Forms, $\mathbf{N} = N_1 N_2 N_3 \dots N_r$.

Two-factor forms, $\mathbf{N} = X^3 + Y^3 = N_1 . N_2$; $N_1 = x_1^3 + y_1^3$, $N_2 = x_2^3 + y_2^3$.

$Y = \frac{1}{3}(N_1 - 1) = -y_2$, $x_2 + y_2 = 1$, $X - 2Y = 1$.

x_1, y_1	x_2, y_2	X, Y	x_1, y_1	x_2, y_2	X, Y
4, 0	22, −21	43, 21			
7, 0	115, −114	229, 114			
2, −1	3, −2	5, 2	3, 1	10, −9	19, 9
5, −1	42, −41	83, 41	6, 1	73, −72	145, 72
3, −2	7, −6	13, 6	2, 2	6, −5	11, 5
6, −2	70, −69	139, 69	5, 2	45, −44	89, 44
4, −3	13, −12	25, 12	4, 3	31, −30	61, 30
7, −3	106, −105	211, 105	7, 3	124, −123	247, 123
5, −4	21, −20	41, 20	6, 4	94, −93	187, 93
8, −4	150, −149	299, 149	9, 4	265, −264	529, 264
6, −5	31, −30	61, 30	5, 5	84, −83	167, 83
9, −5	202, −201	403, 201	8, 5	213, −212	425, 212
12, −5	535, −534	1069, 534	11, 5	486, −485	971, 485
7, −6	43, −42	85, 42	7, 6	187, −186	373, 186
10, −6	262, −261	523, 261	10, 6	406, −405	811, 405
13, −6	661, −660	1321, 660	13, 6	805, −804	1609, 804
8, −7	57, −56	113, 56	9, 7	358, −357	715, 357
9, −8	73, −72	145, 72	8, 8	342, −341	683, 341
10, −9	91, −90	181, 90	10, 9	577, −576	1153, 576
			19, 9	2530, −2529	5059, 2529
			25, 12	5785, −5784	11569, 5784
			89, 44	263385, −263384	526769, 263384

Three-factor forms, $\mathbf{N} = X^3 + Y^3 = N_1 N_2 N_3$; $N_r = x_r^3 + y_r^3$.

From above form, $L = N_1 N_2$, $\mathbf{N} = L N_3 = N_1 N_2 N_3$.

x_1, y_1	x_2, y_2	x_3, y_3	X, Y
2, −1	3, −2	45, −44	89, 44
2, 2	6, −5	486, −485	971, 485
3, −2	7, −6	805, −804	1609, 804
3, 1	10, −9	2530, −2529	5059, 2529
4, −3	13, −12	5785, −5784	11069, 5784

Four-factor forms, $\mathbf{N} = X^3 + Y^3 = N_1 N_2 N_3 N_4$; $N_r = x_r^3 + y_r^3$.

From above form, $L_1 = N_1 N_2$, $L_2 = L_1 N_3$, $\mathbf{N} = L_2 N_4 = N_1 N_2 N_3 N_4$.

x_1, y_1	x_2, y_2	x_3, y_3	x_4, y_4	X, Y
2, −1	3, −2	45, −44	263385, −263384	526769, 263384

Quarto and Half-Quarto Cubans, $[\mathrm{N} = \mathrm{N}_{iii} = \mathrm{N}_{iv} \text{ or } \tfrac{1}{2}\mathrm{N}_{iv}]$.

$\mathrm{N}_1 = \mathrm{N}_{iv} = \mathrm{N}_{iii}$; $\mathrm{N}_2 = \tfrac{1}{2}\mathrm{N}_{iv} = \mathrm{N}_{iii}$;

$\mathrm{N}_{iv} = x^4 + y^4$, $\mathrm{N}_{iii} = (x'^3 \sim y'^3) \div (x' \sim y')$.

	N_1	x, y	x', y'	N_2	x, y	x', y'
Proper.	97	3, 2	8, 3	313	5, 1	16, 3
	337	3, 4	13, 8	1201	7, 1	21, 19
	1297	1, 6	32, 7	7321	11, 1	71, 24
	3697	7, 6	57, 7	8521	11, 7	80, 21
	4177	3, 8	53, 19			
	6577	9, 2	56, 37			
	73·409	13, 6	{112, 87	73·2017	23, 11	{376, 15
			{143, 49			{321, 104
Improper.	17^2·673	21, 2	17.21, 17.8	17^2·1249	29, 11	17.27, 17.13
	7^4·97	21, 14	{477, 11	7^4·313	35, 7	{789, 139
			{293, 264			{576, 421

Duo-Cubics, $\mathrm{N} = \mathrm{N}_3 = \mathrm{N}_2$.

$\mathrm{N} = x^3 + y^3 = \mathrm{L.M}$; $\mathrm{N}_2 = t^2 + u^2$; $\mathrm{L} = x + y = l^2$ or $(\alpha^2 + \beta^2)$; $\mathrm{M} = x^2 - xy + y^2 = a^2 + b^2$.

		$\mathrm{L}' = \xi^2 - 2\eta^2 = x + y$			$\mathrm{L}'' = 2\eta^2 - \xi^2 = x + y$		
L = 1	ξ, η	3, 2	17, 12	99, 70	. .	7, 5	41, 29
	x, y	5, $\overline{4}$	145, $\overline{144}$	4901, $\overline{4900}$	. .	$\overline{24}$, 25	$\overline{840}$, 841
	t, u	5, 6	145, 204	4901, 6930	. .	24, 35	840, 1189
	N	61	62641	72044701	. .	1801	2119321
L = 7^2	ξ, η	9, 4	11, 6	43, 30	1, 5	17, 13	23, 17
	x, y	65, $\overline{16}$	85, $\overline{36}$	949, $\overline{900}$	24, 25	$\overline{120}$, 169	$\overline{240}$, 289
	t, u	7.65, 7.36	7.85, 7.66	7.949, 7.1290	7.24, 7.5	7.120, 7.221	7.240, 7.391
	N	49·5521	49·11581	49·3564701	49·601	49·63241	49·210481
L = 17	ξ, η	5, 2	7, 4	23, 16	1, 3	9, 7	15, 11
	x, y	21, $\overline{4}$	33, $\overline{16}$	273, $\overline{256}$	8, 9	$\overline{32}$, 49	$\overline{104}$, 121
	t, u	{19, 94	79, 160	1199, 1460	29, 20	65, 284	251, 764
		{61, 74	145, 104	1745, 724	35, 4	191, 222	581, 556
	N	17·541	17·1873	17·209953	17·73	17·4993	17·38041

L	$\mathrm{L}' = x + y = \xi^2 - 2\eta^2$		$\mathrm{L}'' = x + y = 2\eta^2 - \xi^2$	
	x, y;	t, u	x, y;	t, u
1	$\xi^2 - \eta^2,\ \eta^2$;	$x,\ \xi\eta$	$\eta^2 - \xi^2,\ \eta^2$;	$x,\ \xi\eta$
l^2	$\xi^2 - \eta^2,\ \eta^2$;	$lx,\ l\xi\eta$	$\eta^2 - \xi^2,\ \eta^2$;	$lx,\ l\xi\eta$
$\alpha^2 + \beta^2$	$\xi^2 - \eta^2,\ \eta^2$;	$\alpha x \mp \beta\xi\eta,\ \beta x \pm \alpha\xi\eta$	$\eta^2 - \xi^2,\ \eta^2$;	$\alpha x \mp \beta\xi\eta,\ \beta x \pm \alpha\xi\eta$

Duo-Cubics, $N = N_3 = N_2$.

$$N_3 = x^3 + y^3 = L.M\,; \quad N_2 = t^2 + u^2\,;$$

$$L = x + y = l^2 \text{ or } (\alpha^2 + \beta^2)\,; \quad M = x^2 + xy + y^2 = a^2 + b^2.$$

L'	$L' = \xi^2 - 3\eta^2 = x + y$: x, y	t	u
1	$\xi^2, -3\eta^2$	1	$3\xi\eta$
l^2	$\xi^2, -3\eta^2$	l	$3l\xi\eta$
$\alpha^2+\beta^2$	$\xi^2, -3\eta^2$	$\alpha L \mp 3\beta\xi\eta$	$\beta L \pm 3\alpha\xi\eta$

L'	$L' = 3\eta^2 - \xi^2 = x + y$: x, y	t	u
2	$-\xi^2, 3\eta^2$	2	$3\xi\eta$
l^2	$-\xi^2, 3\eta^2$	l	$3l\xi\eta$
$\alpha^2+\beta^2$	$-\xi^2, 3\eta^2$	$\alpha L \mp 3\beta\xi\eta$	$\beta L \pm 3\alpha\xi\eta$

$L' = 1$	ξ, η	2, 1	7, 4	26, 15
	x, y	4, $\bar{3}$	49, $\overline{48}$	676, $\overline{675}$
	t, u	1, 6	1, 84	1, 1170
	N	37	7057	1368901
$L' = 13$	ξ, η	4, 1	5, 2	11, 6
	x, y	16, $\bar{3}$	25, $\overline{12}$	121, $\overline{108}$
	t, u	15, 62 63, 10	21, 116 99, 64	357, 620 435, 568
	N	13.313	13.1069	13.39373
$L' = 11^2$	ξ, η		13, 4	14, 5
	x, y		169, $\overline{48}$	196, 75
	t, u		11.121, 11.156	11.121, 11.210
	N		121.38977	121.58741

$L' = 2$	ξ, η	1, 1	5, 3	19, 11
	x, y	$\bar{1}$, 3	$\overline{25}$, 27	$\overline{361}$, 363
	t, u	1, 5	43, 47	625, 629
	N	2.13	2.2029	2.393133
$L' = 26$	ξ, η	1, 3	7, 5	11, 7
	x, y	$\bar{1}$, 27	$\overline{49}$, 75	$\overline{121}$, 147
	t, u	19, 139 71, 121	499, 235 551, 25	101, 1181 361, 1129
	N	26.757	26.11701	26.54037

$$L'' = 2\xi^2 + 3\eta^2 = x + y = \alpha^2 + \beta^2$$

$$\left.\begin{array}{l} x = \xi^2 + 3\eta^2 \\ y = \xi^2 \end{array}\right\}; \quad \left.\begin{array}{l} a = \xi^2 - 3\eta^2 \\ b = 3\xi\eta \end{array}\right\}$$

$$t = \alpha a \mp \beta b, \quad u = \beta a \pm \alpha b$$

L''	ξ, η	x, y	a, b	t, u	N
5	1, 1	4, 1	2, 3	1, 8 7, 4	5.13
29	1, 3	28, 1	26, 9	7, 148 97, 112	29.757

$$L''' = \xi^2 + 6\eta^2 = x + y = \alpha^2 + \beta^2$$

$$\left.\begin{array}{l} x = \xi^2 \quad 3\eta^2 \\ y = 3\eta^2 \end{array}\right\}; \quad \left.\begin{array}{l} a = \xi^2 - 3\eta^2 \\ b = 3\xi\eta \end{array}\right\};$$

$$t = \alpha a \mp \beta b, \quad u = \beta a \pm \alpha b$$

L'''	ξ, η	x, y	a, b	t, u	N
10	2, 1	7, 3	1, 6	17, 9 19, 3	10.37
25	1, 2	13, 12	11, 6	9, 62 57, 26	25.157
49	5, 2	37, 12	13, 30	7.13, 7.30	49.1069

Four-factor Bin-Aurifeuillians, (N).

$$N = x^4 + 4y^4 = X^4 - x'^4 = PQRS.$$

$P = X - x'$, $Q = X + x'$, $R = r - s$, $S = r + s$, $s = x' + 2y$,
$x = \frac{1}{2}\eta^4 - \xi^4$, $y = \xi^3\eta$, $X = \frac{1}{2}\eta^4 + \xi^4$, $x' = \xi\eta^3$, $r = X + 2\xi^2\eta^2$.

$[\xi = 1.]$

ξ, η	x ,	y	X ,	x'	P ,	Q ,	R ,	S
1, 2	7,	2	9,	8	1;	17;	5;	29;
4	127,	4	129,	64	5.13;	193;	89;	233;
6	647,	6	649,	216	433;	5.173;	17.29;	13.73;
8	2047,	8	2049,	512	29.53;	13.197;	17.97;	5.541;
10	4999,	10	5001,	1000	4001;	17.353;	37.113;	6221;
12	10367,	12	10369,	1728	8641;	12097;	5.13.137;	12409;
14	19207,	14	19209,	2744	5.3293;	29.757;	22373;	13.1721;
16	32767,	16	32769,	4096	53.541;	5.73.101;	29153;	37409;
18	52487,	18	52489,	5832	13.37.97;	58321;	47269;	5.11801;
20	79999,	20	80001,	8000	89.809;	88001;	13.29.193;	73.1217;
22	117127,	22	117129,	10648	233.457;	13.9829;	5.21481;	29.4441;
24	165887,	24	165889,	13824	5.17.1789;	29.6197;	89.1721;	113.1601;
26	228487,	26	228489,	17576	210913;	5.29.1697;	137.1549;	17.14557;
28	307327,	28	307329,	21952	285377;	329281;	53.5413;	5.17.17.229;
30	404999,	30	405001,	27000	13.29077;	432001;	433.877;	433861;
32	524287,	32	524289,	32768	17.29.997;	557057;	5.89.1109;	13.43013;
34	668167,	34	668169,	39304	5.29.4337;	13.54421;	37.37.461;	709853;
36	839807,	36	839809,	46656	101.7853;	5.17.10429;	29.27437;	733.1213;
38	1042567,	38	1042569,	54872	987697;	1097441;	13.13.5861;	5.29.7589;
40	1279999,	40	1280001,	64000	509.2389;	229.5869;	17.71713;	13.37.2801;
42	1555847,	42	1555849,	74088	89.16649;	1629973;	5.17.101.173;	1633549;
44	1874047,	44	1874049,	85184	5.13.13.29.73;	17.115249;	101.17749;	1963193;
46	2238727,	46	2238729,	97336	373.5741;	5.467213;	13.165041;	677.3457;
48	2654207,	48	2654209,	110592	2543617;	13.212677;	7.233.1559;	5.7.79283;
50	3124999,	50	3125001,	125000	853.3517;	29.112069;	3004901;	53.61417;
52	3655807,	52	3655809,	140608	181.19421;	157.24181;	5.704101;	29.131101;
54	4251527,	54	4251529,	157464	5.818813;	4408993;	4099789;	653.6761;
56	4917247,	56	4917249,	175616	13.13.28057;	5.37.27529;	29.53.3089;	2089.2441;
58	5658247,	58	5658249,	195112	17.97.3313;	1297.4513;	5469749;	5.13.89.1013;
60	6479999,	60	6480001,	216000	6264001;	13.51577;	6271081;	17.29.13597;
62	7388167,	62	7388169,	238328	7149841;	293.26029;	5.193.7417;	17.449077;
1, 64	8388607,	64	8388609,	262144	5.349.4657;	8650753;	13.29.21577;	37.234029;

Four-factor Bin-Aurifeuillians, (N).

$$N = x^4 + 4y^4 = X^4 - x'^4 = PQRS.$$

$$P = X - x',\quad Q = X + x',\quad R = r - s,\quad S = r + s,\quad s = x' + 2y,$$
$$x = \eta^4 - 2\xi^4,\quad y = 2\xi^3\eta,\quad X = \eta^4 + 2\xi^4,\quad x' = 2\xi\eta^3,\quad r = X + 4\xi^2\eta^2.$$
$$[\xi = 1.]$$

ξ, η	x ,	y	X ,	x'	P ,	Q ,	R ,	S
1, 1	−1,	2	3,	2	1 ;	5 ;	1 ;	13 ;
3	79,	6	83,	54	29 ;	137 ;	53 ;	5·37 ;
5	623,	10	627,	250	13·29 ;	877 ;	457 ;	997 ;
7	2399,	14	2403,	686	17·101 ;	3089 ;	5·13·29 ;	3313 ;
9	6559,	18	6563,	1458	5·1021 ;	13·617 ;	5393 ;	17·17·29 ;
11	14639,	22	14643,	2662	11981 ;	5·3461 ;	12421 ;	17·1049 ;
13	28559,	26	28563,	4394	24169 ;	32957 ;	24793 ;	5·6737 ;
15	50623,	30	50627,	6750	17·29·89 ;	181·317 ;	97·461 ;	58337 ;
17	83519,	34	83523,	9826	13·5669 ;	277·337 ;	5·14957 ;	94573 ;
19	130319,	38	130323,	13718	5·23321 ;	17·37·229 ;	117973 ;	13·11197 ;
21	194479,	42	194483,	18522	175961 ;	5·13·29·113 ;	349·509 ;	214853 ;
23	279839,	46	279843,	24334	197·1297 ;	37·8221 ;	17·15749 ;	5·29·2113 ;
25	390623,	50	390627,	31250	359377 ;	101·4177 ;	13·17·1637 ;	53·8009 ;
27	531439,	54	531443,	39366	492077 ;	17·33577 ;	5·29·3413 ;	13·37·1193 ;
29	707279,	58	707283,	48778	5·131701 ;	73·10357 ;	277·2389 ;	797·953 ;
31	923519,	62	923523,	59582	13·66457 ;	5·353·557 ;	89·9749 ;	29·101·337 ;
33	1185919,	66	1185923,	71874	1114049 ;	137·9181 ;	13·86021 ;	5·252457 ;
35	1500623,	70	1500627,	85750	181·7817 ;	13·122029 ;	29·48953 ;	1591417 ;
37	1874159,	74	1874163,	101306	29·61133 ;	53·37273 ;	5·229·1553 ;	1981093 ;
39	2313439,	78	2313443,	118638	5·438961 ;	97·25073 ;	173·12721 ;	2438321 ;
41	2825759,	82	2825763,	137842	17·158113 ;	5·397·1493 ;	2694481 ;	1229·2417 ;
43	3418799,	86	3418803,	159014	13·250753 ;	29·123373 ;	157·20809 ;	5·17·42181 ;
45	4100623,	90	4100627,	182250	3918377 ;	53·80809 ;	3926297 ;	13·17·19417 ;
47	4879679,	94	4879683,	207646	4672037 ;	13·89·4397 ;	5·37·25301 ;	1409·3617 ;
49	5764799,	98	5764803,	235298	5·17·65053 ;	6000101 ;	29·190997 ;	6009901 ;
51	6765199,	102	6765203,	265302	37·175673 ;	5·1406101 ;	13·500777 ;	29·242797 ;
53	7890479,	106	7890483,	297754	7592729 ;	17·17·29·977 ;	73·104161 ;	5·13·101·1249 ;
1, 55	9150623,	110	9150627,	332750	37·238321 ;	29·349·937 ;	8829757 ;	1361·6977 ;

Four-factor Bin-Aurifeuillians; (N).

$$N = x^4 + 4y^4 = X^4 - x'^4 = PQRS.$$

$$P = X - x', \quad Q = X + x', \quad R = r - s, \quad S = r + s, \quad s = x' + 2y,$$
$$x = 2\eta^4 - \xi^4, \quad y = 2\xi\eta^3, \quad X = 2\eta^4 + \xi^4, \quad x' = 2\xi^3\eta, \quad r = X + 4\xi^2\eta^2.$$
$$[\xi = 1.]$$

ξ, η	x ,	y	X ,	x'	P ,	Q ,	R ,	S
1, 2	31,	16	33,	4	29 ;	37 ;	13 ;	5·17 ;
4	511,	128	513,	8	5·101 ;	521 ;	313 ;	29·29 ;
6	2591,	432	2593,	12	29·89 ;	5·521 ;	1861 ;	3613 ;
8	8191,	1024	8193,	16	13·17·37 ;	8209 ;	5·1277 ;	10513 ;
10	19999,	2000	20001,	20	13·29·53 ;	20021 ;	16381 ;	24421 ;
12	41471,	3456	41473,	24	181·229 ;	17·2441 ;	13·37·73 ;	5·97·101 ;
14	76831,	5488	76833,	28	5·15361 ;	101·761 ;	29·2297 ;	13·17·401 ;
16	131071,	8192	131073,	32	131041 ;	5·13·2017 ;	29·3989 ;	148513 ;
18	209951,	11664	209953,	36	209917 ;	13·29·557 ;	5·53·709 ;	234613 ;
20	319999,	16000	320001,	40	53·6039 ;	320041 ;	17·17033 ;	353641 ;
22	468511,	21296	468513,	44	17·17·1621 ;	468557 ;	427813 ;	5·89·1153 ;
24	663551,	27648	663553,	48	5·132701 ;	663601 ;	181·3373 ;	13·29·1913 ;
26	913951,	35152	913953,	52	709·1289 ;	5·17·10753 ;	37·89·257 ;	987013 ;
28	1229311,	43904	1229313,	56	1229257 ;	1229369 ;	5·13·17609 ;	193·6841 ;
30	1619999,	54000	1620001,	60	1619941 ;	677·2393 ;	1515541 ;	1229·1409 ;
32	2097151,	65536	2097153,	64	461·4549 ;	73·28729 ;	17·17·17·401 ;	5·446477 ;
34	2672671,	78608	2672673,	68	5·13·41117 ;	1613·1657 ;	29·113·769 ;	2834581 ;
36	3359231,	93312	3359233,	72	13·233·1109 ;	5·89·7549 ;	53·59957 ;	17·17·15817 ;
38	4170271,	109744	4170273,	76	1009·4133 ;	4170349 ;	5·13·60869 ;	4395613 ;
40	5119999,	128000	5120001,	80	29·176549 ;	89·57529 ;	101·48221 ;	13·414037 ;
42	6223391,	148176	6223393,	84	17·36677 ;	13·478729 ;	1553·3821 ;	5·29·45013 ;
44	7496191,	170368	7496193,	88	5·1499221 ;	13·576637 ;	7163113 ;	29·270509 ;
1, 46	8954911,	194672	8954913,	92	809·11069 ;	5·17·137·769 ;	1061·8081 ;	181·51673 ;

Four-factor Bin-Aurifeuillians, (N).

$$N = x^4 + 4y^4 = X^4 - x'^4 = PQRS.$$

$$P = X - x',\quad Q = X + x',\quad R = r - s,\quad S = r + s,\quad s = x' + 2y,$$
$$x = 2\eta^4 - \xi^4,\quad y = 2\xi\eta^3,\quad X = 2\eta^4 + \xi^4,\quad x' = 2\xi^3\eta,\quad r = X + 4\xi^2\eta^2.$$
$$[\xi = 1.]$$

ξ, η	x ,	y	X ,	x'	P ,	Q ,	R ,	S
1, 1	1,	2	3,	2	1 ;	5 ;	1 ;	13 ;
3	161,	54	163,	6	157 ;	13·13 ;	5·17 ;	313 ;
5	1249,	250	1251,	10	17·73 ;	13·97 ;	29·29 ;	1861 ;
7	4801,	686	4803,	14	4789 ;	4817 ;	3613 ;	5·1277 ;
9	13121,	1458	13123,	18	5·2621 ;	17·773 ;	10513 ;	16381 ;
11	29281,	2662	29283,	22	29·1009 ;	5·5861 ;	24421 ;	13·37·73 ;
13	57121,	4394	57123,	26	57097 ;	57149 ;	5·97·101 ;	29·2297 ;
15	101249,	6750	101251,	30	101221 ;	101281 ;	13·17·401 ;	29·3989 ;
17	167041,	9826	167043,	34	167009 ;	167077 ;	148513 ;	5·53·709 ;
19	260641,	13718	260643,	38	5·52121 ;	29·89·101 ;	234613 ;	17·17033 ;
21	388961,	18522	388963,	42	13·29917 ;	5·77801 ;	353641 ;	427813 ;
23	559681,	24334	559683,	46	13·43049 ;	29·19301 ;	5·89·1153 ;	181·3373 ;
25	781249,	31250	781251,	50	17·45953 ;	781301 ;	13·29·1913 ;	37·89·257 ;
27	1062881,	39366	1062883,	54	97·10957 ;	29·36653 ;	987013 ;	5·13·17609 ;
29	1414561,	48778	1414563,	58	5·101·2801 ;	13·17·37·173 ;	193·6841 ;	1515541 ;
31	1847041,	59582	1847043,	62	29·63689 ;	5·13·157·181 ;	1229·1409 ;	17·17·17·401 ;
33	2371841,	71874	2371843,	66	193·12289 ;	53·44753 ;	5·446477 ;	29·113·769 ;
35	3001249,	85750	3001251,	70	29·37·2797 ;	3001321 ;	2834581 ;	53·59957 ;
37	3748321,	101306	3748323,	74	1249·3001 ;	3748397 ;	17·208889 ;	5·13·60869 ;
39	4626881,	118638	4626883,	78	5·17·29·1877 ;	37·12553 ;	4395613 ;	101·48221 ;
41	5651521,	137842	5651523,	82	73·77417 ;	5·1130321 ;	13·13·31849 ;	1553·3821 ;
43	6837601,	159014	6837603,	86	113·60509 ;	17·53·7589 ;	5·29·45013 ;	7163113 ;
1, 45	8201249,	182250	8201251,	90	37·221653 ;	8201341 ;	29·270509 ;	1061·8081 ;

Product Quartans, $\mathbf{N}_{iv} = \Pi(N_{iv})$.

$$\mathbf{N} = (X, Y) = (X^4+Y^4);\quad \tfrac{1}{2}\mathbf{N} = \{X, Y\} = \tfrac{1}{2}\{X^4+Y^4\};\quad N_r = (x_r, y_r) = (x_r^4+y_r^4);\quad \tfrac{1}{2}N_r = \{x_r, y_r\} = \tfrac{1}{2}\{x_r^4+y_r^4\}.$$

$(X, Y) =$	$\mathbf{N}$	$=$	$(x_1, y_1).(x_2, y_2)$
$(13, 8) =$	17; 17.113;	$=$	$(1, 2).(5, 6)$

$(X, Y) =$	$\mathbf{N}$	$=$	$\{x_1, y_1\}.\{x_2, y_2\}$
$(15, 4) =$	41; 17.73;	$=$	$\{1, 3\}.\{7, 3\}$

$(X, Y) =$	$\mathbf{N}$	$=$	$(x_1, y_1).\{x_2, y_2\}$
$(23, 14) =$	97; 17.193;	$=$	$(3, 2).\{1, 9\}$
$(27, 22) =$	17; 73.617;	$=$	$(1, 2).\{17, 9\}$

$\{X, Y\} =$	$\tfrac{1}{2}\mathbf{N}$	$=$	$\{x_1, y_1\}.\{x_2, y_2\}$
$\{33, 17\} =$	41; 113.137;	$=$	$\{1, 3\}.\{13, 7\}$

$(X, Y) =$	$\mathbf{N}$	$=$	$\Pi(x_1, y_1).\Pi\{x_2, y_2\}$
$(13, 2) =$	17.41.41;	$=$	$(1, 2).\{1, 3\}.\{1, 3\}$
$(129, 2) =$	41.41.257.641;	$=$	$(1, 4).(5, 2).\{1, 3\}.\{1, 3\}$

$(X, Y) =$	$\mathbf{N}$	$=$	$\Pi(x_1, y_1).\Pi\{x_2, y_2\}$
$(37, 21) =$	17; 17; 17.449;	$=$	$(1, 2).(1, 2).\{11, 5\}$

Dimorph Quartan Products, $\Pi(N_{iv}) = \Pi(N'_{iv})$.

$$N_r = (x_r, y_r) = (x_r^4+y_r^4),\quad N'_r = (x'_r, y'_r) = (x_r'^4+y_r'^4);\quad \tfrac{1}{2}N_r = \{x_r, y_r\} = \tfrac{1}{2}(x_r^4+y_r^4),\quad \tfrac{1}{2}N'_r = \{x'_r, y'_r\} = \tfrac{1}{2}\{x_r'^4+y_r'^4\}.$$

$(x_1, y_1).(x_2, y_2) =$	$\mathbf{N}$	$=$	$(x'_1, y'_1).(x'_2, y'_2)$
$(3, 2).(15, 4) =$	17.41.73.97;	$=$	$(1, 2).(19, 20)$

$(x_1, y_1).(x_2, y_2) =$	$\mathbf{N}$	$=$	$(x'_1, y'_1).\{x'_2, y'_2\}$
$(1, 8).(5, 6) =$	17.17.113.241;	$=$	$(1, 2).\{31, 7\}$
$(3, 10).(7, 8) =$	17.73.89.593;	$=$	$(1, 2).\{47, 41\}$

$(x_1, y_1).\{x_2, y_2\} =$	N	$=$	$(x'_1, y'_1).\{x'_2, y'_2\}$
$(1, 2).\{23, 5\} =$	17.17.73.113;	$=$	$(5, 6).\{7, 3\}$
$(5, 12).\{7, 5\} =$	17.41.89.521;	$=$	$(1, 3).\{33, 25\}$
$(3, 4).\{57, 29\} =$	17.257.337.1289;	$=$	$(1, 4).\{61, 31\}$

$\{x_1, y_1\}.\{x_2, y_2\} =$	N	$=$	$(x'_1, y'_1).\{x'_2, y'_2\}$
$\{1, 3\}.\{1, 43\} =$	17.41.193.521;	$=$	$(5, 12).\{1, 9\}$

Various.	$\mathbf{N}$		Various.
$(1, 2).(1, 2).(1, 64) =$	17.17.97.257.673;	$=$	$(3, 2).(1, 4).(21, 2)$
$(1, 2).(1, 2).\{45, 19\} =$	17.17.97.113.193;	$=$	$(3, 2).(5, 6).\{1, 9\}$

Square Quartan Products, $\Pi(N_{iv}) = \square$.

$N_{iv} = (x, y) = (x^4+y^4)$, $\frac{1}{2}N_{iv} = \{x, y\} = \frac{1}{2}(x^4+y^4)$; $\Pi(N_{iv} \text{ and } \frac{1}{2}N_{iv}) = N^2$.

☞ *See also page* 129.

$\Pi(N_{iv}) = N^2$	$\Pi(N_{iv}.\frac{1}{2}N_{iv}) = N^2$
$(1, 2).(13, 2) = [17.41]^2$	$(1, 2).(5, 6).(13, 8) = [17.113]^2$
$(13, 8).(11, 32) = [17.97.113]^2$	$(1, 2).(31, 18).\{1, 9\} = [17.73.193]^2$
$(1, 2).(43, 38) = [17.569]^2$	$(1, 8).(13, 8).\{31, 7\} = [17.113.241]^2$
	$(13, 8).\{7, 3\}.\{23, 5\} = [17^2.73.113]^2$
	$(2, 5).(2, 129).(1, 4) = [41.257.641]^2$
	$(1, 2).(43, 18).\{7, 5\}.\{15, 13\} = [17^3.89.137]^2$
	$\{3, 5\}.\{7, 3\}.\{15, 13\}.\{87, 89\} = [17^2.73.137.353]^2$

$[N_1.N_2.N_3 \ldots N_r].[\Pi(N')]$	=	$[L_2.L_3.L_r.M_r]^2$
$[\{1, 3\}.(1, 14).\{1, 67\}.\{1, 321\}].[(5, 6).(1, 4)]$	=	$[41; 937; 10753; 17.113.257]^2$
$[(2, 1).(2, 13).(2, 129)].[(1, 4).(5, 2)]$	=	$[17 : 41^2; \quad 257.641]^2$

Polymorph Sum of three 4th-powers, $N = N' = N'' = \&c.$

$$N = x^4+y^4+z^4 = 2u^2,\ N' = x'^4+y'^4+z'^4 = 2u'^2,\ N'' = x''^4+y''^4+z''^4 = 2u''^2 = \&c.$$

$$u = A^2+3B^2,\ u' = A'^2+3B'^2,\ u'' = A''^2+3B''^2 = \&c.;\ u = u' = u'' = \&c. = N_{iii}.$$

$$x = B \sim A,\ y = B+A,\ z = 2B;\ \ x' = B' \sim A',\ y' = B'+A',\ z' = 2B';\ \ \&c.$$

$u = N_{iii}$	A, B	x, y, z	$u = N_{iii}$	A, B	x, y, z
$91 = 7.13$	8, 3	5, 11, 6	$1729 = 7.13.19$	41, 4	37, 45, 8
	4, 5	1, 9, 10		31, 16	15, 47, 32
$133 = 7.19$	11, 2	9, 13, 4		23, 20	3, 43, 40
	5, 6	1, 11, 12		1, 24	23, 25, 48
$217 = 7.31$	13, 4	9, 17, 8	$2611 = 7.373$	44, 15	29, 59, 30
	5, 8	3, 13, 16		32, 23	9, 55, 46
$247 = 13.19$	10, 7	3, 17, 14	$53599 = 7.13.19.31$	46, 131	85, 177, 262
	2, 9	7, 11, 18		118, 115	3, 233, 230
$259 = 7.37$	16, 1	15, 17, 2		82, 125	43, 207, 250
	4, 9	5, 13, 18		206, 61	145, 267, 122
$301 = 7.43$	17, 2	15, 19, 4		134, 109	25, 243, 218
	1, 10	9, 11, 20		226, 29	197, 255, 58
				214, 51	163, 265, 102
				218, 45	173, 263, 90

Dimorph Sum of four or five 4-th powers.

$$\left.\begin{aligned} N_1 &= x_1^4+y_1^4+z_1^4 = x_1'^4+y_1'^4+z_1'^4 = N_1'\,; \\ N_2 &= x_2^4+y_2^4+z_2^4 = x_2'^4+y_2'^4+z_2'^4 = N_2'\,; \end{aligned}\right\}$$ For u_1, u_2 see Table at foot of previous page.

$\mathbf{N} = N_1+N_2 = \Sigma(x^4) = \Sigma(x'^4) = N_1'+N_2' = \mathbf{N}'$; [6 elements in $\mathbf{N}$, $\mathbf{N}'$].

Omit one equal element (v) in $\mathbf{N}$, $\mathbf{N}'$ leaves 5 elements in $\mathbf{N}$, $\mathbf{N}'$.
Omit two equal elements (v, w) in $\mathbf{N}$, $\mathbf{N}'$ leaves 4 elements in $\mathbf{N}$, $\mathbf{N}'$.

n	u_1, u_2	v, w	$x_1, y_1, z_1, x_2, y_2, z_2$	$x_1', y_1', z_1', x_2', y_2', z_2'$	s
5	91, 217	9	1, v, 10, 3, 13, 16	5, 11, 6, v, 17, 8	
	91, 247	11	5, v, 6, 3, 17, 14	1, 9, 10, 7, v, 18	45
	91, 259	5	v, 11, 6, 15, 17, 2	1, 9, 10, v, 13, 18	51
	133, 217	9	v, 13, 4, 3, 13, 16	1, 11, 12, v, 17, 8	49
	133, 247	11	1, v, 12, 3, 17, 14	9, 13, 4, 7, v, 18	
	133, 259	13	9, v, 4, 15, 17, 2	1, 11, 12, 5, v, 18	47
	217, 247	3	v, 13, 16, 7, 11, 18	9, 17, 8, v, 17, 14	65
	217, 301	9	v, 17, 8, 15, 19, 4	3, 13, 16, v, 11, 20	63
	247, 301	11	7, v, 18, 15, 19, 4	3, 17, 14, 9, v, 20	63
	259, 301	15	v, 17, 2, 9, 11, 20	5, 13, 18, v, 19, 4	59
4	217, 259	17, 13	9, v, 8, 5, w, 18	3, w, 16, 15, v, 2	
	247, 259	17, 18	3, v, 14, 5, 13, w	7, 11, w, 15, v, 2	35

s = Sum of elements in $\mathbf{N}$ or $\mathbf{N}'$.

Dimorph Sum of six or seven 4-th powers.

$$N_1 = a^4+b^4+c^4+d^4+e^4 = a'^4+b'^4+c'^4+d'^4+e'^4 = N_1'\,;$$

$$N_3 = x_3^4+y_3^4+z_3^4 = x_3'^4+y_3'^4+z_3'^4 = N_3'.$$

$\mathbf{N} = N_1+N_3 = N_1'+N_3' = \mathbf{N}'$ has 8 elements.

Omit one equal element (q) from $\mathbf{N}$, $\mathbf{N}'$, leaves 7 elements in $\mathbf{N}$, $\mathbf{N}'$.
Omit two equal elements (q, c) from $\mathbf{N}$, $\mathbf{N}'$, leaves 6 elements in $\mathbf{N}$, $\mathbf{N}'$.
For u_1, u_2, u_3 see Table above, and Table at foot of previous page.

n	u_1, u_2, u_3	q	a, b, c, d, e	x_3, y_3, z_3	a, b, c, d, e	x_3', y_3', z_3'
7	91, 217, 1729	3	1, 10, q, 13, 16	15, 47, 32	5, 11, 6, 17, 8	q, 43, 40
	” ” ”	3	” ” ” ” ”	23, 25, 48	” ” ” ” ”	” ” ”
	” ” ”	8	1, 10, 3, 13, 16	37, 45, q	5, 11, 6, 17, q	15, 47, 32
	” ” ”	8	” ” ” ” ”	” ” ”	” ” ” ” ”	23, 25, 48
	91, 247, 1729	3	5, 6, q, 17, 14	37, 45, 8	1, 9, 10, 7, 18	q, 43, 40
	” ” ”	3	” ” ” ” ”	15, 47, 32	” ” ” ” ”	” ” ”
	” ” ”	3	” ” ” ” ”	23, 25, 48	” ” ” ” ”	” ” ”
	91, 259, 1729	15	11, 6, q, 17, 2	37, 45, 8	1, 9, 10, 13, 18	q, 47, 32
	” ” ”	”	” ” ” ” ”	3, 43, 40	” ” ” ” ”	” ” ”
	” ” ”	”	” ” ” ” ”	23, 25, 48	” ” ” ” ”	” ” ”
6	217, 259, 1729	8	9, q, 5, 18	23, 25, 48	3, 16, 15, 2	37, 45, q
	” ” ”	15	9, 8, 5, 18	q, 47, 32	3, 16, q, 2	23, 25, 48
	” ” ”	3	” ” ” ”	q, 43, 40	q, 6, 15, 2	” ” ”
	247, 259, 1729	15	3, 14, 5, 13	q, 47, 32	7, 11, q, 2	37, 45, 8
	” ” ”	”	” ” ” ”	” ” ”	” ” ” ”	23, 25, 48
	” ” ”	3	q, 14, 5, 13	37, 45, 8	7, 11, 15, 2	q, 43, 40
	” ” ”	”	” ” ” ”	23, 25, 48	” ” ” ”	” ” ”

N_1, N_1' come from Table above;
N_3, N_3' come from Table at foot of previous page.

Product-Sextans, $\mathbf{N} = \Pi(N_r)$.

$$\mathbf{N} = (X^6 + Y^6) \div (X^2 + Y^2) \;; \quad N_r = (x_r^6 + y_r^6) \div (x_r^2 + y_r^2).$$

N	N_1	N_2	**N**	N_1	N_2	N_3	N_4	N_5
X, Y	x_1, y_1	x_2, y_2	X, Y	x_1, y_1	x_2, y_2	x_3, y_3	x_4, y_4	x_5, y_5
9, 2	5, 4	1, 2	23, 4	3, 2	3, 2	1, 3	—	—
9, 2	5, 3	1, 2	49, 8	1, 6	3, 2	3, 1	—	—
17, 5	3, 2	1, 6	7, 36	7, 3	1, 2	3, 2	—	—
49, 8	17, 5	3, 1	25, 54	5, 3	3, 2	1, 4	—	—
11, 14	5, 4	3, 2	21, 10	1, 2	1, 2	1, 2	1, 3	—
11, 14	5, 3	3, 2	39, 52	1, 2	1, 2	1, 2	1, 2	3, 4

Dimorph Sextan Products, $N_1N_2N_3 = N_1'N_2'N_3'$.

$$N_r = (x_r^6 + y_r^6) \div (x_r^2 + y_r^2) \;; \quad N_r' = (x_r'^6 + y_r'^6) \div (x_r'^2 + y_r'^2).$$

N_1	N_2	N_3	N_1'	N_2'	N_3'
x_1, y_1	x_2, y_2	x_3, y_3	x_1', y_1'	x_2', y_2'	x_3', y_3'
1, 2	25, 6	—	7, 1	7, 3	—
17, 8	1, 3	—	25, 2	1, 2	—
17, 15	1, 3	—	25, 2	1, 2	—
29, 10	1, 2	—	1, 8	7, 3	—
29, 14	1, 2	—	19, 16	1, 3	—
11, 40	1, 2	—	21, 10	3, 4	—
29, 1	1, 2	—	21, 17	3, 2	—
29, 21	1, 2	—	9, 5	6, 1	—
27, 8	1, 2	1, 2	7, 1	5, 4	3, 1
27, 8	1, 2	1, 2	7, 1	5, 3	3, 1
47, 5	1, 2	1, 2	1, 3	7, 3	9, 8
41, 23	5, 3	1, 2	21, 17	1, 6	3, 1
41, 23	5, 4	1, 2	21, 17	1, 6	3, 1

Dimorph Sums and Differences of N_{iv} *and* N_{vi}.

$$\left.\begin{array}{l} Q(x, x') = x^4 + x'^4, \quad S(x, x') = x^4 - x^2x'^2 + x'^4 \\ X = x + x', \quad X' = x \sim x'\,; \quad Y = y + y', \quad Y' = y \sim y' \end{array}\right\} \begin{array}{c} \text{Take} \\ xx' = yy' = Z. \end{array}$$

Dimorph Differences.

$$\begin{array}{lll} 1^\circ & Q(x, x') - S(x, x') & = Z^2 = y^2y'^2 = Q(y, y') - S(y, y'). \\ 2^\circ & Q(X, X') - 2Q(x, x') & = 12Z^2 \qquad = Q(Y, Y') - 2Q(y, y'). \\ 3^\circ & Q(X, X') - 2S(x, x') & = 14Z^2 \qquad = Q(Y, Y') - 2S(y, y'). \\ 4^\circ & S(X, X') - Q(x, x') & = 14Z^2 \qquad = S(Y, Y') - Q(y, y'). \\ 5^\circ & S(X, X') - S(x, x') & = 15Z^2 \qquad = S(Y, Y') - S(y, y'). \\ 6^\circ & 2S(X, X') - Q(X, X') & = 16Z^2 \qquad = 2S(Y, Y') - Q(Y, Y'). \end{array}$$

Dimorph Sums, (from above).

$$\begin{array}{lll} 1^\circ & Q(x, x') + S(y, y') & = Q(y, y') + S(x, x'). \\ 2^\circ & Q(X, X') + 2Q(y, y') & = Q(Y, Y') + 2Q(x, x'). \\ 3^\circ & Q(X, X') + 2S(y, y') & = Q(Y, Y') + 2S(x, x'). \\ 4^\circ & Q(x, x') + S(y, y') & = Q(y, y') + S(X, X'). \\ 5^\circ & S(X, X') + S(y, y') & = S(Y, Y') + S(x, x'). \\ 6^\circ & Q(X, X') + 2S(Y, Y') & = Q(Y, Y') + 2S(X, X'). \end{array}$$

Dimorph Sums and Differences of N_{viii}, N_{xii}; N_{xvi}, N_{xxiv}, *&c.*

$$\left.\begin{array}{ll} \text{i.} & N_8(x, x') = x^8 + x'^8\,; \quad N_{12}(x, x') = x^8 - x^4x'^4 + x'^8 \\ \text{ii.} & N_{16}(x, x') = x^{16} + x'^{16}\,; \quad N_{24}(x, x') = x^{16} - x^8x'^8 + x'^{16} \end{array}\right\} \begin{array}{c} \text{Take} \\ xx' = yy' = Z. \end{array}$$

$$\left.\begin{array}{ll} \text{iii.} & N_8(x, x') - N_{12}(x, x') = Z^4 = N_8(y, y') - N_{12}(y, y') \\ \text{iv.} & N_8(x, x') + N_{12}(y, y') \qquad = N_8(y, y') + N_{12}(x, x') \end{array}\right\}.$$

$$\left.\begin{array}{ll} \text{v.} & N_{16}(x, x') - N_{24}(x, x') = Z^8 = N_{16}(y, y') - N_{24}(y, y') \\ \text{vi.} & N_{16}(x, x') + N_{24}(y, y') \qquad = N_{16}(y, y') + N_{24}(x, x') \end{array}\right\}.$$

Number of Odd Primes ($p > 1$) of Various Forms.

[$p > 1$, but < the limit stated.]

Limit.	$2\varpi+1$	$4\varpi+1$	$6\varpi+1$	$8\varpi+1$	$12\varpi+1$	$16\varpi+1$	$24\varpi+1$	$32\varpi+1$	$48\varpi+1$
10^3	167,	80,	80,	37,	36,	19,	14,	10,	8
10^4	1228,	609,	611,	295,	300,	144,	143,	73,	69
10^5	9591,	4783,	4784,	2384,	2374,	1188,	1181,	—,	—

Limit.	$8\varpi+1$	$8\varpi+3$	$8\varpi+5$	$8\varpi+7$	$12\varpi+1$	$12\varpi+5$	$12\varpi+7$	$12\varpi+11$
10^3	37,	44,	43,	43	36,	44,	44,	42
10^4	295,	311,	314,	308	300,	309,	311,	307
10^5	2384,	2409,	2399,	2399	2374,	2409,	2410,	2397

Limit.	y^2+1	$\frac{1}{2}(y^2+1)$	y^4+1	$\frac{1}{2}(y^4+1)$	y^8+1	$\frac{1}{2}(y^8+1)$	$\frac{y^3-1}{y-1}$	$\frac{1}{3}\frac{y^3-1}{y-1}$	$\frac{y^6+1}{y^2+1}$	$\frac{y^{12}+1}{y^4+1}$
10^3	9,	11,	2,	3,	1,	0,	15,	12,	4,	1
10^4	18,	27,	3,	5,	1,	0,	32,	29,	6,	2
10^5	50,	69,	4,	8,	2,	0,	76,	65,	9,	2
10^6	111,	176,	7,	11,	2,	0,	189,	173,	13,	3
10^7	315,	462,	12,	15,	2,	0,	520,	438,	21,	4
10^8	840,	1226,	17,	22,	2,	1,	1410,	—,	29,	5
$2\frac{1}{4}.10^8$	1199,	—,	—,	—,	—,	—,	1992,	—,	—,	—
10^9	—,	—,	26,	31,	2,	1,	—,	—,	4,	6

Limit.	x^2+y^2	x^4+y^4	$\frac{1}{2}(x^4+y^4)$	x^8+y^8	$\frac{1}{2}(x^8+y^8)$	$\frac{x^3-y^3}{x-y}$	$\frac{x^6+y^6}{x^2+y^2}$	$\frac{x^{12}+y^{12}}{x^4+y^4}$
10^3	80,	6,	4,	1,	0,	80,	7,	1
10^4	609,	13,	9,	1,	0,	611,	20,	3
10^5	4783,	32,	28,	2,	0,	4784,	54,	4
10^6	—,	89,	72,	2,	1,	—,	133,	7
10^7	—,	240,	172,	3,	1,	—,	369,	11

References to Factorising Tables.

See Dr. C. G. Reuschle's Tafeln Complexer Primzahlen, &c., Berlin, 1875.

☞ Tables of the sets of Least Roots (y) of $F(y) \equiv 0 \pmod{p}$ for all primes $< 1{,}000$ will be found on the pages quoted of the above work.

F (y)	c	Page
y^2+c	− 2	444
	+ 2	445
	− 3	468
	+ 5	474
	− 7	488
	− 11	511
	+ 13	531
	+ 17	571
	− 19	592
	− 23	633
y^4+c^2	± 3	481
	± 5	502
	± 7	546
	11	627
y^8+c^4	± 3	518

F (y)	c	Page
y^2+y+c	− 1	3
	+ 2	7
	+ 3	10
	− 3	16
	− 4	20
	+ 5	27
	+ 6	31
	− 7	37
	+ 8	46
	− 9	55
	− 10	62
	+ 11	71
	+ 12	74
	− 13	81
	+ 15	84
	− 15	98
	+ 17	107
	+ 18	115
	− 18	129
	+ 20	139
	+ 21	144
	− 22	153
	− 24	170
	− 26	440
y^2-y+c	+ 4	196
	− 5	204
	− 8	211
	+ 9	224
	+ 10	237
	+ 13	258
	+ 14	271
	− 14	282
	− 17	319
	− 19	340
	+ 22	360
	− 23	394
	+ 24	409

F (y)	b , c	Page
y^4+by^2+c; $[bc>1]$	+ 1, + 4	558
	+ 2, + 4	480
	− 2, + 4	483
	+ 3, + 1	473
	− 3, + 4	487
	− 4, + 1	477
	+ 4, + 1	478
	− 4, + 2	447
	+ 4, + 2	448
	− 4, + 9	503
	+ 4, + 9	505
	+ 5, + 1	615
	− 5, + 5	472
	− 5, + 9	510
	− 5, + 25	556
	− 6, + 16	547
	+ 6, + 16	548
	+ 7, + 1	500
	+ 7, + 9	530
	− 7, + 16	557
	+ 7, + 49	618
	− 8, + 9	543
	+ 8, + 9	544
	+ 9, + 16	568
	− 9, + 25	591
	− 9, + 36	617
	− 10, + 36	626
	+ 10, + 36	628
	+ 11, + 25	616
	− 11, + 36	631
	− 12, + 25	624
	+ 12, + 25	625
	− 13, + 13	529
	− 17, + 68	569
	c , d	
y^3+y^2+cy+d	− 2, − 1	6
	− 4, + 1	15
	− 6, − 7	26
	− 10, − 8	45
	− 12, + 11	54
	− 14, + 8	69
	− 20, − 9	97
	− 22, + 5	105
	− 24, − 27	128
	− 26, + 41	138
	− 32, − 79	168

F (y)	c , d , e	Page
$y^4+y^3+cy^2+dy+e$	+ 2, − 4, + 3	13
	− 6, − 1, + 1	19
	+ 4, + 20, + 23	36
	+ 5, + 7, + 49	53
	− 15, + 18, − 4	61
	+ 7, − 43, + 47	80
	+ 8, + 42, + 117	96
	− 27, − 41, + 2	126
	− 33, + 39, + 8	152
	− 36, + 91, − 61	167
$y^4-y^3+cy^2+dy+e$	− 4, + 4, + 1	194
	+ 2, + 1, + 1	195
	− 1, − 2, + 4	202
	− 2, − 3, + 9	209
	− 9, + 9, + 11	221
	+ 5, + 2, + 4	222
	− 11, + 17, + 3	235
	+ 4, + 3, + 9	236
	+ 11, + 1, + 52	255
	+ 5, + 4, + 16	257
	− 14, + 14, + 31	268
	+ 8, + 3, + 9	269
	− 4, − 5, + 25	281
	− 7, − 6, + 36	339
	+ 11, − 101, + 171	350
	+ 11, − 69, + 171	351
	+ 21, − 21, + 101	352
	− 25, + 67, − 35	358
	+ 8, + 7, + 49	359

F (y)	c , d	Page
y^3-y^2+cy+d	− 30, + 64	385
	− 30, − 27	385

APPENDIX.

ERRATA IN WORKS CONSULTED.

L. Euler's *Commentationes Arithmeticae*, Petropol., 184 .

Page VOL. I.

104 ; line of 5^5 ; *read* 3^2 (*not* 3^3). Line of 7^{10} ; *read* 1123.293459 *for* $\int 7^{10}$.

105 ; *Add* 79 | $2^4.5$ | 79^2 | $3.7^2.43$ | 79^3 | $2^5.5.3121$ |.
line of 173^2 ; *read* 30103 *for* $\int n^2$.

367 ; *insert* $a = 1080$, $aa+1 = 1166401$ in Table
368 ; *omit* $a = 1234$ [as $aa+1 = 421.3617$]
,, $a = 1320$ gives $aa+1 = 1742401$ (*not* 1747401)
,, $a = 1434$ gives $aa+1 = 2056357$ (*not* 2056351) } in top Table.
,, middle Table, lines 2, 3 ; *interchange* 299 and 199.

369 ; top Table, line 3 ; *read* $a = 698$ (*not* 798).
,, middle Table, line 6 ; *omit* $a = 553$.

370 ; col. 2, $a = 82$; *omit* factor 193. Col. 4, $a = 193$; *omit* factor 3.

375 ; col. 4, $a = 1080$; here $aa+1$ = prime.

378 ; col. 3, $a = 1490$; *omit* factor 313.

475 ; last line ; *for* $\frac{8.150911}{169^2}$, *read* $\frac{8.89736}{169^2} = \frac{64.3739}{169^2}$.

476 ; Correct $x : y$, x, y, p, q, r, s, A, B, C, D to agree with above.
$x = 1014$, $y = 2739$; $p = 11014$, $q = 11217$, $r = 6642$, $s = 3739$;
$A = 12231$, $B = 2903$, $C = 10381$, $D = 10203$.

VOL. II.

495 ; line 4 ; *for* 91, *read* 92. Line 5 ; *insert* $m = 141$.

498 ; lines 1, 2, 3 ; *for* $(m, x, y) = (76, 3, 25)$ *read* (86, 3, 28).
,, line 4 ; *for* $m = 189, 182$; *read* 191, 132.
,, lines 5, 6 ; the values of x, y under $m = 4, 42, 106, 116$ are *incorrect*.
,, line 7 ; *for* $m = 102, 198$; *read* 104, 200.
,, line 12 ; *for* $y = 14.57$ *read* 14.58.

Dr. C. G. Reuschle's *Tafeln Complexer Primzahlen*, Berlin, 1875.

Page	*Line*	*For*	*Read*	*Page*	*Line*	*For*	*Read*
8	9	$\alpha^4 = 55$	$\alpha^4 = 85$	476	7 up	$\omega^{11} = -306$	$\omega^{11} = +295$
106	1	$\lambda = 43$	$\lambda = 67$	487	15	$\varpi^4 - 2\varpi^2 + 4$	$\varpi^4 - 3\varpi^2 + 4$
187	1	$\lambda = 89$	$\lambda = 49$	511	4	$40m - 5$	$44m - 5$
193	4 up	$p = 881$	$p = 811$	513	10	$\omega^{17} = -10$	$\omega^{17} = -11$
282	5 up	$\varpi^2 + \varpi - 14$	$\varpi^2 - \varpi - 14$	,,	13	$\omega^5, \omega^{11}, \omega^{19}$ / $\overline{174}, 57, \overline{154}$	$\omega^5, \omega^{11}, \omega^{19}$ / $163, 38, \overline{153}$
392	8 up	IV, 3	VI, 3				
446	9 up	$\omega = -43$	$\omega = -48$	546	8	$\varpi^4 - 49$	$\varpi^4 + 49$
450	10 up	$\omega = +35$	$\omega = +15$	621	1	$n = 68$	$n = 88$
461	12 up	$\omega^{12} = -275$	$\omega^{12} = +305$	628	1	$n = 76$	$n = 88$
,,	11 up	$\omega^{29} = -138$	$\omega^{12} = -38$	635	9 up	$\omega^{23} = -197$	$\omega^{23} = -196$

CORRIGENDA *in the present Work.*

Page.	*Line.*	*Col.*	*For.*	*Read.*
17	21	y, y	18679, 21030	18179, 21430
101	3		η_β	η^β
108	3 of top Table		4183	4813
112	9 up	5	1151	1181
115	$y = 838$	6	2470 1	247001
126	3 of Heading		$x, y \not> 11$	$\xi, \eta \not> 11$
128	4	3	2048	2047
130	17 and 23	1 and 6	*d* *d*	d d
130	9 up		$n + n' = \xi$	$n_0 + n_0' = \xi$
132, 133	Head-line		SIMPLE	*Omit* SIMPLE
136	4 up	4	34957	34457
141	2 up	4	461	401
144	3 up	$8\varpi - 1$	$p = 977,\ u_4 = 23,\ t_4 = 3$	*Transfer to* Col. $8\varpi + 1$
145	1st formula		$2(x^2 \pm xy + y\ )^2$	$2(x^2 \pm xy + y^2)^2$
145	2 of Heading		$(t_1^4 - u_1^2)$	$(t_1^4 - 2u_1^2)$
145	3 up of left Table	N	839	833
149	7 up		$Y = 3\eta^2$	$Y = y^2$
150	3 of Heading		$2_{3r} - 1$	$2^{3r} - 1$
150	15 up	6	7 . 38336223	3 . 7 . 12778741
150	14 up	6	7 . 38057583	3 . 7 . 12685861
151	10 up	5	419450801	419450881
155	12 up		$(C'^2 - 1)\, y^2$	$(C'^2 - 1)\, x^2$
157–160	2 of Heading		$\not<$	$<$
159	$y = 345$	2	17764857	17864857
159	$y = 422$	6	9122481	10122481
184	22 up	6	60373	4560373
194	$y = 2029$	L	:	*Omit* :
219	Heading	3	$y^{64} \qquad + 0$	$y^{64} + 1 \equiv 0$
221	4 of Heading		$\frac{1}{2}(\lambda' + l'),\ \frac{1}{2}(\lambda' - l')$	$\frac{1}{2}(\lambda' + 2l'),\ \frac{1}{2}(\lambda' - 2l')$
221	3 of top Table	y'	$\overline{199}$	$\overline{197}$
221	9 of top Table	$2l$	440271	440171
221	9 of top Table	x, y	$220136,\ \overline{220135}$	$220086,\ \overline{220085}$
228	11 up		$z = k^2$	$z = 1$
230	2	Side	$x^4 \sim K . y_r^4$	$x_r^4 \sim K . y_r^4$
230	$k = 68$	2, 3	$z = 43,\ y_1 = 172$	$z = 33,\ y_1 = 132$
230	$k = 34$	6	$x_1 = 81$	$x_1 = 1169$
230	$k = 77$	6	$x_1 = 7792$	$x_1 = 7793$
230	$k = 90$	6	$x_1 = 2488$	$x_1 = 3841$
230	$k = 97$	5	$z = 2$	$z = 4$
230	$k = 97$	6	$x = 89,\ y = 6,\ z = 7853,\ +$	$x = 178,\ y = 24,\ z = 31172,\ -$
232	i of Heading		$k = k^2,\ y = k$	$k = \kappa^2,\ y = \kappa$
232	iii of Heading		$k =$	$-k =$
233	1	k	*(blank)*	$k = 140$
261	5 of Heading		$t''u''$	$\frac{1}{2}(t''^2 - u''^2)$

SUPPLEMENT.

High Simple Quartan and Half-Quartan Primes (p).

Quartans, $p = x^4 + y^4$.

	p	$x,\ y$
	29 986 577	1, 74
B	40 960 001	1, 80
	45 212 177	1, 82
	59 969 537	1, 88
B	65 610 001	1, 90
Da	100 000 081	3, 100
	100 006 561	9, 100
	126 247 697	1, 106
	193 877 777	1, 118
	303 595 777	1, 132
	384 160 001	1, 140
	406 586 897	1, 142
	562 448 657	1, 154
	655 360 001	1, 160
	723 394 817	1, 164
	916 636 177	1, 174
	1 049 760 001	1, 180
	1 416 468 497	1, 194
	1 536 953 617	1, 198
	1 731 891 457	1, 204
	1 944 810 001	1, 210
	2 342 560 001	1, 220
	2 702 336 257	1, 228
	3 208 542 737	1, 238
	3 429 742 097	1, 242
	3 782 742 017	1, 248
	4 162 314 257	1, 254
	5 006 411 537	1, 266
	5 473 632 257	1, 272
	5 802 782 977	1, 276
	5 972 816 657	1, 278
	6 879 707 137	1, 288
	7 676 563 457	1, 296
	9 475 854 337	1, 312

Table complete up to $y = 312$, with $x = 1$.

Half-Quartans, $p = \frac{1}{2}(x^4 + y^4)$.

	p	$x,\ y$
B	12 708 841	1, 71
B	14 199 121	1, 73
BJ	21 523 361	1, 81
	56 275 441	1, 103
	60 775 313	1, 105
	81 523 681	1, 113
	87 450 313	1, 115
	100 266 961	1, 119
	107 182 721	9, 121
	138 461 441	1, 129
	273 990 641	1, 153
	370 600 313	1, 165
	407 865 361	1, 169
	427 518 041	1, 171
	784 119 601	1, 199
	849 090 841	1, 203
	883 050 313	1, 205
	1 984 563 001	1, 251
	2 249 930 281	1, 259
	2 541 060 761	1, 267
	2 859 570 313	1, 275
	4 558 310 681	1, 309
	4 798 962 481	1, 313
	5 049 019 561	1, 317
	6 148 185 161	1, 333
	6 448 958 881	1, 337
	6 603 418 121	1, 339
	7 763 701 441	1, 353
	8 681 534 681	1, 363

Table complete up to $y = 399$, with $x = 1$.

High Prime Factors (p) of Simple Quartans.

$p = (x^4 + y^4) \div f; \quad [x = 1, f > 1].$

	p	y	f		p	y	f
	10 088 489	934	241.313		40 514 561	162	17
	11 165 137	528	6961		44 669 593	736	6569
	11 505 017	942	89.769		44 711 201	548	2017
	11 966 641	252	337		49 916 473	378	409
	12 321 041	474	17.241		50 855 561	486	1097
	13 294 121	502	17.281		50 897 897	330	233
	13 374 089	336	953		51 244 313	572	2089
	14 394 409	362	1193		51 483 121	172	17
	14 579 681	970	41.1481		52 216 841	608	2617
	14 641 849	456	2953		57 734 881	676	3617
	15 120 673	766	22769		60 880 681	872	9497
L	15 790 321	128	17		63 798 737	668	3121
	16 673 401	674	12377		65 798 849	700	41.89
	16 782 449	326	673		73 805 233	680	2897
	16 898 729	684	12953		73 853 993	644	17.137
	17 137 129	514	4073		74 046 641	666	2657
	17 957 969	956	193.241		74 524 553	728	3769
	18 145 313	388	1249		75 297 473	724	41.89
	18 468 497	434	17.113		77 938 409	586	17.89
	19 050 289	750	17.977		78 374 441	816	5657
	20 260 553	402	1289		82 509 577	814	17.313
	20 361 377	458	2161		86 631 049	282	73
	20 905 193	726	97.137		94 106 561	422	337
	21 333 761	138	17		97 089 257	686	2281
	22 925 033	806	41.449		97 905 289	630	1609
	24 132 457	416	17.73		98 672 257	446	401
	24 290 249	398	1033	Lo,R	99 990 001	1000	73.137
	25 068 521	960	17.1993		113 607 841	324	97
	25 397 761	770	13841		113 947 529	302	73
	25 737 017	464	1801		118 821 361	212	17
	25 744 921	366	17.41		123 993 929	958	6793
	27 126 929	862	20353		126 041 329	908	5393
	27 475 081	372	17.41		126 431 801	976	7177
	29 497 513	544	2969		134 472 673	444	17.17
	31 142 473	756	17.617		137 123 009	534	593
	31 582 673	592	3889		141 456 017	722	17.113
	36 268 129	354	433		157 341 673	344	89
	36 269 593	950	17.1321		165 991 393	468	17.17
	39 818 929	392	593		194 213 177	564	521
	40 054 897	356	401		201 796 057	626	761

High Prime Factors (p) of Simple Quartans.

p	y	f	p	y	f
205 048 201	904	3257	1 505 882 353	400	17
206 063 593	368	89	1 598 288 641	406	17
238 275 601	994	17.241	1 644 737 441	632	97
241 632 361	740	17.73	1 705 386 889	594	73
273 148 633	890	2297	1 723 043 929	796	233
287 803 777	834	41.41	2 139 412 049	990	449
334 140 193	762	1009	2 398 657 561	560	41
361 562 353	280	17	3 173 244 601	812	137
389 961 553	830	1217	3 227 992 561	484	17
400 495 049	802	1033	3 421 541 657	612	41
431 830 177	470	113	3 441 994 489	708	73
463 504 289	636	353	3 497 620 921	832	137
463 891 201	298	17	3 557 705 897	618	41
535 609 489	496	113	3 703 804 177	972	241
563 676 649	602	233	4 143 394 217	642	41
599 786 777	396	41	4 840 780 177	860	113
606 454 393	482	89	4 914 907 561	670	41
611 416 873	870	937	5 132 694 433	840	97
613 350 137	460	73	5 183 822 921	918	137
613 775 969	718	433	5 461 442 801	552	17
670 464 121	744	457	5 657 883 817	694	41
707 646 281	558	137	6 677 102 713	878	89
714 666 481	332	17	6 946 100 401	906	97
775 275 233	688	17.17	7 286 326 409	854	73
788 278 297	424	41	7 704 575 369	866	73
825 799 841	532	97	7 828 865 521	604	17
937 534 777	836	521	8 144 612 353	610	17
1 150 215 097	754	281	8 360 352 001	614	17
1 175 716 081	376	17	8 691 962 353	620	17
1 293 339 569	822	353	8 908 046 729	898	73
1 366 540 169	562	73	9 121 014 697	782	41
1 493 089 193	884	409	9 746 165 761	638	17

High Prime Factors (p) *of Simple Half Quartans.*

$$p = \tfrac{1}{2}(x^4 + y^4) \div f; \quad [x = 1, \ y \text{ odd}, \ f > 1].$$

p	y	f
10 316 017	197	73
10 509 841	303	401
10 771 417	347	673
11 616 697	433	17.89
11 756 681	831	17.1193
12 084 217	215	89
12 452 641	797	17.953
12 602 857	915	27809
12 732 529	327	449
13 001 489	145	17
13 068 697	209	73
13 974 721	621	17.313
14 042 233	907	24097
14 314 841	993	33961
14 414 377	457	17.89
14 751 089	983	31649
14 896 841	631	17.313
15 290 753	151	17
15 499 417	599	4153
15 601 081	901	21121
16 230 041	191	41
17 137 793	385	641
17 522 137	483	1553
17 808 841	921	20201
19 007 873	439	977
20 253 553	391	577
23 019 641	511	1481
23 754 217	255	89
24 840 737	535	17.97
24 953 633	633	3217
27 093 617	803	7673
27 766 481	175	16273
28 271 569	425	577
D,B 29 423 041	625	2593
31 308 961	779	5881
33 510 401	295	113
33 621 673	813	73.89
34 040 569	279	89
37 529 113	189	17
39 606 769	767	17.257
39 785 017	369	233
40 124 537	755	4049
41 912 953	877	7057
B 42 521 761	243	41
42 526 489	195	17
43 026 433	585	1361
43 068 329	495	17.41
45 509 137	697	2593
45 721 937	663	2113
50 088 697	695	17.137
51 909 329	493	569
52 048 313	519	17.41
52 333 297	377	193
53 152 753	709	2377
53 203 889	955	7817
58 175 849	319	89
60 539 593	213	17
60 665 273	867	4657
65 886 001	943	17.353
66 062 657	793	41.73
68 530 937	469	353
69 593 033	903	17.281
87 748 937	429	193
93 621 401	851	2801
95 392 169	541	449
100 104 161	301	41
108 003 089	873	2689
116 490 961	885	2633
119 577 209	953	3449
128 307 953	257	17
131 579 017	581	433
135 447 881	375	73
183 377 633	281	17
186 643 993	825	17.73
210 907 993	291	17
219 192 097	737	673
233 726 369	937	17.97
240 110 729	887	1289
268 083 401	727	521
288 959 497	967	17.89

High Prime Factors (p) of Simple Half Quartans.

p	y	f	p	y	f
319 585 921	651	281	1 355 128 457	989	353
334 629 161	407	41	1 355 659 057	667	73
436 337 753	349	17	1 398 906 233	467	17
468 260 633	549	97	1 591 050 761	601	41
468 571 633	899	17.41	1 627 376 489	485	17
478 014 457	687	233	1 709 413 193	491	17
492 387 713	759	337	2 234 386 489	525	17
499 445 449	733	17.17	2 299 999 841	659	41
504 988 801	897	641	2 653 804 721	683	41
508 142 377	751	313	2 750 563 073	553	17
522 026 489	365	17	2 960 153 873	949	137
552 784 657	533	73	3 082 976 033	569	17
563 402 449	785	337	3 217 692 553	969	137
635 151 689	849	409	3 559 061 593	735	41
701 849 009	775	257	3 736 099 273	597	17
793 707 041	985	593	3 888 573 673	603	17
889 334 833	417	17	4 842 602 393	637	17
1 142 991 841	815	193	6 628 236 113	689	17
1 200 229 417	647	73	7 265 701 489	705	17
1 351 728 281	577	41	8 036 635 513	723	17

High Simple Sextan Primes.

$p = (x^6 + y^6) \div (x^2 + y^2)$, $[x = 1]$.

p	x, y	p	x, y	p	x, y
BC 13 842 121	1, 61	815 702 161	1, 169	4 640 402 521	1, 261
BC 14 772 493	1, 62	875 183 473	1, 172	4 784 281 393	1, 263
BC 17 846 401	1, 65	1 121 479 633	1, 183	5 236 041 961	1, 269
47 451 433	1, 83	1 171 316 401	1, 185	5 314 337 101	1, 270
71 630 833	1, 92	1 303 173 901	1, 190	5 473 558 273	1, 272
78 066 061	1, 94	1 416 430 861	1, 194	5 972 739 373	1, 278
96 049 801	1, 99	1 475 750 641	1, 196	7 170 787 081	1, 291
Lo 99 990 001	1, 100	1 536 914 413	1, 198	7 676 475 841	1, 296
116 975 041	1, 104	1 907 986 081	1, 209	7 992 449 401	1, 299
121 539 601	1, 105	2 517 580 801	1, 224	8 099 910 001	1, 300
141 146 281	1, 109	2 562 840 001	1, 225	8 318 078 413	1, 302
168 883 021	1, 114	2 750 006 041	1, 229	8 999 083 633	1, 308
193 863 853	1, 118	2 847 342 961	1, 231	9 354 855 121	1, 311
252 031 501	1, 126	3 262 751 521	1, 239	9 597 826 993	1, 313
294 482 761	1, 131	3 544 475 761	1, 244		
759 305 581	1, 166	4 362 404 353	1, 257	Lo 999 999 000 001	1. 1000
796 565 953	1, 168	4 569 692 401	1, 260		

Table complete up to $y = 319$.

High Prime Aurifeuillian Factors (L, M) *of Simple Sextans.*

Of Bin-Aurifeuillians, $[x = 1,\ y = 2\eta^2]$.

η	p	L, M	y
41	11 030 641	L	3 362
45	16 771 141	M	4 050
47	19 107 757	L	4 418
50	24 504 901	L	5 000
50	25 505 101	M	5 000
51	27 596 713	M	5 202
54	33 388 093	L	5 832
55	35 942 941	L	6 050
55	37 274 161	M	6 050
58	46 053 277	M	6 728
59	47 654 773	L	6 962
61	54 482 761	L	7 442
65	72 509 581	M	8 450
67	79 410 277	L	8 978
68	86 792 617	M	9 248
72	106 012 657	L	10 368
73	112 047 409	L	10 658
75	124 886 101	L	11 250
75	128 261 401	M	11 250
78	146 174 029	L	12 168
80	165 900 961	M	12 800
81	170 074 081	L	13 122
83	187 559 749	L	13 778
84	201 533 641	M	14 112
86	216 273 661	L	14 792
87	226 539 997	L	15 138
88	242 619 697	M	15 488
89	248 164 753	L	15 842
91	271 301 941	L	16 562
94	315 639 781	M	17 672
95	329 250 241	M	18 050
96	343 296 193	M	18 432
100	396 019 801	L	20 000
101	412 140 601	L	20 402
102	437 238 709	M	20 808
104	472 464 721	M	21 632
112	623 812 897	L	25 088
114	669 683 653	L	25 992
119	795 423 133	L	28 322
120	822 556 561	L	28 800
124	938 089 513	L	30 752

Table complete to $\eta = 128$.

Of Sext-Aurifeuillians, $[x = 1,\ y = 6\eta^2]$.

η	p	L, M	y
26	17 096 197	M	4 056
28	21 351 289	L	4 704
32	36 587 329	L	6 144
32	38 947 009	M	6 144
33	41 418 829	L	6 534
33	44 006 689	M	6 534
35	55 588 261	M	7 350
38	73 115 269	L	8 664
39	85 446 973	M	9 126
41	99 276 253	L	10 086
42	109 385 389	L	10 584
43	120 247 609	L	11 094
45	150 939 721	M	12 150
47	171 970 369	L	13 254
48	187 162 849	L	13 824
49	211 811 713	M	14 406
52	258 204 649	L	16 224
52	268 329 049	M	16 224
55	323 487 121	L	18 150
56	347 775 793	L	18 816
59	443 681 653	M	20 886
60	458 848 441	L	21 600
61	506 688 937	M	22 326
67	714 693 289	L	26 934
68	758 492 809	L	27 744
70	876 796 621	M	29 400
72	954 114 769	L	31 104
72	980 989 489	M	31 104

Table complete to $\eta = 75$.

High Prime Factors (p) *of Simple Sextans.*

$p = N_{vi} \div f,\ [f > 1];\quad N_{vi} = (x^6 + y^6) \div (x^2 + y^2),\ [x = 1].\quad p > 10^7.$

p	y	f	p	y	f	p	y	f
10 066 321	834	13.3697	21 889 297	945	36433	59 341 273	925	13.13.73
10 122 481	422	13.241	La 22 253 377	256	193	59 828 341	167	13
10 201 693	411	2797	22 745 929	249	13.13	61 380 769	286	109
10 452 577	775	34513	23 225 533	352	661	62 375 569	577	1777
10 566 001	547	37.229	23 352 181	132	13	64 571 173	787	13.457
10 613 929	817	13.3229	23 552 257	928	31489	65 510 521	694	3541
10 636 513	616	13537	24 263 377	603	5449	65 997 769	553	13.109
11 407 993	556	8377	24 362 557	957	34429	67 285 993	669	13.229
11 676 517	111	13	25 588 837	581	61.73	68 119 489	789	5689
11 691 709	999	13.6553	25 683 817	892	157.157	68 570 329	918	10357
12 242 017	526	13.13.37	25 949 893	330	457	73 550 329	514	13.73
12 480 913	592	13.757	26 650 453	391	877	73 806 277	176	13
12 590 113	440	13.229	27 425 833	444	13.109	75 631 273	230	37
12 951 793	460	3457	27 464 293	786	13.1069	79 076 209	506	829
13 090 873	237	241	27 692 173	627	5581	80 282 557	471	613
13 123 009	333	937	27 809 581	672	7333	80 375 089	759	4129
13 303 621	814	61.541	27 980 989	827	73.229	81 271 753	439	457
13 895 449	418	13.13.13	28 080 553	617	13.397	81 385 921	566	13.97
14 242 321	867	97.409	28 390 981	204	61	83 575 993	529	937
14 429 521	642	61.193	29 412 529	946	73.373	84 041 641	666	2341
15 593 593	891	13.3109	29 573 209	408	937	84 991 813	829	5557
15 651 913	569	37.181	30 939 253	264	157	86 327 149	535	13.73
15 688 837	898	181.229	30 955 129	414	13.73	87 746 821	355	181
15 712 849	624	9649	32 936 341	884	18541	88 168 837	184	13
15 998 977	504	37.109	33 113 413	212	61	90 865 189	941	8629
16 005 853	156	37	33 272 101	222	73	92 697 193	902	37.193
16 020 997	548	13.433	34 002 277	145	13	95 126 341	276	61
16 156 597	975	55933	34 208 509	312	277	96 113 137	357	13.13
16 253 437	247	229	36 197 893	970	37.661	100 712 509	709	13.193
16 410 829	881	36709	36 562 777	473	37.37	103 532 053	678	13.157
16 864 789	979	54469	37 230 241	994	13.2017	111 815 653	364	157
17 605 501	123	13	39 760 921	683	13.421	113 225 953	711	37.61
17 864 857	345	13.61	41 066 089	360	409	114 877 921	984	8161
18 185 077	124	13	41 465 521	720	6481	115 544 389	335	109
18 359 713	425	1777	42 383 773	199	37	125 553 877	201	13
18 462 421	680	37.313	43 707 541	342	313	125 685 421	937	6133
18 639 013	650	61.157	47 763 361	243	73	126 064 093	454	337
18 937 693	935	40357	47 936 641	158	13	128 071 201	202	13
19 199 821	310	13.37	48 006 457	462	13.73	132 892 369	930	13.433
19 284 073	714	13477	48 682 873	901	13537	133 114 357	570	13.61
19 323 793	824	23857	49 961 341	501	13.97	133 370 989	966	6529
19 459 717	741	15493	50 796 841	448	13.61	136 590 697	316	73
19 663 681	934	13.13.229	51 470 401	712	4993	140 908 081	497	433
19 806 337	195	73	51 605 161	784	7321	143 960 581	863	3853
20 254 777	950	40213	54 298 861	163	13	148 696 897	880	37.109
21 033 541	574	13.397	55 463 437	373	349	153 308 581	848	3373
21 153 361	878	13.2161	55 576 681	841	9001	156 248 161	806	37.73
21 710 461	498	2833	58 343 977	763	31.157	164 529 709	981	13.433

High Prime Factors (p) of Simple Sextans.

p	y	f	p	y	f	p	y	f
169 289 641	888	3673	438 517 393	818	1021	1 380 313 537	366	13
170 918 569	282	37	466 087 957	279	13	1 457 300 557	371	13
176 939 197	219	13	469 299 013	521	157	1 521 173 077	375	13
181 080 349	821	13.193	474 946 957	477	109	1 545 367 933	489	37
185 155 441	326	61	498 789 913	469	97	1 650 999 529	932	457
188 178 937	664	1033	505 557 721	550	181	1 761 994 177	662	109
192 191 221	819	2341	518 118 697	441	73	1 773 652 357	887	349
196 708 177	427	13.13	551 775 529	378	37	1 828 425 673	510	37
201 586 597	620	733	559 831 549	965	1549	1 871 932 921	608	73
204 010 321	334	61	561 377 497	996	1753	1 932 856 969	756	13.13
204 245 101	227	13	563 176 981	677	373	1 959 874 177	885	313
206 273 401	936	61.61	566 920 381	293	13	2 069 541 277	405	13
207 868 021	228	13	588 057 577	655	313	2 164 927 069	532	37
210 246 697	639	13.61	599 173 273	491	97	2 190 890 677	910	313
213 163 477	904	13.241	612 054 637	873	13.73	2 269 810 633	685	97
222 455 881	844	2281	635 399 977	790	613	2 289 164 253	971	397
224 176 669	476	229	640 468 813	649	277	2 315 185 813	541	37
238 265 017	546	373	644 711 869	393	37	2 462 723 701	423	13
240 050 257	538	349	665 658 277	305	13	2 599 272 937	660	73
248 433 613	394	97	684 994 573	399	37	2 654 381 797	431	13
252 828 109	976	37.97	698 838 577	846	733	2 715 379 189	563	37
261 931 693	519	277	708 806 101	456	61	2 754 437 029	826	13.13
262 259 653	980	3517	734 634 097	751	433	3 239 271 421	453	13
264 568 741	760	13.97	747 774 817	314	13	3 355 207 381	457	13
268 799 581	580	421	749 535 361	986	13.97	3 474 227 917	461	13
280 790 413	679	757	791 764 741	809	541	3 759 184 453	692	61
289 006 981	573	373	805 898 161	628	193	3 883 006 177	474	13
291 357 697	688	769	879 515 701	327	13	4 056 283 489	792	97
294 885 673	804	13.109	912 284 521	508	73	4 186 424 941	483	13
307 854 997	428	109	919 019 293	746	337	4 507 287 541	492	13
315 160 621	253	13	923 346 397	331	13	4 563 007 093	920	157
318 987 289	601	409	923 995 273	430	37	4 807 673 077	500	13
320 173 057	254	13	934 555 381	332	13	5 002 884 277	505	13
320 233 009	870	1789	955 452 061	802	433	5 044 594 417	779	73
335 575 897	488	13.13	1 014 206 161	947	13.61	5 270 469 493	753	61
342 009 721	572	313	1 049 940 313	843	13.37	5 538 269 281	518	13
355 383 913	643	13.37	1 059 328 573	736	277	5 560 975 249	857	97
367 714 813	387	61	1 078 748 269	705	229	5 585 253 061	764	61
375 702 673	652	13.37	1 128 105 949	452	37	5 933 316 901	527	13
382 395 061	897	1693	1 134 793 633	576	97	6 168 031 669	992	157
385 103 137	266	13	1 163 908 321	865	13.37	6 439 242 193	889	97
388 336 537	715	673	1 182 826 237	815	373	6 540 789 877	540	13
391 843 429	347	37	1 194 406 021	353	13	7 759 643 869	732	37
405 837 121	597	313	1 232 356 369	588	97	7 893 135 253	833	61
408 831 757	530	193	1 263 529 441	358	13	8 987 660 137	900	73
411 338 833	398	61	1 274 102 293	528	61	9 384 374 437	591	13
424 505 401	845	1201	1 347 186 937	560	73	9 963 805 853	883	61

MATHEMATICAL WORKS

By Lt.-Col. ALLAN J. C. CUNNINGHAM, R.E.

1, published by Taylor & Francis, Red Lion Court, Fleet St., London, E.C.4.

A Binary Canon ... ... Reduced price 5*s*., 1900,

showing Residues of Powers of 2 for divisors under 1,000, and Indices to Residues.

2 to 7, published by Francis Hodgson, 89 Farringdon St., London, E.C.4.

2. *Quadratic Partitions* ... ... Reduced price 7*s*. 6*d*., 1904,

giving the partitions $p = \mathrm{a}^2+\mathrm{b}^2$, $\mathrm{c}^2+2\mathrm{d}^2$, $\mathrm{A}^2+3\mathrm{B}^2$, $\frac{1}{4}(\mathrm{L}^2+27\mathrm{M}^2)$, up to $p \not> 100{,}000$, and $p = \mathrm{e}^2-2\mathrm{f}^2$ up to $p \not> 25{,}000$, and many others up to $p \not> 10{,}000$, &c.

3. *Haupt-Exponents, Residue-Indices, Primitive Roots, and Standard Congruences*, [Joint Authors, H. J. Woodall and T. G. Creak] ... ... price 10*s*., 1922,

giving Haupt-Exponents and Residue-Indices of 2, 3, 5, 6, 7, 10, 11, 12 for all primes up to $p \not> 25{,}049$, and the Least Primitive Roots of those primes; also solutions of the Congruences

$$\left.\begin{aligned} 2^{x_0} &\equiv \pm y^{\alpha_0}, \quad 2^{x_0'}.y^{\alpha_0} \equiv \pm 1 \\ 10^{x_0} &\equiv \pm y^{\alpha_0}, \quad 10^{x_0'}.y^{\alpha_0} \equiv \pm 1 \end{aligned}\right\} \begin{aligned} &(\mathrm{mod}\ p),\ \text{up to } p \not> 10{,}000. \\ &\qquad [y = 3, 5, 7, 11.] \end{aligned}$$

4. *Fundamental Congruence Solutions* ... price 10*s*., 1923,

giving one root (y) of every Congruence $y^{\xi} \equiv +1$ (mod p and p^{κ}), up to p and $p^{\kappa} \not> 10{,}000$. [Joint Author, T. G. Creak.]

5. *Binomial Factorisations*, [7 Volumes] ... ... 1923, &c.,

giving extensive Tables of solutions of $\phi(y^n \mp 1) \equiv 0$ (mod p and p^{κ}), up to p and $p^{\kappa} \not> 100{,}000$, with $n = 2$ to 30, and extensive Tables of Factorisation of $(x^n \mp y^n)$, and allied Forms, up to $n = 30$.

Vol. I, price 15*s*.; Vol. IV, price 5*s*. 1923,
contain the above Tables for $n = 2, 4, 8, 16$; 3, 6, 12, 24, &c.

Vol. II, price 15*s*.; Vol. VI, price 5*s*. 1924,
contain similar Tables for $n = 5, 10, 15, 20, 25, 30$, &c.

Vol. III, price 15*s*. 1924,
contains similar Tables for $n = 7, 14, 21$, &c.; 9, 18, 27, &c.

Vol. V, price 15*s*. 1925,
contains similar Tables for $n = 11, 22, 33$, &c.; 13, 26, 39, &c.

Vol. VII, price 5*s*. (Supplemy. to Vols. III and V) ... ... 1925.

6. *Factorisation of* $(y^n \mp 1)$, [$y = 2$ *to* 12] price 10*s*., 1925.
[Joint Author, H. J. Woodall.]

7. *Quadratic and Linear Tables* ... ... price 10*s*., 1927
contains numerous Tables useful in factorisation.

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